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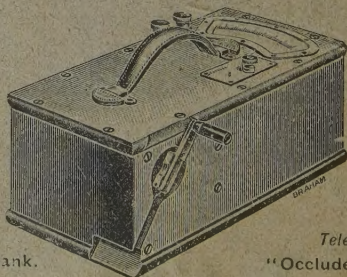
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EDITED BY
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WITH 181 ILLUSTRATIONS

THIRD EDITION THOROUGHLY REVISED

WHITTAKER & CO.
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1911

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PREFACE TO THE THIRD EDITION

The following extract from the Preface to the First Edition will indicate the scope of the present volume.

“IN view of the large number of ‘pocket-books’ which have already appeared, it may be well to point out in what respects the present volume differs from previous publications.

“In the first place, it does not merely give a number of formulæ, rules and data collected from various sources, and often of a contradictory nature, but it will be found to contain a kind of synopsis on each subject brought thoroughly up to date.

“Owing to the great specialization which the rapid growth of electrical engineering has necessitated it is impossible for any one individual to keep pace with all the new methods and systems coming into use, in other branches than his own. It is often, however, of the utmost importance to know where to turn for reliable and up-to-date information.

“It is believed that the present volume will fulfil this purpose, for while the various sections will be found quite sufficiently detailed for all ordinary requirements, at the same time copious references are given to various recent papers and publications, where a full treatment of the subject can be found, in case it should be wished to pursue the matter further.”

The success attending the publication of the first two editions has led to precisely the same lines being followed as regards general arrangement, but full advantage has been taken of the opportunity afforded by the issue of a new edition to carry into effect the various improvements suggested by users of the Pocket-book.

At the same time a very large part of the whole has been entirely re-written, so as to bring it thoroughly up to date, and every line has moreover been most carefully scrutinized, with a view to eliminating any of those misprints or inaccuracies, the presence of which is so detrimental to the usefulness of a volume such as this.

The present opportunity is taken of thanking those engineers who have so kindly made suggestions for the improvement of the book, particular thanks being due to Mr. Thomas Carter in this respect, and to Mr. Maurice Gheury for the preparation of the very complete index to the present volume.

KENELM EDGCUMBE.

1911.

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ELECTRICAL ENGINEER'S POCKET-BOOK

USEFUL NUMBERS, ETC.

Feet $\times$ 0·00019 ...	...	... = miles.
Yards $\times$ 0·00057 ...	...	... = miles.
Square inches $\times$ 0·007 ...	...	... = square feet.
Cubic feet $\times$ 0·037 ...	...	... = cubic yards.
Cubic inches $\times$ 0·00058 ...	...	... = cubic feet.
Circular inches $\times$ 0·0055 ...	...	... = square feet.
Lb. avoirdupois $\times$ 0·009 ...	...	... = cwt.
Lb. avoirdupois $\times$ 0·00045 ...	...	... = tons.
Millimetres $\times$ 0·03937 ...	...	... = inches.
Metres $\times$ 39·37 ...	...	... = inches.
Metres $\times$ 3·281 ...	...	... = feet.
Metres $\times$ 1·094 ...	...	... = yards.
Kilometres $\times$ 0·6214 ...	...	... = miles.
Grammes $\times$ 15·44 ...	...	... = grains.
Kilogrammes $\times$ 2·205 ...	...	... = lb.
Litres $\times$ 61 ...	...	... = cubic inches.
Foot-pounds $\times$ 0·13825 ...	...	... = kilogrammetres.
Kilogrammetres $\times$ 7·233 ...	...	... = foot-lb.
Horse-power $\times$ 0·9863 ...	...	... = force de cheval.
Force de cheval $\times$ 1·01385 ...	...	... = horse-power.
Horse-power $\times$ 0·166 ...	...	... = man-power.
Grains $\times$ 0·0648 ...	...	... = grammes.
Pounds $\times$ 0·4535 ...	...	... = kilogrammes.
Inches $\times$ 25·399 ...	...	... = millimetres.
Pounds per square foot $\times$ 4·882 ...	...	... = kilogrammes per square metre.
Kilogrammes per sq. metre $\times$ 0·2048 ...	...	... = pounds per square foot.
Tons per mile run $\times$ ·786 ...	...	... = pounds per yard.
Kilogrammes per metre run $\times$ 1·488 ...	...	... = pounds per yard.
Pounds per square inch $\times$ ·0703 ...	...	... = kilogrammes per square centimetre.
Kilos. per sq. centimetre $\times$ 14·223 ...	...	... = pounds per sq. inch.
Heat-units (B.Th.U.) $\times$ ·25202 ...	...	... = kg. calories.
Feet per second $\times$ ·3048 ...	...	... = metres per second.
Miles per hour $\times$ ·447 ...	...	... = metres per second.

- 1 British Thermal Unit (B.Th.U.) = 778 foot pounds (latest determination).
 1 ampere hour = 3600 coulombs.
 1 kilowatt = 10 Hecto watts = 1000 watts = 101·98 mkg. per second = 1·346 Eng. H.P. = 1·36 met. H.P.
 1 Eng. H.P. = 550 foot pounds per sec. = 746 watts = 76 mkg. per sec. = 1·013 met H.P.
 1 watt second = ·24014 gcal. = ·10198 mkg.
 1 kilowatt hour (B.T.U.) = 1000 watt hours = 864·5 kgcals. = 367·114 mkg.
 1 kgal. = 1·157 watt hours = 424·7 mkg.

Logarithms of Useful Numbers.*

$e = 2·7183$	$\log. = 0·4343$	$\sqrt{\pi} = 1·7725$	$\log. = 0·2486$
$\pi = 3·1416$	$,, = 0·4971$	$\frac{1}{\sqrt{\pi}} = 0·5642$	$,, = 9·7514-10 \uparrow$
$\frac{\pi}{4} = 0·7854$	$,, = 9·8951-10 \uparrow$	$\sqrt[3]{\pi} = 1·4646$	$,, = 0·1657$
$\frac{4}{3}\pi = 4·1888$	$,, = 0·6221$	$\frac{1}{\sqrt[3]{\pi}} = 0·6828$	$,, = 9·8343-10 \uparrow$
$\frac{\pi}{6} = 0·5236$	$,, = 9·7190-10 \uparrow$	$\sqrt[3]{\frac{6}{\pi}} = 1·2407$	$,, = 0·0937$
$\frac{1}{6}\pi = 0·5236$	$,, = 9·7190-10 \uparrow$		
$\frac{1}{\pi} = 0·3183$	$,, = 9·5028-10 \uparrow$		
$\pi^2 = 9·8696$	$,, = 0·9943$		
$\frac{1}{\pi^2} = 0·1013$	$,, = 9·0056-10 \uparrow$		
		1 H.P. (met.) = 736 V.A. ;	$\log. = 2·8669$
		1 H.P. (Eng.) = 746	$,, , , = 2·8727$

A radian (see p. 55) = $57·2958^\circ$ ($\log. = 1·7581$)

Hyperbolic logarithm = common logarithm $\times 2·30258509$

Common logarithm = hyperbolic logarithm $\times 0·43429448$

Approximations—

4 in.	=	10 centimetres	within 2 per cent.
2 ft.	=	61	$,, , , \frac{1}{16} , ,$
5 miles	=	8 kilometres	$,, , , \frac{1}{7} , ,$
11 sq. in.	=	71 sq. centimetres	within $\frac{1}{10}$ per cent.
1 $,,$ ft	=	930 $,, , ,$	$,, , , \frac{1}{9} , ,$
6 $,,$ yds.	=	5 $,,$ metres	$,, , , \frac{1}{2} , ,$
2 acres	=	81 ares	$,, , , \frac{1}{12} , ,$
1 acre	=	69½ yards square	$,, , , \frac{1}{4} , ,$
11 cub. in.	=	180 cub. centimetres	within $\frac{1}{10}$ per cent.
600 $,,$ ft.	=	17 $,,$ metres	$,, , , \frac{1}{10} , ,$
4 $,,$ yds.	=	3 $,, , ,$	$,, , , 2 , ,$
44 pints	=	25 litres	$,, , , \frac{1}{10} , ,$
1 pint	=	35 cub. inches	$,, , , 1 , ,$
1 gallon	=	277 $,, , ,$	$,, , , \frac{1}{5} , ,$
7 bushels	=	9 $,,$ ft.	$,, , , \frac{1}{10} , ,$

* See p. 13.

† See p. 22.

Thermometers (see pp. 35-37) $C = \frac{5}{9}(F - 32) = \frac{5}{9}R$;
 $F = \frac{9}{5}C + 32 = \frac{9}{5}R + 32$;
 $R = \frac{4}{9}(F - 32) = \frac{4}{9}C$.

Circumference of circle $= \pi d$.

Area of circle $= \frac{1}{4}\pi d^2 = .7854 d^2$.

Water. 1 gallon (Imp.) = 10 lbs. = 4.547 litres.

1 cubic foot = 62.35 lbs. = 28.378 litres.

Air. 1 atmosphere (nominal)* = 14.7 lbs. per square inch.
 = 1.0335 kgms. per square cm.
 = 760 mm. of mercury at 0° C.
 = 29.922 inch "
 = 34.947 feet water at 62° F.
 1 pound at 62° F. = 13.141 cubic feet at 760 mm.
 mercury.

Wrought Iron. 1 cubic foot = 466 to 487 lbs.

Cast Iron. 1 cubic foot = 380 to 465 lbs.

Steel. 1 cubic foot = 435 to 490 lbs.

Brass. 1 cubic foot = 505 lbs.

Copper. 1 cubic foot = 550 lbs.

For other Metals, see p. 34.

General Measures, etc.

A gross = 12 dozen.

A quire (of paper) = 24 sheets = $\frac{1}{2}$ ream.

A ton of coal = 12 sacks.

A chaldron = 12 sacks.

A ton of freight = 40 cubic feet.†

A faggot of steel = 120 lbs.

A barrel of tar = 25 gallons.

A new penny weighs $\frac{1}{3}$ ounce.

A new penny has a diameter of $\frac{1}{10}$ foot.

A new halfpenny has a diameter of 1 inch.

* This unit is used in England, United States, and all European countries except France, Germany and Austria, where 1 kg. per square cm. is taken as 1 atmosphere.

† The cubic feet are in such cases obtained by multiplying together the length, breadth and height, each being measured at the widest part.

MATHEMATICS AND PHYSICS.

WEIGHTS AND MEASURES.

Long Measure.

Ins.	Feet.	Yards.	Fath.	Poles.	Furl.	Mile.	Metres.
1 = '0833	= '0278	= '0139	= '00505	= '000126	= '0000158	= '0253998	
12 = 1 = '333	= '167	= '0606	= '00152	= '000189	= '3047973		
36 = 3 = 1 = '5	= '182	= '00455	= '000568	= '9143918			
72 = 6 = 2 = 1 = '364	= '00909	= '001136	= '1'828784				
198 = 16½ = 5½ = 2¾ = 1 = '025	= '003125	= 5'029155					
7,920 = 660 = 220 = 110 = 40 = 1 = '125	= 201'1662						
63,360 = 5,280 = 1,760 = 880 = 320 = 8 = 1 = 1609'330							

1 mil = '001 inch.
1 perch = 1 rod = 1 pole.

Decimal Equivalents of Fractions.

Fractions.	Decimals.	Fractions.	Decimals.	Fractions.	Decimals.	Fractions.	Decimals.
$\frac{1}{64}$	0'015625	$\frac{17}{64}$	0'265625	$\frac{33}{64}$	0'515625	$\frac{49}{64}$	0'765625
$\frac{3}{32}$	0'03125	$\frac{5}{8}$	0'28125	$\frac{17}{32}$	0'53125	$\frac{25}{32}$	0'78125
$\frac{3}{64}$	0'046875	$\frac{9}{64}$	0'296875	$\frac{35}{64}$	0'546875	$\frac{51}{64}$	0'796875
$\frac{1}{8}$	0'0625	$\frac{15}{64}$	0'3125	$\frac{9}{16}$	0'5625	$\frac{13}{16}$	0'8125
$\frac{5}{64}$	0'078125	$\frac{21}{64}$	0'328125	$\frac{17}{64}$	0'578125	$\frac{53}{64}$	0'828125
$\frac{3}{32}$	0'09375	$\frac{11}{32}$	0'34375	$\frac{19}{32}$	0'59375	$\frac{71}{32}$	0'84375
$\frac{7}{64}$	0'109375	$\frac{23}{64}$	0'359375	$\frac{39}{64}$	0'609375	$\frac{55}{64}$	0'859375
$\frac{1}{8}$	0'125	$\frac{3}{8}$	0'375	$\frac{5}{8}$	0'625	$\frac{7}{8}$	0'875
$\frac{9}{64}$	0'140625	$\frac{25}{64}$	0'390625	$\frac{41}{64}$	0'640625	$\frac{57}{64}$	0'890625
$\frac{5}{32}$	0'15625	$\frac{13}{32}$	0'40625	$\frac{21}{32}$	0'65625	$\frac{29}{32}$	0'90625
$\frac{11}{64}$	0'171875	$\frac{27}{64}$	0'421875	$\frac{43}{64}$	0'671875	$\frac{59}{64}$	0'921875
$\frac{3}{8}$	0'1875	$\frac{7}{8}$	0'4375	$\frac{15}{16}$	0'6875	$\frac{15}{16}$	0'9375
$\frac{13}{64}$	0'203125	$\frac{29}{64}$	0'453125	$\frac{45}{64}$	0'703125	$\frac{61}{64}$	0'953125
$\frac{7}{32}$	0'21875	$\frac{15}{16}$	0'46875	$\frac{23}{32}$	0'71875	$\frac{31}{32}$	0'96875
$\frac{15}{64}$	0'234375	$\frac{31}{64}$	0'484375	$\frac{47}{64}$	0'734375	$\frac{63}{64}$	0'984375
$\frac{1}{4}$	0'25	$\frac{1}{2}$	0'5	$\frac{5}{4}$	0'75	1	1'0

Cubic Measure.

Cub. Ins.	Cub. Ft.	Cub. Yds.	Cub. Metres.
1,728.....	= 1.....	= '03704.....	= '02832
46,656.....	= 27.....	= 1.....	= '76451

2 pints = 1 quart... = 8 gills.

4 quarts ... = 1 gallon.

2 gallons ... = 1 peck.

4 pecks = 1 bushel.

8 bushels ... = 1 quarter.

5 quarters... = 1 wagon-load.

2 loads = 1 last.

1 cubic foot = '7785 bushel.

1 pint = 20 fluid ounces (= 20 ounces or water).

1 fluid ounce = 8 fluid drachms = 480 minims = 720 drops.

Avoirdupois Weight.

16 drams = 1 ounce (oz.).

16 ounces = 1 pound (lb.).

14 pounds = 1 stone.

2 stones = 1 quarter.

4 quarters..... = 1 hundredweight (cwt.).

20 hundredweight... = 1 ton.

NOTE.

1 grain troy = 1 grain avdps. = 1 grain apoth.

1 lb. ,, = '82286 lb. avoirdupois.

1 oz. ,, = 1'0974 oz. ,,

1 dram (avdps.) = 27'34375 grains.

1 drachm (apoth.) = 60 grains.

1 oz. apoth. = 1 oz. troy.

1 lb. ,, = 1 lb. ,,

METRIC MEASURES.**Weight.**

	Grammes.		
Milligramme ...	= '001	=	'01543 English grs. troy.
Centigramme ...	= '01	=	'15432 ,, ,, ,,
Decigramme ...	= '1	=	1'54323 ,, ,, ,,
Gramme	= 1	=	15'43235 ,, ,, ,,
Decagramme ...	= 10	=	'35274 ,, oz. avdps.
Hectogramme ...	= 100	=	3'52739 ,, ,, ,,
Kilogramme ...	= 1,000	=	2'20462 ,, lbs. ,,
Myriagramme ...	= 10,000	=	'78736 ,, grs. ,,
Quintal.....	= 100,000	=	1'96841 ,, cwts. ,,
Millier or Tonne	= 1,000,000	=	'98421 ,, tons ,,

Long Measure.

Millimetre	=	·001 metres	=	·0393704 English inches.
Centimetre	=	·01 ,,	=	·39370 ,, ,,
Decimetre	=	·1 ,,	=	3·93704 ,, ,,
Metre	=	1 ,,	=	3·28087 ,, feet.
Decametre	=	10 ,,	=	10·9362 ,, yards.
Hectometre	=	100 ,,	=	109·362 ,, ,,
Kilometre	=	1,000 ,,	=	·62138 ,, mile.

Square Measure.

Sq. Millimetre	=	·000001 sq. metres	=	·00155 sq. inches.
,, Centimetre	=	·0001 ,, ,,	=	·15500 ,, ,,
,, Decimetre	=	·01 ,, ,,	=	15·5003 ,, ,,
Centiare ...	=	1 ,, ,,	=	10·7641 ,, feet.
Deciare ...	=	10 ,, ,,	=	11·9601 ,, yards.
Are.....	=	100 ,, ,,	=	3·95376 poles.
Hectare ...	=	10,000 ,, ,,	=	2·47110 acres.

Solid Measure.

Cub. Millimetre	=	·000000001 cub. mtrs.	=	·000061 cub. ins.
,, Centimetre	=	·000001 ,, ,,	=	·061025 ,, ,,
,, Decimetre	=	·001 ,, ,,	=	61·0254 ,, ,,
Decistere	=	·1 ,, ,,	=	3·53156 ,, feet.
Stere	=	1 ,, ,,	=	1·30799 ,, yds.
Decastere	=	10 ,, ,,	=	13·07986 ,, ,,
Hectastere	=	100 ,, ,,	=	130·7936 ,, ,,
Kilostere	=	1,000 ,, ,,	=	1307·9858 ,, ,,

Capacity.

Millilitre	=	·001 litre	=	·00176 English pints.
Centilitre	=	·01 ,,	=	·01760 ,, ,,
Decilitre	=	·1 ,,	=	·17598 ,, ,,
Litre	=	1 ,,	=	1·7598 ,, ,,
Decalitre	=	10 ,,	=	2·1998 ,, gallons.
Hectolitre	=	100 ,,	=	2·7498 ,, bushels.
Kilolitre	=	1,000 ,,	=	3·4372 ,, quarters.

Inches to Centimetres.

In.	0	1	2	3	4	5	6	7	8	9
	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.
0	0	2.54	5.08	7.62	10.16	12.7	15.24	17.78	20.32	22.86
10	25.4	27.94	30.48	33.02	35.56	38.1	40.64	43.18	45.72	48.26
20	50.8	53.34	55.88	58.42	60.96	63.5	66.04	68.58	71.12	73.66
30	76.2	78.74	81.28	83.82	86.36	88.9	91.44	93.98	96.52	99.06
40	101.6	104.14	106.68	109.22	111.76	114.3	116.84	119.38	121.92	124.46
50	127.0	129.54	132.08	134.62	137.16	139.7	142.24	144.78	147.32	149.86
60	152.4	154.94	157.48	160.02	162.56	165.1	167.64	170.18	172.72	175.26
70	177.8	180.34	182.88	185.42	187.96	190.5	193.04	195.58	198.12	200.66
80	203.2	205.74	208.28	210.82	213.36	215.9	218.44	220.98	223.52	226.06
90	228.6	231.14	233.68	236.22	238.76	241.3	243.84	246.38	248.92	251.46
100	254.0	256.54	259.08	261.62	264.16	266.7	269.24	271.78	274.32	276.85

4 inches = rather more than 10 centimetres, see p. 2.

Centimetres to Inches.

Cm.	0	1	2	3	4	5	6	7	8	9
	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
0	.000	.394	.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.742	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.662	9.055	9.449	9.843	10.236	10.630	11.024	11.418
30	11.811	12.205	12.599	12.992	13.386	13.780	14.173	14.567	14.961	15.355
40	15.748	16.142	16.536	16.929	17.323	17.717	18.111	18.504	18.898	19.292
50	19.685	20.079	20.473	20.867	21.260	21.654	22.048	22.441	22.835	23.229
60	23.622	24.016	24.410	24.804	25.197	25.591	25.985	26.378	26.772	27.166
70	27.560	27.953	28.347	28.741	29.134	29.528	29.922	30.316	30.709	31.103
80	31.497	31.890	32.284	32.678	33.071	33.465	33.859	34.253	34.646	35.040
90	35.434	35.827	36.221	36.615	37.009	37.402	37.796	38.190	38.583	38.977
100	39.370	39.764	40.158	40.552	40.945	41.339	41.733	42.126	42.520	42.914

2.5 centimetres or 25 millimetres = nearly 1 inch, see p. 2.

AREAS AND CIRCUMFERENCES OF CIRCLES $\frac{1}{32}$ IN. TO 6 IN. IN
DIAMETER.

Dia.	Circum.	Area.	Dia.	Circum.	Area.	Dia.	Circum.	Area.
$\frac{1}{32}$	·0981	·00077	$\frac{13}{32}$	4·3197	1·4848	$3\frac{1}{32}$	11·584	10·679
$\frac{1}{16}$	·1963	·00307	$1\frac{7}{16}$	4·516	1·6229	$3\frac{3}{4}$	11·781	11·044
$\frac{3}{32}$	·2945	·0069	$1\frac{1}{2}$	4·7124	1·7671	$3\frac{1}{2}$	11·977	11·416
$\frac{1}{8}$	·3927	·01227	$1\frac{9}{16}$	4·9087	1·9175	$3\frac{7}{8}$	12·173	11·793
$\frac{5}{32}$	·4908	·0192	$1\frac{5}{8}$	5·1051	2·0739	$3\frac{1}{2}$	12·369	12·177
$\frac{3}{16}$	·589	·02761	$1\frac{1}{4}$	5·3014	2·2365	4	12·566	12·566
$\frac{7}{32}$	·6872	·0376	$1\frac{3}{4}$	5·4978	2·4052	$4\frac{1}{16}$	12·762	12·962
$\frac{1}{4}$	·7854	·04909	$1\frac{1}{2}$	5·6941	2·58	$4\frac{1}{8}$	12·959	13·364
$\frac{9}{32}$	·8835	·0621	$1\frac{7}{8}$	5·8905	2·7611	$4\frac{3}{16}$	13·155	13·772
$\frac{5}{16}$	·9817	·0767	$1\frac{1}{2}$	6·0868	2·9483	$4\frac{1}{4}$	13·351	14·186
$1\frac{1}{32}$	1·0799	·0928	2	6·2832	3·1416	$4\frac{5}{16}$	13·547	14·606
$\frac{3}{8}$	1·1781	·1104	$2\frac{1}{16}$	6·4795	3·3410	$4\frac{3}{8}$	13·744	15·033
$1\frac{1}{32}$	1·2762	·1296	$2\frac{1}{8}$	6·6759	3·5465	$4\frac{7}{16}$	13·94	15·465
$\frac{7}{16}$	1·3744	·1503	$2\frac{3}{16}$	6·8722	3·7584	$4\frac{1}{2}$	14·137	15·904
$1\frac{5}{32}$	1·4726	·1725	$2\frac{1}{4}$	7·0686	3·976	$4\frac{9}{16}$	14·333	16·349
$\frac{1}{2}$	1·5708	·1963	$2\frac{5}{16}$	7·2649	4·2	$4\frac{5}{8}$	14·529	16·8
$1\frac{1}{16}$	1·6689	·2216	$2\frac{3}{8}$	7·4613	4·4302	$4\frac{1}{2}$	14·725	17·257
$\frac{9}{16}$	1·7771	·2485	$2\frac{7}{16}$	7·6576	4·6604	$4\frac{3}{4}$	14·922	17·72
$1\frac{1}{2}$	1·8653	·2768	$2\frac{1}{2}$	7·854	4·9087	$4\frac{1}{2}$	15·119	18·19
$\frac{5}{8}$	1·9635	·3068	$2\frac{9}{16}$	8·0503	5·1573	$4\frac{7}{8}$	15·315	18·665
$2\frac{1}{32}$	2·0616	·3382	$2\frac{5}{8}$	8·2467	5·4119	$4\frac{1}{2}$	15·511	19·147
$\frac{11}{16}$	2·1598	·3712	$2\frac{1}{2}$	8·443	5·6723	5	15·708	19·635
$2\frac{3}{32}$	2·258	·4057	$2\frac{3}{4}$	8·6394	5·9395	$5\frac{1}{16}$	15·904	20·129
$\frac{3}{4}$	2·3562	·4417	$2\frac{1}{2}$	8·8357	6·2126	$5\frac{1}{8}$	16·1	20·629
$2\frac{5}{32}$	2·4543	·4793	$2\frac{7}{8}$	9·0321	6·4918	$5\frac{3}{16}$	16·296	21·135
$1\frac{1}{2}$	2·5525	·5185	$2\frac{1}{2}$	9·2284	6·7772	$5\frac{1}{4}$	16·493	21·647
$\frac{7}{16}$	2·6507	·5591	3	9·4248	7·0686	$5\frac{5}{16}$	16·689	22·166
$2\frac{7}{32}$	2·7489	·6013	$3\frac{1}{16}$	9·6211	7·3662	$5\frac{3}{8}$	16·886	22·69
$\frac{9}{16}$	2·847	·645	$3\frac{1}{8}$	9·8175	7·6699	$5\frac{7}{16}$	17·082	23·221
$2\frac{9}{32}$	2·9452	·6903	$3\frac{3}{16}$	10·014	7·9798	$5\frac{1}{2}$	17·278	23·758
$\frac{11}{16}$	3·0434	·737	$3\frac{1}{4}$	10·21	8·2957	$5\frac{9}{16}$	17·474	24·301
1	3·1416	·7854	$3\frac{5}{16}$	10·406	8·618	$5\frac{1}{2}$	17·671	24·85
$1\frac{1}{16}$	3·3379	·8866	$3\frac{3}{8}$	10·602	8·9462	$5\frac{1}{4}$	17·867	25·406
$\frac{1}{8}$	3·5343	·994	$3\frac{7}{16}$	10·799	9·2807	$5\frac{3}{8}$	18·064	25·967
$1\frac{3}{16}$	3·7306	1·1075	$3\frac{1}{2}$	10·995	9·6211	$5\frac{1}{2}$	18·261	26·535
$\frac{1}{4}$	3·927	1·2271	$3\frac{9}{16}$	11·191	9·968	$5\frac{5}{8}$	18·457	27·108
$1\frac{5}{16}$	4·1233	1·353	$3\frac{5}{8}$	11·388	10·32	$5\frac{1}{4}$	18·653	27·688

TABLE OF SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS,
RECIPROCAL, NATURAL (HYPERBOLIC) LOGARITHMS,
CIRCUMFERENCES AND AREAS OF CIRCLES.

n	n ²	n ³	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	log nat } n	$\pi \frac{n}{10}$	$\frac{1}{4} \pi \left(\frac{n}{10}\right)^2$
0	0	0	0.0000	0.0000	∞	— ∞	0.000	0.0000
1	1	1	1.0000	1.0000	1.00000	0.0000	0.314	0.0079
2	4	8	1.4142	1.2599	0.50000	0.6931	0.628	0.0314
3	9	27	1.7321	1.4422	0.33333	1.0986	0.942	0.0707
4	16	64	2.0000	1.5874	0.25000	1.3863	1.257	0.1257
5	25	125	2.2361	1.7100	0.20000	1.6094	1.571	0.1964
6	36	216	2.4495	1.8171	0.16667	1.7918	1.885	0.2827
7	49	343	2.6458	1.9129	0.14286	1.9459	2.199	0.3848
8	64	512	2.8284	2.0000	0.12500	2.0794	2.513	0.5026
9	81	729	3.0000	2.0801	0.11111	2.1972	2.827	0.6362
10	100	1000	3.1623	2.1544	0.10000	2.3026	3.142	0.7854
11	121	1331	3.3166	2.2240	0.09091	2.3979	3.456	0.9503
12	144	1728	3.4641	2.2894	0.08333	2.4840	3.770	1.1310
13	169	2197	3.6056	2.3518	0.07692	2.5649	4.084	1.3273
14	196	2744	3.7417	2.4101	0.07143	2.6391	4.398	1.5394
15	225	3375	3.8730	2.4662	0.06667	2.7081	4.712	1.7671
16	256	4096	4.0000	2.5198	0.06250	2.7723	5.027	2.0106
17	289	4913	4.1231	2.5718	0.05882	2.8332	5.341	2.2628
18	324	5832	4.2426	2.6207	0.05556	2.8904	5.655	2.5447
19	361	6859	4.3589	2.6684	0.05263	2.9444	5.969	2.8353
20	400	8000	4.4721	2.7144	0.05000	2.9957	6.283	3.1416
21	441	9261	4.5836	2.7589	0.04762	3.0445	6.597	3.4636
22	484	10648	4.6904	2.8020	0.04545	3.0910	6.912	3.8018
23	529	12167	4.7958	2.8439	0.04348	3.1355	7.226	4.1548
24	576	13824	4.8990	2.8845	0.04167	3.1781	7.540	4.5239
25	625	15625	5.0000	2.9240	0.04000	3.2189	7.854	4.9087
26	676	17576	5.0990	2.9625	0.03846	3.2581	8.168	5.3093
27	729	19683	5.1963	3.0000	0.03704	3.2958	8.482	5.7256
28	784	21952	5.2915	3.0366	0.03571	3.3322	8.796	6.1575
29	841	24389	5.3852	3.0723	0.03448	3.3673	9.111	6.6052
30	900	27000	5.4772	3.1072	0.03333	3.4012	9.425	7.0656
31	961	29791	5.5678	3.1414	0.03226	3.4340	9.739	7.5477
32	1024	32768	5.6569	3.1748	0.03125	3.4657	10.05	8.0425
33	1089	35937	5.7446	3.2075	0.03030	3.4965	10.37	8.5530
34	1156	39304	5.8310	3.2396	0.02941	3.5264	10.68	9.0792
35	1225	42875	5.9161	3.2711	0.02857	3.5553	11.00	9.6211
36	1296	46656	6.0000	3.3019	0.02778	3.5835	11.31	10.1790
37	1369	50653	6.0828	3.3322	0.02703	3.6109	11.62	10.752
38	1444	54872	6.1644	3.3620	0.02632	3.6376	11.94	11.341
39	1521	59319	6.2450	3.3912	0.02564	3.6636	12.25	11.946
40	1600	64000	6.3246	3.4200	0.02500	3.6889	12.57	12.566
41	1681	68921	6.4031	3.4482	0.02439	3.7136	12.88	13.203
42	1764	74088	6.4807	3.4760	0.02381	3.7377	13.19	13.854
43	1849	79507	6.5574	3.5034	0.02326	3.7612	13.51	14.522
44	1936	85184	6.6332	3.5303	0.02273	3.7842	13.82	15.205
45	2025	91125	6.7082	3.5569	0.02222	3.8067	14.14	15.904
46	2116	97336	6.7823	3.5830	0.02174	3.8286	14.45	16.619
47	2209	103823	6.8557	3.6088	0.02128	3.8501	14.77	17.349
48	2304	110592	6.9282	3.6342	0.02083	3.8712	15.08	18.096
49	2401	117649	7.0000	3.6593	0.02041	3.8918	15.39	18.857
50	2500	125000	7.0711	3.6840	0.02000	3.9120	15.73	19.635

TABLE OF SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, RECIPROCALs, NATURAL (HYPERBOLIC) LOGARITHMS, CIRCUMFERENCES AND AREAS OF CIRCLES, *continued*.

n	n ²	n ³	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	log nat } n	$\pi \frac{n}{10}$	$\frac{1}{4} \pi \left(\frac{n}{10}\right)^2$
51	2601	132651	7.1414	3.7084	0.01961	3.9318	16.02	20.428
52	2704	140608	7.2111	3.7325	0.01923	3.9512	16.34	21.237
53	2809	148877	7.2801	3.7563	0.01887	3.9703	16.65	22.062
54	2916	157464	7.3485	3.7798	0.01852	3.9890	16.96	22.902
55	3025	166375	7.4162	3.8030	0.01818	4.0073	17.28	23.758
56	3136	175616	7.4833	3.8259	0.01786	4.0254	17.59	24.630
57	3249	185193	7.5498	3.8485	0.01754	4.0431	17.91	25.518
58	3364	195112	7.6158	3.8709	0.01724	4.0604	18.22	26.421
59	3481	205379	7.6811	3.8930	0.01695	4.0775	18.54	27.340
60	3600	216000	7.7460	3.9149	0.01667	4.0943	18.85	28.274
61	3721	226981	7.8102	3.9365	0.01639	4.1109	19.16	29.225
62	3844	238328	7.8740	3.9579	0.01613	4.1271	19.48	30.191
63	3969	250047	7.9373	3.9791	0.01587	4.1431	19.79	31.172
64	4096	262144	8.0000	4.0000	0.01563	4.1589	20.11	32.170
65	4225	274625	8.0623	4.0207	0.01538	4.1744	20.42	33.183
66	4356	287496	8.1240	4.0412	0.01515	4.1897	20.73	34.212
67	4489	300763	8.1854	4.0615	0.01493	4.2047	21.05	35.257
68	4624	314432	8.2462	4.0817	0.01471	4.2195	21.36	36.317
69	4761	328509	8.3066	4.1016	0.01449	4.2341	21.68	37.393
70	4901	343000	8.3666	4.1213	0.01429	4.2485	21.99	38.485
71	5041	357911	8.4261	4.1408	0.01408	4.2627	22.31	39.592
72	5184	373248	8.4853	4.1602	0.01389	4.2767	22.62	40.715
73	5329	389017	8.5440	4.1793	0.01370	4.2905	22.93	41.854
74	5476	405224	8.6023	4.1983	0.01351	4.3041	23.25	43.008
75	5625	421875	8.6603	4.2172	0.01333	4.3175	23.56	44.179
76	5776	438976	8.7178	4.2358	0.01316	4.3307	23.88	45.365
77	5929	456533	8.7750	4.2543	0.01299	4.3438	24.19	46.566
78	6084	474552	8.8318	4.2727	0.01282	4.3567	24.50	47.784
79	6241	493039	8.8882	4.2908	0.01266	4.3694	24.82	49.017
80	6400	512000	8.9443	4.3089	0.01250	4.3820	25.13	50.265
81	6561	531441	9.0000	4.3267	0.01235	4.3944	25.45	51.530
82	6724	551368	9.0554	4.3445	0.01220	4.4067	25.76	52.810
83	6889	571787	9.1104	4.3621	0.01205	4.4188	26.08	54.106
84	7056	592704	9.1652	4.3795	0.01190	4.4308	26.39	55.418
85	7225	614125	9.2195	4.3968	0.01176	4.4427	26.70	56.745
86	7396	636056	9.2736	4.4140	0.01163	4.4543	27.02	58.088
87	7569	658503	9.3274	4.4310	0.01149	4.4659	27.33	59.447
88	7744	681472	9.3808	4.4480	0.01136	4.4773	27.65	60.821
89	7921	704969	9.4340	4.4647	0.01124	4.4886	27.96	62.211
90	8100	729000	9.4868	4.4814	0.01111	4.4998	28.27	63.617
91	8281	753571	9.5394	4.4979	0.01099	4.5109	28.59	65.039
92	8464	778688	9.5917	4.5144	0.01087	4.5218	28.90	66.476
93	8649	804357	9.6437	4.5307	0.01075	4.5326	29.22	67.929
94	8836	830584	9.6954	4.5468	0.01064	4.5433	29.53	69.398
95	9025	857375	9.7468	4.5629	0.01053	4.5539	29.85	70.882
96	9216	884736	9.7980	4.5789	0.01042	4.5643	30.16	72.382
97	9409	912673	9.8489	4.5947	0.01031	4.5747	30.47	73.898
98	9604	941192	9.8995	4.6104	0.01020	4.5850	30.79	75.430
99	9801	970299	9.9499	4.6261	0.01010	4.5951	31.10	76.977
100	10000	1000000	10.0000	4.6416	0.01000	4.6052	31.42	78.540

Algebraic Symbols.

In addition to the more common algebraic signs the following may also be met with—

$\sim$ Approximately equal to (also often denotes "periods").	$\angle$ an angle.
$\neq$ not equal to.	L a right angle.
$>$ greater than.	$\triangle$ a triangle.
$\nless$ not greater than.	$\odot$ a circle.
$<$ less than.	$\bigcirc$ the circumference.
$\nless$ not less than.	$\propto$ proportional to.
$\parallel$ parallel to.	∞ infinity.
$\nparallel$ not parallel to.	$\square$ " square inch.
$\perp$ perpendicular to.	m^2 square metre.
	cm^3 cubic centimetre.

$a > b > c$ means that a is greater than b and that b is greater than c ; that is, that b lies between a and c in value.

In **continental** publications the following will be found:—6,75 for 6·75, also 0,75 or 0·75 is commonly given for ·75.

If the **error** of an instrument or the necessary correction is given as (*e. g.*) + ·684, this quantity is to be added to the observed value, or if - ·684 is given, it is to be subtracted.

$\theta = \sin^{-1} \frac{1}{2}$ means that θ is an angle whose sine is $\frac{1}{2}$. Such expressions are known as "**Inverse Functions.**" $L \sin$, etc. = logarithmic sine = log of sine + 10 (see p. 2). The expression $a \pm b = p \mp q$ means that $a + b = v - q$ and that $a - b = p + q$.

The letter Σ usually stands for "the sum of such quantities as" —*e. g.* $\Sigma (mr^2)$ means that corresponding values of m and r^2 are to be multiplied together and the sum of all the products taken.

[n or n! (called "Factorial n") denotes the product of the first n consecutive numbers — *e. g.* $3! = 1 \times 2 \times 3 = 6$.

The following letters are often used without explanation:—

π = Ratio of circumference to diameter = 3·141593, or $\frac{22}{7}$ within $\frac{1}{20}$ %, or since 2π radians = 4 right angles (see p. 55)

$\therefore \pi = 2$ right angles, or 180° .

e or e^* base of hyperbolic or Napierian logarithms = 2·718282.

E Modulus of elasticity.

dt (used in the differential calculus) in such an expression as dt denotes an infinitely small quantity (of time), and does not mean $d \times t$.

* e also often represents the "eccentricity" of a conic section.

g acceleration due to gravity = 32.1908 feet per second per second in London (see p. 55).

The following relations will be found useful—

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$n^2 + (a + b)n + ab = (n + a)(n + b)$$

$$n^2 + (a - b)n - ab = (n + a)(n - b)$$

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc; \frac{a}{c} = \frac{b}{d}; \frac{b}{a} = \frac{d}{c}; \frac{a+b}{b} = \frac{c+d}{d};$$

$$\frac{a-b}{b} = \frac{c-d}{d}; \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{1}{a^n} = \sqrt[n]{a}$$

$$\frac{a^n}{a^n} = \sqrt[n]{a^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1$$

$$\text{If } n^2 + an + b = 0, \text{ then } n = -\frac{a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$$

If the roots of the equation $x^2 + ax + b = 0$ are α and β (that is if $x = \alpha$ or $x = \beta$) then $\alpha + \beta = -a$ and $\alpha\beta = b$.

Logarithms.

If $a^m = x$, then m is called the logarithm of x to the base a (written $m = \log_a x$) and x is known as the antilogarithm of m .

The application of logarithms is based on the following properties—

$$\log m \cdot n = \log m + \log n; \log \frac{m}{n} = \log m - \log n;$$

$$\log x^m = m \log x; \log \sqrt[m]{x} = \frac{\log x}{m}.$$

$$\log_b N = \frac{\log_a N}{\log_a b} = \log_a N \times \log_a b$$

Logarithms are primarily calculated to base 2.71828... which is usually denoted by e or e ("Natural" or "Napierian" or "Hyperbolic" logarithms), but for convenience in use, tables are drawn up to base 10 ("Common" or "Briggs" logarithms).

To convert common to natural, multiply by $\log_e 10 = 2.30258509$, and to perform the converse process multiply by $.43429448$.

The following **examples** will sufficiently illustrate the method of using the logarithm tables.

(1) To find the logarithm of 5134

$$\log 5130 = 3.7101^*$$

$$\log 5140 = 3.7110$$

$$\text{diff. for 1} = .0009 \text{ (or from diff. col. 9 to 8)}$$

$$\therefore \text{diff. for } .4 = .0009 \times .4 = .00036$$

$$\therefore \log. 5134 = \left\{ \begin{array}{r} 3.7101 \\ .00036 \\ \hline 3.71046 \end{array} \right\} = 3.71046$$

(True value is 3.710456).

(2)

$$\sqrt[7]{.0543}$$

$$\log .0543 = 2.7348 = 7 + 5.7348$$

$$\text{and } \frac{7 + 5.7348}{7} = \bar{1}.815$$

$$\text{Antilog. } \bar{1}.815 = .6531.$$

* 3 is known as the "characteristic" of the logarithm and .7101 as the "mantissa." The former is always represented by one less than the number of digits before the decimal place in the original number, while the latter is found from the table. In the case of a fraction (example 2) the characteristic is negative (written $\bar{2}$) and is counted from the decimal place. The mantissa is always positive.

COMMON LOGARITHMS.

$\log$	0	1	2	3	4	5	6	7	8	9	Diff.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	43—40
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	39—36
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	36—34
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	34—31
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	31—29
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	29—27
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	27—26
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	26—25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	24—23
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	23—22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	22—20
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	21—19
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	20—19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	19—18
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18—17
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	18—17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	17—16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16—15
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	16—15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15—14
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	15—14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14—13
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	14—13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13—12
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	13—12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12—11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12—11
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	12—11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	12—11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11—10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	11—10
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	11—10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10—9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10—9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	10—9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	10—9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9—8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9—8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	9—8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	9—8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	9—8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8—7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8—7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8—7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	8—7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	8—7
$\log$	0	1	2	3	4	5	6	7	8	9	Diff.

COMMON LOGARITHMS (*continued*).

No.	0	1	2	3	4	5	6	7	8	9	Diff.
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	8—7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	8—7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7—6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7—6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7—6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7—6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7—6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	7—6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	7—6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	7—6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	7—6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	7—6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6—5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6—5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6—5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6—5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6—5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6—5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	6—5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	6—5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	6—5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	6—5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	6—5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	6—5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	6—5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5—4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5—4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5—4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5—4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5—4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5—4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5—4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5—4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	5—4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	5—4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	5—4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	5—4
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	43
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817	43
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242	43
103	01384	01326	01368	01410	01452	01494	01536	01578	01620	01662	42
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078	42
105	02119	02160	02202	02243	02284	02325	02366	02408	02449	02490	41
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898	41
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302	41—40
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703	41—40
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100	41—40
No.	0	1	2	3	4	5	6	7	8	9	Diff.

ANTILOGARITHMS.

Log.	0	1	2	3	4	5	6	7	8	9	Diff.
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	2-3
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	2-3
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	2-3
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	2-3
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	2-3
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	2-3
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	2-3
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	2-3
08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	2-3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	2-3
10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	2-3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	3
12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	3-4
13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	3-4
14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	3-4
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	3-4
16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	3-4
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	3-4
18	1514	1517	1521	1524	1528	1531	1536	1538	1542	1545	3-4
19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	3-4
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	3-4
21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	3-4
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	3-4
23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	3-4
24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	4-5
26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	4-5
27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	4-5
28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	4-5
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	4-5
30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	4-5
31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	4-5
32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	4-5
33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	4-5
34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	5-6
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	5-6
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	5-6
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	5-6
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	5-6
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	5-6
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	6
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	6-7
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	6-7
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	6-7
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	6-7
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	6-7
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	6-7
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	7
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	7-8
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	7-8
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	7-8
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	7-8
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	7-8
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	8-9
Log.	0	1	2	3	4	5	6	7	8	9	Diff.

ANTILOGARITHMS (*continued*).

Log.	0	1	2	3	4	5	6	7	8	9	Diff.
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	8-9
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	8-9
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	8-9
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	8-9
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	9-10
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	9-10
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	9-10
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	9-10
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	9-10
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	10-11
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	10-11
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	10-11
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	10-11
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	11-12
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	11-12
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	11-12
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	11-12
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	12-13
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	12-13
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	12-13
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	13-14
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	13-14
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	13-14
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	13-14
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	14-15
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	14-15
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	14-15
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	15-16
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	15-16
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	16-17
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	16-17
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	16-17
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	17-18
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	18-19
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	18-19
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	18-19
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	18-19
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	19-20
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	19-20
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	20-21
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	20-21
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	21-22
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	21-22
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	22-23
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	22-23
00	10000	10023	10046	10069	10093	10116	10139	10162	10186	10209	23-24
01	10233	10257	10280	10304	10328	10351	10375	10399	10423	10447	23-25
02	10471	10495	10520	10544	10568	10593	10617	10641	10666	10691	24-25
03	10715	10740	10765	10789	10814	10839	10864	10889	10914	10940	24-25
04	10965	10990	11015	11041	11066	11092	11117	11143	11169	11194	25-26
05	11220	11246	11272	11298	11324	11350	11376	11402	11429	11455	26-27
06	11482	11508	11535	11561	11588	11614	11641	11668	11695	11722	26-27
07	11749	11776	11803	11830	11858	11885	11912	11940	11967	11995	27-28
08	12023	12050	12078	12106	12134	12162	12190	12218	12246	12274	27-28
09	12303	12331	12359	12388	12417	12445	12474	12503	12531	12560	28-29
Log.	0	1	2	3	4	5	6	7	8	9	Diff.

Trigonometry.

Each angle has a set of properties known as its **Trigonometrical Ratios**. If θ be the angle in question and if BA (Fig. 1) be drawn perpendicular to AC , then the ratios are—

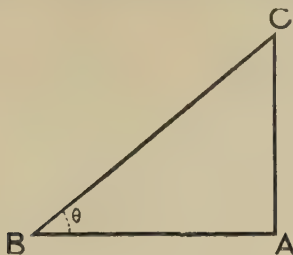


FIG. 1.

sine (written $\sin \theta$) $\frac{AC}{BC}$	tangent (written $\tan \theta$) $\frac{AC}{AB}$
cosine (, $\cos \theta$) $\frac{AB}{BC}$	cotangent (, $\cot \theta$) $\frac{AB}{AC}$

Less used ratios are—

secant ($\sec \theta$) = $\frac{1}{\cosine \theta}$	versed sine ($\text{versin } \theta$) = $1 - \cosine \theta$
cosecant ($\text{cosec } \theta$) = $\frac{1}{\sin \theta}$	coverved sine ($\text{coversin } \theta$) = $1 - \sin \theta$

Note that $AC = BC \sin \theta$, $BC = \frac{AC}{\sin \theta}$, etc.

The following relations are also of value—

$$\begin{aligned} \tan \alpha &= \frac{\sin \alpha}{\cos \alpha} = \frac{1}{\cot \alpha} \\ \sin^2 \alpha + \cos^2 \alpha &= 1 \\ \sin \alpha &= 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \\ \cos \alpha &= \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \\ 1 - \cos \alpha &= 2 \sin^2 \frac{\alpha}{2} \\ 1 + \cos \alpha &= 2 \cos^2 \frac{\alpha}{2} \end{aligned}$$

$$\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin \alpha \sin \pm \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$$

$$\cos \alpha - \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \cos (\alpha - \beta) - \frac{1}{2} \cos (\alpha + \beta)$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \cos (\alpha + \beta) + \frac{1}{2} \cos (\alpha - \beta)$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \sin (\alpha - \beta) + \frac{1}{2} \sin (\alpha + \beta)$$

Use of the Table of Trigonometrical Ratios.

For angles up to 45° the values are as indicated at the head of each column, while for angles between 45° and 90° they are indicated at the foot. The columns of **differences** (each being the difference for 1°) are used as explained above for logarithms, except that, in the case of cosines and cotangents, since the values decrease with increasing angles the differences must be subtracted, instead of being added.

If the line BC (Fig. 1) be supposed to revolve in a **positive** (counter-clockwise) **direction** round B , it will sweep out a larger and larger angle up to 360° (4 right angles), and if it continue to revolve, the angle at any point is to be regarded as the entire angle which has been swept out, and not merely that described since passing through the point A or O (which is the same as the point 360°). If BC revolve in a clockwise direction the angle swept out is regarded as negative. Note that angle $-\alpha$ is the same as angle $= 360^\circ - \alpha$. In every position the ratios are determined by the same lines, the only change being one of sign. BC is throughout regarded as positive, AB is positive when A is to the right of B , but negative when it is to the left, while AC is positive when C is above A , and negative when it is below.

Fig. 2 shows graphically the change of value in sine, cosine and tangent for various **angles** from 0° to 360° . Ordinates below the middle horizontal line are to be taken as negative. From the curve the values of the ratios for 0° , 90° , 180° , etc., can be seen at a glance. In the table, p. 23, the ratios for angles from 0° to 90° are given, and from the curve it will be seen that the values repeat themselves for other angles up to 360° , *e. g.* $\cos 250^\circ$ is the same as $\cos 70^\circ$ with the sign changed or $- \cdot 342$. After 360° the cycle begins again and is identical with that shown in the figure; thus, $\sin 420^\circ = \sin (360^\circ + 60^\circ) = \sin 60^\circ = \cdot 8660$. Hence, with the help of the curve, the trigonometrical ratio for any angle can be found from the table.

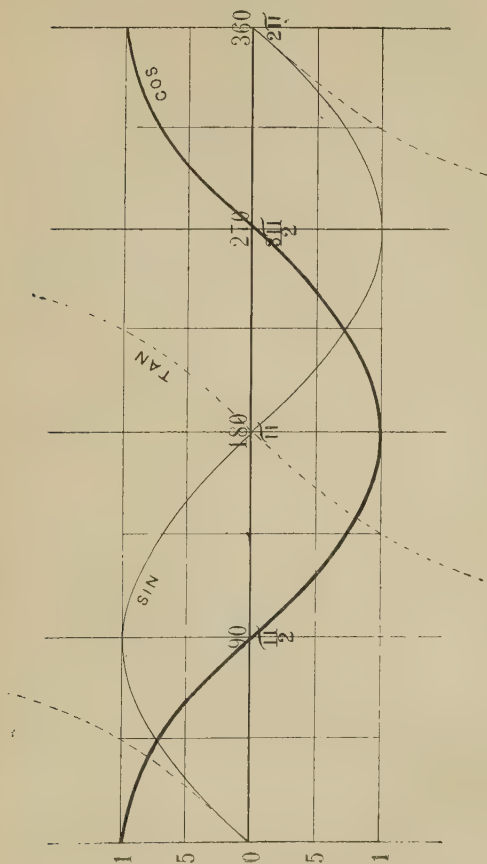


FIG. 2.

If any one ratio is known for an angle less than 90° , the others can be readily found for that angle. Suppose, for example, the sine of θ (Fig. 1) be known to be $\cdot 25 (= \frac{1}{4})$, then we know that if $AC = 1$, CB must $= 4$, and $\therefore BA = \sqrt{4^2 - 1^2} = \sqrt{15}$, hence $\cos \theta = \frac{\sqrt{15}}{4}$.

For measurement of angles in "**Radians**" see p. 55.

Use of Table of Logarithmic Trigonometrical Ratios.

In this case, in order to **avoid negative characteristics**, 10 has been added to each logarithm before tabulating. In other respects the table is similar to that just described. Natural sines, etc., can be obtained from the logarithms by subtracting 10 and finding the corresponding number from the anti-logarithm table. The logarithmic sine, etc., is often written $L \sin$, etc.

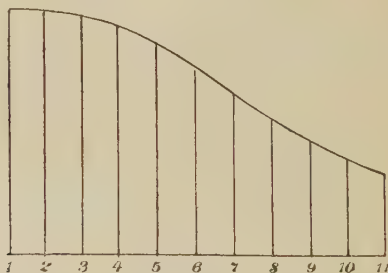


FIG. 3.

Simpson's Rule for Computing Areas.

This rule is applicable only to a four-sided figure of which one side alone is curved, the other three sides being at right angles to each other (any figure can be split up into two or more such figures). Divide up the base into an even number (n) of equal parts (Fig. 3), and erect perpendiculars from each point to meet the curved bounding line. Let the sum of the first and last ordinates be p , the sum of the even ordinates q , and the sum of the remaining (*i. e.* omitting first and last) odd ordinates r ; then if b = length of base, the area $= \frac{p + 4q + 2r}{3} \times \frac{b}{n}$.

A simple though less accurate rule is as follows: Let the sum of first and last ordinates be p and the sum of all the remaining ordinates s , the area $= \frac{p + 2s}{2} \times \frac{b}{n}$. In each case the greater the number of ordinates the more accurate the result.

TRIGONOMETRICAL RATIOS.

Deg.	Sine.	Diff.	Cosine.	Diff.	Tangut.	Diff.	Cotngt.	Diff.	
0	0.0000		1.0000		0.0000		inf.		90
1	0.0175	175	0.9998	2	0.0175	175	57.2900	. . .	89
2	0.0349	174	0.9994	4	0.0349	174	28.6363	28.6537	88
3	0.0523	174	0.9986	8	0.0524	175	19.0811	9.5552	87
4	0.0698	175	0.9976	10	0.0699	175	14.3007	4.7804	86
		174		14		176		2.8706	
5	0.0872	173	0.9962	17	0.0875	176	11.4301	1.9157	85
6	0.1045	174	0.9945	17	0.1051	177	9.5144	1.3701	84
7	0.1219	173	0.9925	20	0.1228	177	8.1443	1.0289	83
8	0.1392	172	0.9903	22	0.1405	177	7.1154	8016	82
9	0.1564	172	0.9877	26	0.1584	179	6.3138	6425	81
		172		29		179			
10	0.1736	172	0.9848	32	0.1763	181	5.6713	5267	80
11	0.1908	171	0.9816	32	0.1944	181	5.1446	4400	79
12	0.2079	171	0.9781	35	0.2126	182	4.7046	3731	78
13	0.2250	169	0.9744	37	0.2309	183	4.3315	3207	77
14	0.2419	169	0.9703	41	0.2493	184	4.0108		76
		169		44		186		2787	
15	0.2588	168	0.9659	46	0.2679	188	3.7321	2447	75
16	0.2756	168	0.9613	50	0.2867	190	3.4874	2165	74
17	0.2924	166	0.9563	52	0.3057	192	3.2709	1932	73
18	0.3090	166	0.9511	56	0.3249	194	3.0777	1735	72
19	0.3256	164	0.9455	58	0.3443	197	2.9042	1567	71
		164		61	0.3640	199	2.7475	1424	70
20	0.3420	162	0.9397	64	0.3839	201	2.6051	1300	69
21	0.3584	161	0.9336	67	0.4040	205	2.4751	1192	68
22	0.3746	160	0.9272	70	0.4245	207	2.3559	1099	67
23	0.3907	159	0.9205	72	0.4452	211	2.2460	1015	66
24	0.4067	158	0.9135	75	0.4663	214	2.1445		
		158		78	0.4877	218	2.0503	942	65
25	0.4226	156	0.9063	81	0.5095	222	1.9626	877	64
26	0.4384	155	0.8988	83	0.5317	226	1.8807	819	63
27	0.4540	152	0.8910	86	0.5543	231	1.8040	767	62
28	0.4695	152	0.8829	88	0.5774	235		719	61
29	0.4848	150	0.8746	92	0.6009	240	1.7321		
		149		93	0.6249	245	1.6643	678	60
30	0.5000	147	0.8660	97	0.6494	251	1.6003	640	59
31	0.5150	146	0.8572	98	0.6745	257	1.5399	604	58
32	0.5299	144	0.8480	102	0.7002	263	1.4826	573	57
33	0.5446	142	0.8387	104	0.7265	271		545	56
34	0.5592	140	0.8290	106	0.7536	277	1.4281		
		139		109	0.7813	285	1.3764	517	55
35	0.5736	136	0.8192	111	0.8098	293	1.3270	494	54
36	0.5878	135	0.8090	113	0.8391	302	1.2799	471	53
37	0.6018	133	0.7986	116	0.8693	311	1.2349	450	52
38	0.6157	129	0.7880	117	0.9004	321		431	51
39	0.6293	127	0.7771	121	0.9325	332	1.1918		
		124		122	0.9657	343	1.1504	414	50
40	0.6428	124	0.7660				1.1106	398	49
41	0.6561		0.7547				1.0724	382	48
42	0.6691		0.7431				1.0355	369	47
43	0.6820		0.7314					355	46
44	0.6947		0.7193						
45	0.7071		0.7071		1.0000		1.0000		45
	Cosine	Diff.	Sine.	Diff.	Cotngt.	Diff.	Tangnt.	Diff.	Deg.

LOGARITHMS OF TRIGONOMETRICAL RATIOS.

Deg.	Min.	L. Sin.	Diff.	L. Cos.	L. Tan.	Diff.	L. Cot.		
0	0	neg. inf.		0.0000	neg. inf.		pos. inf.	90	0
	10	7.4637		0.0000	7.4637		2.5363	89	50
	20	7.7648	3011	0.0000	7.7648	3011	2.2352		40
	30	7.9408	1760	0.0000	7.9409	1761	2.0591		30
	40	8.0658	1250	0.0000	8.0658	1249	1.9342		20
	50	8.1627	969	0.0000	8.1627	969	1.8373		10
1			792			792			
	0	8.2419	669	9.9999	8.2419	670	1.7581	89	0
	10	8.3088	580	9.9999	8.3089	580	1.6911	88	50
	20	8.3668	511	9.9999	8.3669	512	1.6331		40
	30	8.4179	458	9.9999	8.4181	457	1.5819		30
	40	8.4637	413	9.9998	8.4638	415	1.5362		20
2	50	8.5050		9.9998	8.5053		1.4947		10
			378			378			
	0	8.5428	348	9.9997	8.5431	348	1.4569	88	0
	10	8.5776	321	9.9997	8.5779	322	1.4221	87	50
	20	8.6097	300	9.9996	8.6101	300	1.3899		40
	30	8.6397	280	9.9996	8.6401	281	1.3599		30
3	40	8.6677	263	9.9995	8.6682	263	1.3318		20
	50	8.6940		9.9995	8.6945		1.3055		10
			248			249			
	0	8.7188	235	9.9994	8.7194	235	1.2806	87	0
	10	8.7423	222	9.9993	8.7429	223	1.2571	86	50
	20	8.7645	212	9.9993	8.7652	213	1.2348		40
4	30	8.7857	202	9.9992	8.7865	202	1.2135		30
	40	8.8059	192	9.9991	8.8067	202	1.1933		20
	50	8.8251		9.9990	8.8261	194	1.1739		10
			185			185			
	0	8.8436	177	9.9989	8.8446	178	1.1554	86	0
	10	8.8613	170	9.9989	8.8624	171	1.1376	85	50
5	20	8.8783	163	9.9988	8.8795	165	1.1205		40
	30	8.8946	158	9.9987	8.8960	158	1.1040		30
	40	8.9104	152	9.9986	8.9118	154	1.0882		20
	50	8.9256		9.9985	8.9272		1.0728		10
			147			148			
	0	8.9403	142	9.9983	8.9420	143	1.0580	85	0
6	10	8.9545	137	9.9982	8.9563	138	1.0437	84	50
	20	8.9682	134	9.9981	8.9701	135	1.0299		40
	30	8.9816	129	9.9980	8.9836	130	1.0164		30
	40	8.9945	125	9.9979	8.9966	127	1.0034		20
	50	9.0070		9.9977	9.0093		0.9907		10
			122			123			
7	0	9.0192	119	9.9976	9.0216	120	0.9784	84	0
	10	9.0311	115	9.9975	9.0336	117	0.9664	83	50
	20	9.0426	113	9.9973	9.0453	114	0.9547		40
	30	9.0539	109	9.9972	9.0567	111	0.9433		30
	40	9.0648	107	9.9971	9.0678	108	0.9322		20
	50	9.0755		9.9969	9.0786		0.9214		10
8			104			105			
	0	9.0859	102	9.9968	9.0891	104	0.9109	83	0
	10	9.0961	99	9.9966	9.0995	101	0.9005	82	50
9	20	9.1060		9.9964	9.1096		0.8904		40
		L. Cos.	Diff.	L. Sin.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

LOGARITHMS OF TRIGONOMETRICAL RATIOS (*continued*).

Deg.	Min.	L. Sin.	Diff.	L. Cos.	L. Tan.	Diff.	L. Cot.		
7	30	9.1157	95	9.9963	9.1194	97	0.8806	82	30
	40	9.1252	93	9.9961	9.1291	94	0.8709		20
	50	9.1345		9.9959	9.1385		0.8615		10
8			91			93			
	0	9.1436	89	9.9958	9.1478	91	0.8522	82	0
	10	9.1525	87	9.9956	9.1569	89	0.8431	81	50
	20	9.1612	85	9.9954	9.1658	87	0.8342		40
	30	9.1697	84	9.9952	9.1745	86	0.8255		30
	40	9.1781	82	9.9950	9.1831	84	0.8169		20
9	50	9.1863		9.9948	9.1915		0.8085		10
			80			82			
	0	9.1943	79	9.9946	9.1997	81	0.8003	81	0
	10	9.2022	78	9.9944	9.2078	80	0.7922	80	50
	20	9.2100	76	9.9942	9.2158	78	0.7842		40
	30	9.2176	75	9.9940	9.2236	77	0.7764		30
10	40	9.2251	73	9.9938	9.2313	76	0.7687		20
	50	9.2324		9.9936	9.2389		0.7611		10
			73			74			
	0	9.2397	71	9.9934	9.2463	73	0.7537	80	0
	10	9.2468	70	9.9931	9.2536	73	0.7464	79	50
	20	9.2538	68	9.9929	9.2609	71	0.7391		40
11	30	9.2606	68	9.9927	9.2680	70	0.7320		30
	40	9.2674	66	9.9924	9.2750	69	0.7250		20
	50	9.2740		9.9922	9.2819		0.7181		10
			66			68			
	0	9.2806	64	9.9919	9.2887	66	0.7113	79	0
	10	9.2870	64	9.9917	9.2953	67	0.7047	78	50
12	20	9.2934	63	9.9914	9.3020	65	0.6980		40
	30	9.2997	61	9.9912	9.3085	64	0.6915		30
	40	9.3058	61	9.9909	9.3149	63	0.6851		20
	50	9.3119		9.9907	9.3212		0.6788		10
			60			63			
	0	9.3179	59	9.9904	9.3275	61	0.6725	78	0
13	10	9.3238	58	9.9901	9.3336	61	0.6664	77	50
	20	9.3296	57	9.9899	9.3397	61	0.6603		40
	30	9.3353	57	9.9896	9.3458	61	0.6542		30
	40	9.3410	56	9.9893	9.3517	59	0.6483		20
	50	9.3466		9.9890	9.3576		0.6424		10
			55			58			
14	0	9.3521	54	9.9887	9.3634	57	0.6366	77	0
	10	9.3575	54	9.9884	9.3691	57	0.6309	76	50
	20	9.3629	53	9.9881	9.3748	56	0.6252		40
	30	9.3682	52	9.9878	9.3804	55	0.6196		30
	40	9.3734	52	9.9875	9.3859	55	0.6141		20
	50	9.3786		9.9872	9.3914		0.6086		10
15			51			54			
	0	9.3837	50	9.9869	9.3968	53	0.6032	76	0
	10	9.3887	50	9.9866	9.4021	53	0.5979	75	50
	20	9.3937	49	9.9863	9.4074	53	0.5926		40
	30	9.3986	49	9.9859	9.4127	51	0.5873		30
	40	9.4035	48	9.9856	9.4178	51	0.5822		20
16	50	9.4083		9.9853	9.4230	52	0.5770		10
		L. Cos.	Diff.	L. Sin.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

LOGARITHMS OF TRIGONOMETRICAL RATIOS (*continued*).

Deg.	Min.	L. Sin.	Diff.	L. Cos.	Diff.	L. Tan.	Diff.	L. Cot.		
15	0	9.4130	47	9.9849	3	9.4281	50	0.5719	75	0
	10	9.4177	46	9.9846	3	9.4331	50	0.5669	74	50
	20	9.4223	46	9.9843	4	9.4381	49	0.5619		40
	30	9.4269	45	9.9839	3	9.4430	49	0.5570		30
	40	9.4314	45	9.9836	4	9.4479	48	0.5521		20
	50	9.4359	44	9.9832	4	9.4527	48	0.5473		10
16	0	9.4403	44	9.9828	3	9.4575	47	0.5425	74	0
	10	9.4447	44	9.9825	4	9.4622	47	0.5378	73	50
	20	9.4491	42	9.9821	4	9.4669	47	0.5331		40
	30	9.4533	43	9.9817	3	9.4716	46	0.5284		30
	40	9.4576	42	9.9814	4	9.4762	46	0.5238		20
	50	9.4618	41	9.9810	4	9.4808	45	0.5192		10
17	0	9.4659	41	9.9806	4	9.4853	45	0.5147	73	0
	10	9.4700	41	9.9802	4	9.4898	45	0.5102	72	50
	20	9.4741	40	9.9798	4	9.4943	44	0.5057		40
	30	9.4781	40	9.9794	4	9.4987	44	0.5013		30
	40	9.4821	40	9.9790	4	9.5031	44	0.4969		20
	50	9.4861	39	9.9786	4	9.5075	43	0.4925		10
18	0	9.4900	39	9.9782	4	9.5118	43	0.4882	72	0
	10	9.4939	38	9.9778	4	9.5161	42	0.4839	71	50
	20	9.4977	38	9.9774	4	9.5203	42	0.4797		40
	30	9.5015	37	9.9770	5	9.5245	42	0.4755		30
	40	9.5052	38	9.9765	4	9.5287	42	0.4713		20
	50	9.5090	36	9.9761	4	9.5329	41	0.4671		10
19	0	9.5126	37	9.9757	5	9.5370	41	0.4630	71	0
	10	9.5163	36	9.9752	4	9.5411	40	0.4589	70	50
	20	9.5199	36	9.9748	5	9.5451	40	0.4549		40
	30	9.5235	35	9.9743	4	9.5491	40	0.4509		30
	40	9.5270	36	9.9739	5	9.5531	40	0.4469		20
	50	9.5306	35	9.9734	4	9.5571	40	0.4429		10
20	0	9.5341	34	9.9730	5	9.5611	39	0.4389	70	0
	10	9.5375	34	9.9725	4	9.5650	39	0.4350	69	50
	20	9.5409	34	9.9721	5	9.5689	38	0.4311		40
	30	9.5443	34	9.9716	5	9.5727	39	0.4273		30
	40	9.5477	33	9.9711	5	9.5766	38	0.4234		20
	50	9.5510	33	9.9706	4	9.5804	38	0.4196		10
21	0	9.5543	33	9.9702	5	9.5842	37	0.4158	69	0
	10	9.5576	33	9.9697	5	9.5879	38	0.4121	68	50
	20	9.5609	32	9.9692	5	9.5917	37	0.4083		40
	30	9.5641	32	9.9687	5	9.5954	37	0.4046		30
	40	9.5673	31	9.9682	5	9.5991	37	0.4009		20
	50	9.5704	32	9.9677	5	9.6028	36	0.3972		10
22	0	9.5736	31	9.9672	6	9.6064	36	0.3936	68	0
	10	9.5767	31	9.9667	5	9.6100	36	0.3900	67	50
	20	9.5798		9.9661		9.6136		0.3864		40
		L. Cos.	Diff.	L. Sin.	Diff.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

LOGARITHMS OF TRIGONOMETRICAL RATIOS (*continued*).

Deg.	Min.	L. Sin.	Diff.	L. Cos.	Diff.	L. Tan.	Diff.	L. Cot.		
22	30	9.5328		9.9656	5	9.6172		0.3828	67	30
	40	9.5359	31	9.9651	5	9.6208	36	0.3792		20
	50	9.5389	30	9.9646		9.6243	35	0.3757		10
23			30		6		36			
	0	9.5919		9.9640	5	9.6279		0.3721	67	0
	10	9.5948	29	9.9635	6	9.6314	35	0.3686	66	50
	20	9.5978	30	9.9629	6	9.6348	34	0.3652		40
	30	9.6007	29	9.9624	5	9.6383	35	0.3617		30
	40	9.6036	29	9.9618	6	9.6417	34	0.3583		20
	50	9.6065	29	9.9613	5	9.6452	35	0.3548		10
24			28		6		34			
	0	9.6093		9.9607		9.6486		0.3514	66	0
	10	9.6121	28	9.9602	5	9.6520	34	0.3480	65	50
	20	9.6149	28	9.9596	6	9.6553	33	0.3447		40
	30	9.6177	28	9.9590	6	9.6587	34	0.3413		30
	40	9.6205	28	9.9584	6	9.6620	33	0.3380		20
	50	9.6232	27	9.9579	5	9.6654	34	0.3346		10
25			27		6		33			
	0	9.6259		9.9573		9.6687		0.3313	65	0
	10	9.6286	27	9.9567	6	9.6720	33	0.3280	64	50
	20	9.6313	27	9.9561	6	9.6752	32	0.3248		40
	30	9.6340	27	9.9555	6	9.6785	33	0.3215		30
	40	9.6366	26	9.9549	6	9.6817	32	0.3183		20
	50	9.6392	26	9.9543	6	9.6850	33	0.3150		10
26			26		6		32			
	0	9.6418		9.9537		9.6882		0.3118	64	0
	10	9.6444	26	9.9530	7	9.6914	32	0.3086	63	50
	20	9.6470	26	9.9524	6	9.6946	32	0.3054		40
	30	9.6495	25	9.9518	6	9.6977	31	0.3023		30
	40	9.6521	26	9.9512	6	9.7009	32	0.2991		20
	50	9.6546	25	9.9505	7	9.7040	31	0.2960		10
27			24		6		32			
	0	9.6570		9.9499		9.7072		0.2928	63	0
	10	9.6595	25	9.9492	7	9.7103	31	0.2897	62	50
	20	9.6620	25	9.9486	6	9.7134	31	0.2866		40
	30	9.6644	24	9.9479	7	9.7165	31	0.2835		30
	40	9.6668	24	9.9473	6	9.7196	31	0.2804		20
	50	9.6692	24	9.9466	7	9.7226	30	0.2774		10
28			24		7		31			
	0	9.6716		9.9459		9.7257		0.2743	62	0
	10	9.6740	24	9.9453	6	9.7287	30	0.2713	61	50
	20	9.6763	23	9.9446	7	9.7317	30	0.2683		40
	30	9.6787	24	9.9439	7	9.7348	31	0.2652		30
	40	9.6810	23	9.9432	7	9.7378	30	0.2622		20
	50	9.6833	23	9.9425	7	9.7408	30	0.2592		10
29			23		7		30			
	0	9.6856		9.9418		9.7438		0.2562	61	0
	10	9.6878	22	9.9411	7	9.7467	29	0.2533	60	50
	20	9.6901	23	9.9404	7	9.7497	30	0.2503		40
	30	9.6923	22	9.9397	7	9.7526	29	0.2474		30
	40	9.6946	23	9.9390	7	9.7556	30	0.2444		20
	50	9.6968	22	9.9383	7	9.7585	29	0.2415		10
		L. Cos.	Diff.	L. Sin.	Diff.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

LOGARITHMS OF TRIGONOMETRICAL RATIOS (*continued*).

Deg.	Min.	L. Sin.	Diff.	L. Cos.	Diff.	L. Tan.	Diff.	L. Cot.		
30	0	9.6990		9.9375		9.7614		0.2386	60	0
	10	9.7012	22	9.9368	7	9.7644	30	0.2356	59	50
	20	9.7033	21	9.9361	7	9.7673	29	0.2327		40
	30	9.7055	22	9.9353	8	9.7701	28	0.2299		30
	40	9.7076	21	9.9346	7	9.7730	29	0.2270		20
	50	9.7097	21	9.9338	8	9.7759	29	0.2241		10
31			21		7		29			
	0	9.7118		9.9331		9.7788		0.2212	59	0
	10	9.7139	21	9.9323	8	9.7816	28	0.2184	58	50
	20	9.7160	21	9.9315	8	9.7845	29	0.2155		40
	30	9.7181	21	9.9308	7	9.7873	28	0.2127		30
	40	9.7201	20	9.9300	8	9.7902	29	0.2098		20
32	50	9.7222	21	9.9292	8	9.7930	28	0.2070		10
			20		8		28			
	0	9.7242		9.9284		9.7958		0.2042	58	0
	10	9.7262	20	9.9276	8	9.7986	28	0.2014	57	50
	20	9.7282	20	9.9268	8	9.8014	28	0.1986		40
	30	9.7302	20	9.9260	8	9.8042	28	0.1958		30
33	40	9.7322	20	9.9252	8	9.8070	28	0.1930		20
	50	9.7342	20	9.9244	8	9.8097	27	0.1903		10
			19		8		28			
	0	9.7361		9.9236		9.8125		0.1875	57	0
	10	9.7380	19	9.9228	8	9.8153	28	0.1847	56	50
	20	9.7400	20	9.9219	9	9.8180	27	0.1820		40
34	30	9.7419	19	9.9211	8	9.8208	28	0.1792		30
	40	9.7438	19	9.9203	8	9.8235	27	0.1765		20
	50	9.7457	19	9.9194	9	9.8263	28	0.1737		10
			19		8		27			
	0	9.7476		9.9186		9.8290		0.1710	56	0
	10	9.7494	18	9.9177	9	9.8317	27	0.1683	55	50
35	20	9.7513	19	9.9169	8	9.8344	27	0.1656		40
	30	9.7531	18	9.9160	9	9.8371	27	0.1629		30
	40	9.7550	19	9.9151	9	9.8398	27	0.1602		20
	50	9.7568	18	9.9142	9	9.8425	27	0.1575		10
			18		8		27			
	0	9.7586		9.9134		9.8452		0.1548	55	0
36	10	9.7604	18	9.9125	9	9.8479	27	0.1521	54	50
	20	9.7622	18	9.9116	9	9.8506	27	0.1494		40
	30	9.7640	18	9.9107	9	9.8533	27	0.1467		30
	40	9.7657	17	9.9098	9	9.8559	26	0.1441		20
	50	9.7675	18	9.9089	9	9.8586	27	0.1414		10
			17		9		27			
37	0	9.7692		9.9080		9.8613		0.1387	54	0
	10	9.7710	18	9.9070	10	9.8639	26	0.1361	53	50
	20	9.7727	17	9.9061	9	9.8666	27	0.1334		40
	30	9.7744	17	9.9052	9	9.8692	26	0.1308		30
	40	9.7761	17	9.9042	10	9.8718	26	0.1282		20
	50	9.7778	17	9.9033	9	9.8745	27	0.1255		10
38			17		10		26			
	0	9.7795		9.9023		9.8771		0.1229	53	0
	10	9.7811	16	9.9014	9	9.8797	26	0.1203	52	50
39	20	9.7828	17	9.9004	10	9.8824	27	0.1176		40
		L. Cos.	Diff.	L. Sin.	Diff.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

LOGARITHMS OF TRIGONOMETRICAL RATIOS (*continued*).

Deg.	Min.	L. Sin.	Diff.	L. Cos.	Diff.	L. Tan.	Diff.	L. Cot.		
37	30	9.7844	17	9.8995	10	9.8850	26	0.1150	52	30
	40	9.7861	16	9.8985	10	9.8876	26	0.1124		
	50	9.7877	16	9.8975	10	9.8902	26	0.1098		
38	0	9.7893	17	9.8965	10	9.8928	26	0.1072	52	0
	10	9.7910	16	9.8955	10	9.8954	26	0.1046		
	20	9.7926	15	9.8945	10	9.8980	26	0.1020	51	50
	30	9.7941	16	9.8935	10	9.9006	26	0.0994		
	40	9.7957	16	9.8925	10	9.9032	26	0.0968		
	50	9.7973	16	9.8915	10	9.9058	26	0.0942		
39	0	9.7989	15	9.8905	10	9.9084	26	0.0916	51	0
	10	9.8004	16	9.8895	11	9.9110	25	0.0890		
	20	9.8020	15	9.8884	10	9.9135	26	0.0865	50	50
	30	9.8035	15	9.8874	10	9.9161	26	0.0839		
	40	9.8050	16	9.8864	11	9.9187	26	0.0813		
	50	9.8066	15	9.8853	10	9.9212	26	0.0788		
40	0	9.8081	15	9.8843	11	9.9238	26	0.0762	50	0
	10	9.8096	15	9.8832	11	9.9264	25	0.0736		
	20	9.8111	14	9.8821	11	9.9289	26	0.0711	49	50
	30	9.8125	15	9.8810	10	9.9315	26	0.0685		
	40	9.8140	15	9.8800	11	9.9341	25	0.0659		
	50	9.8155	14	9.8789	11	9.9366	26	0.0634		
41	0	9.8169	15	9.8778	11	9.9392	25	0.0608	49	0
	10	9.8184	14	9.8767	11	9.9417	26	0.0583		
	20	9.8198	15	9.8756	11	9.9443	25	0.0557	48	50
	30	9.8213	14	9.8745	12	9.9468	26	0.0532		
	40	9.8227	14	9.8733	11	9.9494	25	0.0506		
	50	9.8241	14	9.8722	11	9.9519	25	0.0481		
42	0	9.8255	14	9.8711	12	9.9544	26	0.0456	48	0
	10	9.8269	14	9.8699	11	9.9570	25	0.0430		
	20	9.8283	14	9.8688	12	9.9595	26	0.0405	47	50
	30	9.8297	14	9.8676	11	9.9621	25	0.0379		
	40	9.8311	14	9.8665	12	9.9646	25	0.0354		
	50	9.8324	13	9.8653	12	9.9671	25	0.0329		
43	0	9.8338	14	9.8641	12	9.9697	26	0.0303	47	0
	10	9.8351	13	9.8629	12	9.9722	25	0.0278		
	20	9.8365	14	9.8618	11	9.9747	25	0.0253	46	50
	30	9.8378	13	9.8606	12	9.9772	25	0.0228		
	40	9.8391	13	9.8594	12	9.9798	26	0.0202		
	50	9.8405	14	9.8582	12	9.9823	25	0.0177		
44	0	9.8418	13	9.8569	13	9.9848	25	0.0152	46	0
	10	9.8431	13	9.8557	12	9.9874	25	0.0126		
	20	9.8444	13	9.8545	13	9.9899	25	0.0101	45	50
	30	9.8457	12	9.8532	12	9.9924	25	0.0076		
	40	9.8469	13	9.8520	13	9.9949	26	0.0051		
	50	9.8482	13	9.8507	13	9.9975	26	0.0025		
45	0	9.8495	13	0.8495	12	0.0000	25	0.0000	45	0
		L. Cos.	Diff.	L. Sin.	Diff.	L. Cot.	Diff.	L. Tan.	Deg.	Min.

Figure.	Area.	Volume.	Centre of Gravity.
<i>Triangle.</i>  $s = \frac{1}{2}$ sum of sides, $a d = c d$ <i>Parallelogram.</i> 	$\frac{\sqrt{s(s-a)(s-b)(s-c)}}{= \frac{1}{2} a b \times a c \times \sin \theta.}$ $= a c \times \frac{h}{2}$ $a b \times b c \times \sin b = h \times b c$		$g d = \frac{1}{3} b d$
<i>Trapezoid.</i>  $a i = i b, d k = k c, b e = d c, f d = a b$ <i>Annulus.</i> 	$\frac{1}{2} h (a b + d c)$ difference in area of 2 concentric circles. $= \pi t (R + r)$ $= \pi t$ (mean radius)		Intersection of diagonals Intersection of $f e$ and $i k$ Centre of circles

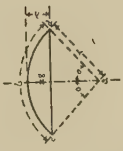
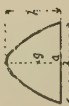
Figure.	Area.	Volume.	Centre of Gravity.
<i>Sector of Circle.</i>  $b = r \pi \theta$ $b = 90'$	$\frac{b r}{2} = \frac{r^2 \pi \theta^2}{180}$ $= .0174 r^2 \theta^2$		$a g = \frac{2}{3} r \frac{c d}{b}$ $= \frac{2}{3} r \frac{\sin \theta^{\circ}}{\theta}$
<i>Segment of Circle.</i> 	$\frac{r^2}{2} \left(\frac{\theta^{\circ} \pi}{90} - \sin 2 \theta^{\circ} \right)$ $= \frac{b r - c d h}{2}$		$a g = \frac{(c d)}{12 \times \text{area}}$ $= \frac{\frac{2}{3} r \sin^3 \theta^{\circ}}{\theta - \sin \theta^{\circ} \cos \theta^{\circ}}$
<i>Part of Annulus.</i> 	$\frac{\pi \theta^{\circ}}{180} (R^2 - r^2)$		$a g = \frac{\frac{2}{3} R^3 - r^3}{R^2 - r^2} \times \frac{\sin \theta^{\circ}}{\theta}$

Figure.	Area.	Volume.	Centre of Gravity.
<i>Cylinder.</i>  <i>Cylinder with one end cut off on slant.</i> 	$2 \pi r h = \pi d h$ (exclusive of ends) $\pi r (h + h')$ (exclusive of ends)	$\pi r^2 h = .785 d^2 h$ $\pi r^2 \frac{h + h'}{2}$	Centre of line joining centres of ends. $h + \frac{h'}{4} + \frac{r^2 \tan^2 \theta}{4(h + h')}$ from the flat end.
<i>Sphere.</i>	$4 \pi r^2 = 12.56 r^2 = \pi d^2$	$\pi r^3 = .5236 d^3$	Centre.
<i>Segment of Sphere.</i>	$2 \pi r h$ (Perimeter $\times$ height) + areas of ends	$\frac{1}{3} \pi h^2 (3r - h)$ Area of base $\times$ height	$ag = \frac{3}{4} \frac{(2r - h)^2}{3r - h}$
<i>Pyramid or cone.</i>	Surface of cone = $\pi r l = \pi r \sqrt{r^2 + h^2}$ (exclusive of base)	Area of base $\times$ $\frac{1}{3} h$	$\frac{1}{4}$ up line from centre of base to apex.

Figure.	Area.	Volume.	Centre of Gravity.
Ring. 	$\pi D \cdot \pi d = 9 \cdot 8696 D d$	$\pi D \left(\frac{\pi d^2}{4} \right) = 2 \cdot 46741 D d^2$	centre of ring
Circle.	$\pi r^2 = \frac{1}{4} \pi d^2 = \cdot 7854 d^2$		Centre
Ellipse.  $[Periphery = 2 \sqrt{D^2 + 1 \cdot 4674 d^2}]$	$\frac{1}{4} \pi (d_1 d_2) = \cdot 7854 d_1 d_2$		Intersection of axes
Parabola.  $[Periphery = b \sqrt{b^2 + 1 \cdot 3333 h^2}]$ exclusive of base.]	$b \times \frac{2}{3} h$		$ag = \frac{2}{5} h$

Tables of Specific Gravities.

METALS.

METAL.	Specific Gravity.	Weight of Cubic Foot. ¹
	Water = 1	Lbs.
Platinum	21·522	1344
Gold	19·245	1200
Mercury	13·596	849
Lead, sheet	11·418	712
„ wire	11·282	704
Silver	10·505	655
Bismuth	9·90	617
Copper, sheet	8·805	549
„ hammered	8·917	556
„ wire	8·880	554
Bronze:—84 copper, 16 tin, gun-metal	8·56	534
83 „ 17 „ „	8·46	528
81 „ 19 „ „	8·46	528
79 „ 21 „ mill bearings	8·73	544
91 „ 10 „	8·50	530·6
Nickel, hammered	8·67	541
„ cast	8·28	516
Brass:—Cast	8·10	505
75 copper, 25 zinc, sheet	8·45	527
66 „ 34 „ yellow ...	8·30	518
60 „ 40 „ Muntz's metal	8·20	511
Brass, wire	8·548	533
Phosphor-bronze, cast	8·60	536·8
Manganese	8·00	499
Steel	7·729 to 7·904	435 to 493
Homogeneous metal	7·904	493
Blistered steel	7·823	488
Crucible steel average	7·842	489
Cast steel average	7·848	489·3
Bessemer steel average	7·852	489·6
Mean	7·852	489·6
Iron, wrought	7·47 to 7·808	466 to 487
Common bar	7·55	471
Puddled slab	7·53 to 7·60	469·5 to 474
Various (tested by Mr. Kirkaldy)	7·5 to 7·8	468 to 486
„ average	7·65	477
Yorkshire iron bar	7·758	484
Lowmoor plates, 1½ to 3 in. thick	7·808	487
Mean	7·698	480
Iron, cast	6·900 to 7·500	378·25 to 467·66
White	7·50	468
Grey	7·20	449
Eglinton hot-blast, 1st melting	6·969	435
„ „ 2nd „	6·970	435
Rennie	6·977 to 7·113	435 to 444
Mean	7·217	450
Tin	7·409	462
Zinc, sheet	7·20	449
„ cast	6·86	428
Antimony	6·71	418
Aluminium, wrought	2·67	167
„ cast	2·56	160

¹ Weight of a cubic foot = specific gravity $\times \frac{1000}{16}$ lbs. (roughly).

SPECIFIC GRAVITIES OF SOLUTIONS AT 15° C. (59° F.).¹

Percentage Strength. ²	Sulphuric Acid.	Nitric Acid.	Hydrochloric Acid.	Silver Nitrate.	Zinc Sulphate.	Potassium Hydrate.	Sodium Hydrate.	Sodium Chloride.	Copper Sulphate.
0	·9991	·999	·9991		·999	·999	·999	·9991	·999
5	1·0334	1·029	1·0242	1·043	1·052	1·045	1·056	1·0354	1·050
10	1·0687	1·058	1·0490	1·090	1·108	1·092	1·111	1·0724	1·103
15	1·1048	1·089	1·0744	1·141	1·168	1·141	1·166	1·1105	1·161
20	1·1430	1·121	1·1001	1·197	1·236	1·191	1·222	1·1501	(1·225)
25	1·1816	1·154	1·1262	1·257	1·307	1·242	1·277	1·1913	
30	1·223	1·187	1·1524	1·323	1·382	1·295	1·333		
35	1·264	1·220	1·1775	1·396		1·349	1·387		
40	1·307	1·253	1·2007	1·479		1·406	1·442		
45	1·352	1·287		1·572		1·466	1·496		
50	1·399	1·320		1·677		1·528	1·548		
55	1·449	1·350		1·792					
60	1·503	1·377		1·919					
65	1·559	1·402							
70	1·616	1·424							
75	1·675	1·443							
80	1·733	1·461							
85	1·785	1·479							
90	1·819	1·497							
95	1·839	1·514							
100	1·8384	1·530							

¹ F. Kohlrausch.

² By weight.

HEAT.

Measurement of High Temperatures.—The ordinary mercurial thermometer can be used up to 500° F. (or for short readings up to 600° F.), or if of special glass and filled above the mercury with compressed nitrogen (so-called **Nitrogen Thermometers**) can be used up to 800° F.³ For high temperatures (and in fact for all readings where accuracy is required) electrical thermometers should be used (see p. 167). An **approximate method**, based upon the specific heat of water, is as follows:—Place a ball, say of fire-clay, of known specific heat (h) and weight (w), in such a position as to acquire the temperature to be measured (*e. g.* of a furnace). Remove it and place it in a quantity of water of weight (w') and temperature (t'). Note the rise of temperature of the water (to t''). Then the temperature to be measured = $\frac{(t'' - t') w'}{h \times w} + t''$ (to whichever thermometer scale has been chosen).

³ Recently quartz mercurial thermometers have been introduced for high temperatures.

TABLE FOR CONVERTING TEMPERATURES CENT. TO FAHR.¹

°C.	°F.	°C.	°F.	°C.	°F.	°C.	°F.	°C.	°F.
0	32·0	27	80·6	53	127·4	79	174·2	125	257·0
+1	33·8	28	82·4	54	129·2	80	176·0	130	266·0
2	35·6	29	84·2	55	131·0	81	177·8	135	275·0
3	37·4	30	86·0	56	132·8	82	179·6	140	284·0
4	39·2	31	87·8	57	134·6	83	181·4	145	293·0
5	41·0	32	89·6	58	136·4	84	183·2	150	302·0
6	42·8	33	91·4	59	138·2	85	185·0	155	311·0
7	44·6	34	93·2	60	140·0	86	186·8	160	320·0
8	46·4	35	95·0	61	141·8	87	188·6	165	329·0
9	48·2	36	96·8	62	143·6	88	190·4	170	338·0
10	50·0	37	98·6	63	145·4	89	192·2	175	347·0
11	51·8	38	100·4	64	147·2	90	194·0	180	356·0
12	53·6	39	102·2	65	149·0	91	195·8	185	365·0
13	55·4	40	104·0	66	150·8	92	197·6	190	374·0
14	57·2	41	105·8	67	152·6	93	199·4	195	383·0
15	59·0	42	107·6	68	154·4	94	201·2	200	392·0
16	60·8	43	109·4	69	156·2	95	203·0	210	410·0
17	62·6	44	111·2	70	158·0	96	204·8	220	428·0
18	64·4	45	113·0	71	159·8	97	206·6	230	446·0
19	66·2	46	114·8	72	161·6	98	208·4	240	464·0
20	68·0	47	116·6	73	163·4	99	210·2	250	482·0
21	69·8	48	118·4	74	165·2	100	212·0	260	500·0
22	71·6	49	120·2	75	167·0	105	221·0	270	518·0
23	73·4	50	122·0	76	168·8	110	230·0	280	536·0
24	75·2	51	123·8	77	170·6	115	239·0	290	554·0
25	77·0	52	125·6	78	172·4	120	248·0	300	572·0
26	78·8								

¹ See also p. 3.

TABLE FOR CONVERTING TEMPERATURES FAHR. TO CENT.¹

°F.	°C.	°F.	°C.	°F.	°C.	°F.	°C.	°F.	°C.
0	-17.78	50	10.00	100	37.78	150	65.56	200	93.34
+1	17.23	51	10.56	101	38.34	151	66.11	201	93.90
2	16.67	52	11.11	102	38.90	152	66.67	202	94.45
3	16.11	53	11.67	103	39.45	153	67.23	203	95.00
4	15.56	54	12.23	104	40.00	154	67.78	204	95.56
5	15.00	55	12.78	105	40.56	155	68.34	205	96.11
6	14.45	56	13.34	106	41.11	156	68.90	206	96.67
7	13.90	57	13.90	107	41.67	157	69.45	207	97.23
8	13.34	58	14.45	108	42.23	158	70.00	208	97.78
9	12.78	59	15.00	109	42.78	159	70.56	209	98.34
10	12.23	60	15.56	110	43.34	160	71.11	210	98.90
11	11.67	61	16.11	111	43.90	161	71.67	211	99.45
12	11.11	62	16.67	112	44.45	162	72.23	212	100.00
13	10.56	63	17.23	113	45.00	163	72.78	213	100.56
14	10.00	64	17.78	114	45.56	164	73.34	214	101.11
15	9.45	65	18.34	115	46.11	165	73.90	215	101.67
16	8.89	66	18.89	116	46.67	166	74.45	216	102.23
17	8.34	67	19.45	117	47.23	167	75.00	217	102.78
18	7.78	68	20.00	118	47.78	168	75.56	218	103.34
19	7.23	69	20.56	119	48.34	169	76.11	219	103.90
20	6.67	70	21.11	120	48.90	170	76.67	220	104.45
21	6.11	71	21.67	121	49.45	171	77.23	225	107.23
22	5.56	72	22.23	122	50.00	172	77.78	230	110.00
23	5.00	73	22.78	123	50.56	173	78.34	235	112.78
24	4.45	74	23.34	124	51.11	174	78.90	240	115.56
25	3.90	75	23.90	125	51.67	175	79.45	245	118.34
26	3.34	76	24.45	126	52.23	176	80.00	250	121.11
27	2.78	77	25.00	127	52.78	177	80.56	255	123.90
28	2.23	78	25.56	128	53.34	178	81.11	260	126.67
29	1.67	79	26.12	129	53.90	179	81.67	265	129.45
30	1.11	80	26.67	130	54.45	180	82.23	270	132.23
31	-0.56	81	27.23	131	55.00	181	82.78	275	135.00
32	0	82	27.78	132	55.56	182	83.34	280	137.78
33	+0.56	83	28.34	133	56.11	183	83.90	285	140.56
34	1.11	84	28.89	134	56.67	184	84.45	290	143.34
35	1.67	85	29.45	135	57.23	185	85.00	295	146.11
36	2.23	86	30.00	136	57.78	186	85.56	300	148.90
37	2.78	87	30.55	137	58.34	187	86.11	310	154.45
38	3.34	88	31.11	138	58.90	188	86.67	320	160.00
39	3.90	89	31.67	139	59.45	189	87.23	330	165.56
40	4.45	90	32.22	140	60.00	190	87.78	340	171.11
41	5.00	91	32.78	141	60.56	191	88.34	350	176.67
42	5.56	92	33.33	142	61.11	192	88.90	360	182.23
43	6.11	93	33.89	143	61.67	193	89.45	370	187.78
44	6.67	94	34.45	144	62.23	194	90.00	380	193.34
45	7.23	95	35.00	145	62.78	195	90.56	390	198.90
46	7.78	96	35.56	146	63.34	196	91.11	400	204.45
47	8.34	97	36.11	147	63.90	197	91.67	450	232.23
48	8.89	98	36.67	148	64.45	198	92.23	500	260.00
49	9.45	99	37.23	149	65.00	199	92.78		

¹ See also p. 3.

The **first visible light** is emitted by iron at 390°C . (H. F. Weber), dark red = 525°C ., bright cherry red = 1000°C ., White heat = 1300°C ., Blinding whiteness = 1500°C .

The following will enable the preceding tables to be employed to find the value of any number of degrees and a fraction by simple addition :—

C.	F.	C.	F.	F.	C.	F.	C.
·1	·18	·6	1·08	·1	·056	·6	·333
·2	·36	·7	1·26	·2	·111	·7	·389
·3	·54	·8	1·44	·3	·167	·8	·444
·4	·72	·9	1·62	·4	·222	·9	·500
·5	·90	1·0	1·80	·5	·278	1·0	·560

Absolute Temperature.—If a perfect gas were to obey the same law as regards expansion throughout the temperature scale it would have zero volume at -460°F ., and all its heat would have been taken from it. This point (-460°F .) is known as the “absolute zero of temperature,” and is of great importance in the theory of the heat engine.

The **specific Heat** of a substance may be defined in several ways, perhaps the simplest definition being “that quantity of heat which is required to raise unit mass of a substance through 1°C at any particular temperature.” The **unit of Heat**, *i.e.* the quantity of heat required to raise unit mass (1 gm.) of water through 1°C varies slightly according to the temperature at which it is measured. Callender suggests 20°C being taken as a standard, since it is practically equal to the mean value between 0° and 100°C .

SPECIFIC HEATS OF METALS.¹

Metal.	Specific heat at		
	0° C. or 32° F.	50° C. or 122° F.	100° C. or 212° F.
Aluminium	0·2070	0·2185	0·2300
Copper	0·0901	0·0923	0·0966
German silver	0·0941	0·0947	0·0952
Iron	0·1060	0·1130	0·1200
Lead	0·0300	0·0315	0·0331
Platinum	0·0320	0·0326	0·0333
Platinum silver ...	0·0473	0·0487	0·0501
Silver	0·0547	0·0569	0·0591
Tin	0·0523	0·0568	0·0595
Zinc	0·0901	0·0938	0·0976

¹ Tomlinson.

TABLE OF MELTING AND BOILING POINTS.

	Melting point cent.	Boiling point cent.	Meas at pressure mm.		Melting point cent.	Boiling point cent.	Meas at pressure mm.
Acetic acid . .	16·5	118	760	Mercury . .	-38·5	357	760
Alcohol . .	- 90	78·3	760	Naphthaline . .	80	216	747
Aluminium . .	624	—	—	Nickel . .	1400	—	—
Amylacetate . .	—	140	754	Nitric acid (sp. gr. 1·52) . .	—	86	760
Antimony . .	440	—	—	Osmium . .	2500	—	—
Benzol . .	5·4	80	764	Palladium . .	1700	—	—
Bismuth . .	260	1700	—	Paraffin . .	46	370	760
Bronze . .	900	—	—	Petroleum (refined) . .	—	150	760
Cadmium . .	315	—	—	Phenol . .	18	184	760
Carbonic acid . .	—	- 80	—	Phosphorous . .	44·2	290	760
Chloroform . .	—	63	760	Platinum . .	1772	—	—
Chromium . .	over 1800	—	—	Roses metal (bis. 4, tin 1, lead 1)	94	—	—
Cobalt . .	1500	—	—	Silver . .	962	—	—
Copper ² . .	1070	—	—	Steel cast . .	1375	—	—
Ether . .	- 32	34·2	760	Solder (tin 17, lead 10) . .	170	—	—
Gold . .	1064	—	—	Sulphur . .	115	448	760
Glycerine . .	—	290	754	Sulphurous acid	-79·2	- 10	760
Iridium . .	2200	—	—	Tin . .	230	1500	760
Iron cast (grey, " (white)	1220 1135	—	—	Turpentine . .	- 10	160	754
Lead . .	326	1500	760	Wax . .	62·5	—	—
Linseed oil . .	- 20	316	760	Zinc . .	412	1000	760
Magnesium . .	—	1100	760				
Manganese . .	1900	—	—				

² But see Richards, Am. El. Science 13, May 1902, p. 377.

The **latent heat** of a change of state is the quantity of heat taken up in the transformation. Thus the latent heat of water is 80 gramme calories per gramme (*i.e.* each gramme of ice in becoming water takes up heat enough to raise 80 grammes of water through 1° C.). The latent heat of steam (at 100° C.) = 535.9 gramme calories per gramme, or 965.7 B. Th. U. per lb.

TABLE OF LINEAR COEFFICIENTS OF EXPANSION (average).

Material.	Co-efficient of expansion.	
	Per $^{\circ}$ F.	Per $^{\circ}$ C.*
Aluminium	.0000114	.0000206
Brass.....	.0000104	.0000187
Brick	.00000306	.00000551
Bronze	.0000100	.0000180
Cement and } Concrete }	.0000055 to .0000078	.000010 to .000014
Copper	.00000961	.0000173
Glass..... { "	.00000399 to .00000521	.00000719 to .00000938
Gold	.00000841	.0000151
Granite.....	.0000046	.0000083
Iron, cast.....	.00000587	.0000106
" wrought.....	.00000677	.0000122
Lead	.0000158	.0000284
Marble	.000004	.000007
Masonry { "	.0000026 to .0000049	.0000047 to .0000088
Platinum	.00000494	.00000890
Porcelain	.0000020	.0000036
Sandstone { "	.0000040 to .0000067	.0000070 to .000012
Silver.....	.0000108	.0000194
Slate	.0000056	.0000102
Steel, untempered	.00000611	.0000110
" tempered.....	.00000689	.0000124
Tin	.0000116	.0000209
Wood (pine).....	.00000276	.00000496
Zinc	.0000163	.0000293

The "**linear coefficient**" of expansion is that fraction by which a rod elongates, for each degree rise of temperature.

Thus $l_t = l_o(1 + kt)$.

when l_t = length at temperature t° | t = rise of temperature
 l_o = " " " " | k = linear coefficient of expansion

$a_t = a_o (1 + 2kt)$ when a_t and a_o denote areas of a plate (say) at t° and o° respectively.

$v_t = v_o (1 + 3kt)$ when v_t and v_o denote volumes at t° and o° respectively.

Conduction of Heat.—If two parallel surfaces, each of area A sq. cm., which face one another at a distance apart of l cm., be kept at temperatures t_1 and t_2 respectively, the quantity of heat flowing through each square cm. of the intervening medium,

$$H = k \frac{(t_1 - t_2) A}{l} \text{ gramme calories.}$$

k is a constant which has the following values—

Silver.....	1.0	German silver	.07 to .1
Copper.....	.5 to .9	Constantan...	.054
Steel.....	.5 to .19	Mercury.....	0.2
Aluminium....	.34	Marble.....	.001 to .002
Zinc.....	.26 to .3	Glass.....	.0005 to .001
Platinum.....	.16	Carbon..	.0003 to .0004
Iron.....	.15 to .18	Hard rubber..	.00026
Brass.....	.15 to .3	Paraffin.....	.0001
Tin.....	.14	Air (and most	
Lead.....	.083	gases).....	.00005

For radiation of Heat, see p. 52.

To express work done in heat units Joule's conversion factor or "**Mechanical Equivalent of Heat**" must be used. Joule stated that 772 foot-pounds of work were required to raise one lb. of water through 1° F. From the latest experiments* it would seem that 777 to 778 foot-pounds is the correct equivalent. 778 foot-pounds per B. Th. U. is equivalent to 426.9 meter kilograms per kg. calorie.

LIGHT.

The velocity of light is about 300,574,000 metres or 186,700 miles per sec. The **quantity of light** coming from a source of **intensity** I , which falls on a unit area perpendicular to its path, is $\frac{I}{r^2}$, where r is the distance from source. The total light emitted by the source = $4\pi I$. The **illumination** of a surface = $\frac{\text{total light falling upon it}}{\text{area}}$, if its plane be at right

* Professor Rowland in America and Mr. Morley with Professor Osborne Reynolds in England are the chief experimenters.

angles to the beam. If the beam makes an angle θ with the normal to the surface, then the illumination will be $\cos \theta$ times that in the above expression. In England, the "**standard or Parliamentary candle**" was long the legal standard of intensity (I), though on account of its variableness the candle itself was seldom used.

The following **standards** are amongst the more important—

The 10 cp. **Vernon-Harcourt Pentane** lamp which forms the British standard.

The **Carcel** lamp (colza) which is roughly equivalent to the Harcourt (see below) and is the French standard.

The **Hefner** (an amyl-acetate lamp due to von Hefner-Alteneck) which forms the German standard, and is roughly equivalent to 1 cp. (see below).

The **Violle** standard which is represented by the light emitted from 1 sq. cm. of pure platinum at its melting point. One-twentieth of this was called the **Bongie décimale** (see below).

The following table gives the **relation existing between these various units** as laid down by the International Commission on Photometry in 1907—

	Bongie décimale.	Carcel.	Hefner.	Vernon- Harcourt.
Bongie décimale.....	1	0.104	1.12	0.102
Carcel	9.6	1	10.7 ₅	0.98 ₀
Hefner	0.89 ₅	0.093 ₀	1	0.091 ₅
Vernon-Harcourt	9.8	1.02 ₀	10.9 ₅	1

Since then, however, the inconvenience of having three units, each differing by a few per cent., and yet all nominally representing the "candle-power," has led England, France and the United States to bring their units into agreement (by making this small adjustment), as closely as the present state of photometry would admit. This arrangement has resulted in bringing both the Bongie décimale and the American candle-power into line with the British or Pentane standard, and while the agreement, at present, only applies to the three countries named, it seems probable that others will subsequently acquiesce, and that this unit will form the "**International Candle-power.**"

The relations of the various units to this candle-power are as follows—

1 International candle-power = 1 Pentane candle-power,
 = 1 Bongie décimale = 1 American candle-power,
 = 1.11 Hefner unit = 0.104 Carcels,
 consequently 1 Hefner unit = 0.90 International candle-power.

The intensity of a source of light is measured in practice by means of a **photometer**. This consists, in its simplest form, of a bench some 5 to 10 feet long, having the standard and the lamp under test fixed one at either end of it. The photometer screen usually slides along the bench, and when equality of illumination is obtained, the intensities are proportional to the squares of the distances from the screen to the sources of light.

One of the simplest photometers, and one still much used, is the **Bunsen**, which consists merely of a sheet of white paper having a grease spot in its centre. If illuminated from both sides, the spot will appear either darker or lighter than the rest of the screen, according as the light in front or behind is the brighter of the two, and when both sides are equally illuminated the spot disappears. In a modification (the Leeson) the grease spot is replaced by a star-shaped opening cut in Bristol board, and covered on each side by thin tissue paper.

In the **Rumford** photometer the shadows cast by objects held in the path of each source of light are compared.

In the grid photometer of **Trotter** a sheet of Bristol board illuminated by one source has a hole cut in it in the form of a grid or star, through which a corresponding surface illuminated by the other source can be seen and compared with it.

Another excellent photometer is the **Lummer-Brodhun** prism type, in which the same effect is produced by reflection.

A well-known photometer is the **Paraffin-block**. In this two square blocks of Paraffin wax are separated by a sheet of tinfoil, and each is illuminated by one of the two sources.

Of the above the simplest, and in most cases the best, is the **Trotter** form.

Instead of moving the photometer itself it will be found best to fix this and one of the lamps, and to move the other up and down the bench, which for convenience can be graduated direct in squares.

In order to get accurate results with any of the standard burners mentioned on the preceding page considerable time and skill is necessary. Hence in photometering incandescent lamps a glow lamp substandard will be found most useful.* A further advantage lies in the fact that any increase or decrease of voltage during the test affects both lamps equally, and so need

not be allowed for. When it is remembered that the candle-power increases roughly as the sixth power of the voltage the importance of this will be realized.

In order to compensate for any possible irregularities in the two sides of the photometer this should be turned round and a mean taken, or, better, when using an incandescent lamp standard, the lamp to be tested can be placed where the standard was previously. By this means stray light is also allowed for.

In using a photometer it should be moved backwards and forwards through smaller and smaller distances until a balance is obtained. Several readings should be made and the mean taken.

Care must be taken that no light is reflected from the walls of the photometer room, etc. For this purpose screens with a hole at the centre to enable the rays of light to reach the photometer will be found useful.

In **Arc lamp photometry** it is usually necessary, owing to the high intensity, to place the arc at a greater distance from the screen than the end of the bench, and in order to be able to take readings in various directions, a small mirror is often fixed to the bench.

Owing to its colour and the small effect on the candle-power of variations of voltage a Nernst lamp forms a very good standard for this purpose.

A photometer which to some extent gets over the colour difficulty is the **flicker photometer** suggested in 1893 by Professor Rood and since improved upon. A simple form consists of a white card disc cut into the shape of a Maltese cross, which is rotated rapidly by a motor or spring, and is illuminated by one of the two sources of light, while a white card seen through the gaps between the arms of the cross is illuminated by the other. When the flickering ceases the illuminations are equal. The flicker photometer has of late been somewhat discredited, however.

It is often useful to be able to determine the actual illumination (in a street, for example), and for this purpose an **illumination photometer** is employed. The first practical form of portable instrument was that described by Messrs. Trotter and Preece * in 1892. In this the illumination of a horizontal card exposed to the light to be measured was balanced against that falling from a standardized glow lamp upon another card whose angle could be varied, and with it the illumination. Assuming the candle-power of the glow lamp to remain constant this angle is a measure of the illumination, and a direct reading scale can be provided.

* *Electrician*, vol. xxxv. p. 671.

Fig. 4 shows, in diagrammatic section, the latest form of Trotter "universal" photometer.* The glow lamp (L) illuminates the adjustable screen (S_1) by means of the mirror (M). The outer screen (S_2), which has a slit cut in it, receives the illumination to be measured, and the screen S_1 is rotated about its axis, until the illumination of S_1 is seen to be equal to that of S_2 . The lamp L is worked off a portable 4-volt accumulator, and is set at such a distance from the inner screen that the circular and almost evenly divided scale indicates direct the illumination of S_1 , and consequently of S_2 . The lamp L can be at any time replaced by another, without in

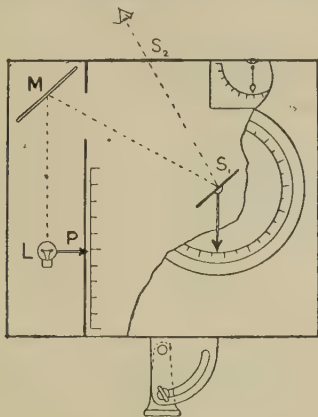


FIG. 4.

any way interfering with the calibration of the instrument, so long as it is set at a point along the scale P corresponding to its candle-power.† The range of this photometer is usually 0 to 4 foot-candles.† A small spirit level indicates when the screen S_2 is horizontal, and if the illumination at any other angle (*e. g.* 45° with the horizontal) is required, the screen can be set accordingly, by means of a plumb-bob. The instrument is equally suitable for the measurement of candle-power. For this purpose the screen S_2 is set at right angles to the rays of

* See *Electrical Engineering*, July 4, 1907.

† A foot-candle (or candle-foot) is the illumination produced on a surface at right angles to the rays of light of a source of one candle-power at a distance of one foot (or four candle-power at two feet, or n^2 candle-power at n feet).

light by means of the adjustable tripod head, the correct setting being determined by means of a small self-contained view-finder. Under these conditions—

$$\text{candle-power} = \text{scale reading} \times D^2,$$

where D = distance from lamp to screen.

The glow lamp L is so chosen as to give a whitish light which forms a convenient mean between that of a carbon glow lamp and (say) an arc. Owing to the ease and rapidity with which the illumination of S_1 can be varied, no difficulty is found in comparing lights differing enormously in colour. As a further aid in this direction Mr. A. P. Trotter has proposed the use of coloured screens, applying Creva's law, but in most cases this is found to be unnecessary.

The indications are only strictly correct when a pressure of 4 volts is applied to the terminals of the instrument. A simple means of correcting for any variation in the voltage of the

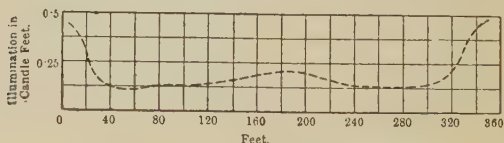


FIG. 5.

accumulator is provided by altering the position of the lamp, but for most purposes such an adjustment is unnecessary.

The question of how **street illumination should be compared** has been much discussed. It is now, however, pretty well agreed that the illumination on a horizontal surface 3 to 4 feet from the ground should be taken. To find an expression representative of the illumination of a particular street is a matter of great difficulty, some authorities proposing the minimum illumination as the criterion, and others the average. The only complete method of comparing two streets is to take readings every few yards, and plot a curve showing the illumination at each point. For most purposes, such a curve representing the illumination down the centre of the road will be found quite sufficient, but if greater exactness is aimed at, a second curve showing the illumination along the curb should be plotted. Fig. 5* shows such an illumination curve taken in a London thoroughfare lit by high-pressure gas lamps.

The readiest way of representing graphically the **intensity of**

* See *Electrical Engineering*, July 4, 1907.

a source of light in various directions is by plotting a polar curve. Though such a curve (see Fig. 6) shows the intensity of the source (*e. g.* an arc lamp) in each direction, it is for many purposes more useful to know the average intensity, or **mean spherical intensity** (say in **candle-power**); that is, the average illumination on a sphere of unit radius, which has the source of light at its centre. To this end it will clearly be of no use to find the mean intensity from the curve, as that represents one

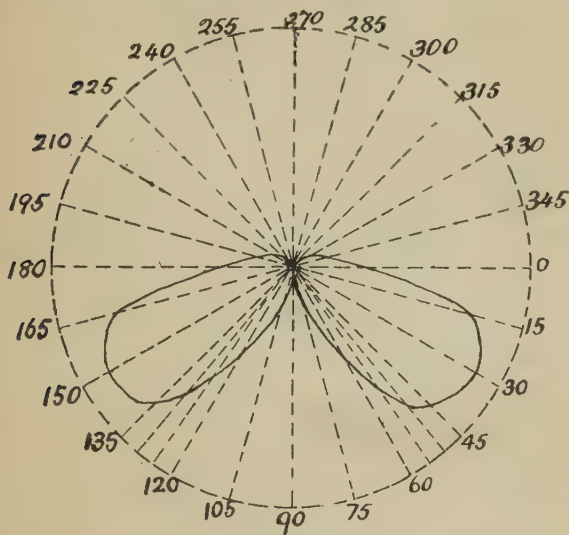


FIG. 6.

plane only, and the surface upon which the illumination falls becomes smaller and smaller as the angle made with the vertical decreases, until, in the vertical line, the surface becomes a point. It is therefore necessary to project the intensities on to a circle. Fig. 7 shows the method to be employed. For simplicity only half a polar curve is shown, the other half being assumed to be identical with it. From the point at which each radius cuts the circumference, drop a perpendicular on to a line parallel to *AB*, and up it set off a distance equal to the corresponding polar ordinate (*i. e.* candle-power). It is not, of course, necessary to

plot the polar curve at all if sufficient points are available. The mean height of the curve so obtained will represent the mean spherical candle-power, to the same scale as that chosen for

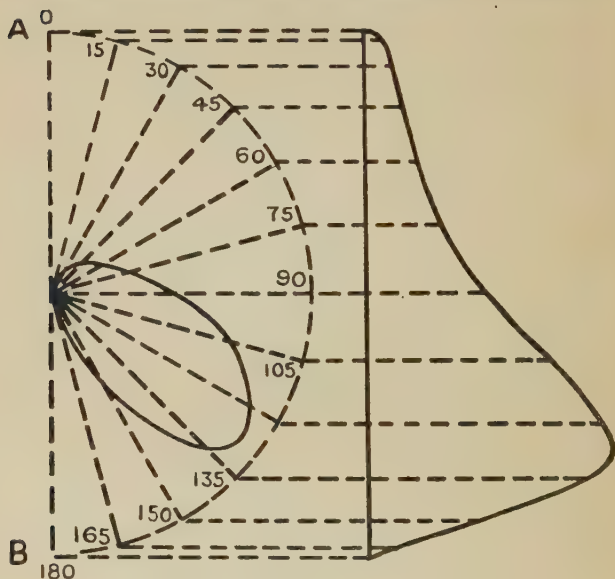


FIG. 7.

the polar ordinates. In the case shown in Fig. 7 it will be the mean hemispherical candle-power, and for most practical cases it is sufficient to determine this.

The following table shows approximately the **energy required by various illuminants**—*

* W. Wedding, *Verhandlung des Vereins zur Beförderung des Gewerbfleißes, and Wirkungsgrad d. Gebräuchlichsten Lichtquellen.*

Illuminant.	Calories per hour per candle-power.
Ordinary gas-burner (in horizontal plane)	66.5
Argand " " " "	50.0
Petroleum lamp " " "	36.4
Incandescent gas " " "	11.0
Acetylene gas " " "	8.9
High Pressure Incandescent gas "	6.5
Glow lamps—Carbon " "	3.0
Nernst lamps " " "	1.6
Glow lamps—metal " " "	1.4
Arc lamp " " "	1.0
Arc lamp Flame " " "	0.2

MIRRORS AND LENSES.

Where a ray of light is **reflected** at a surface (as distinguished from being diffused), the angle of incidence = angle of reflection.* This holds good whether the surface be plane or curved. The "image" of an "object" given by a plane mirror lies symmetrically behind the mirror. A **spherical mirror** is either "concave" or "convex." In either case the focal length (the "**focus**" is the point through which all rays parallel to the axis of the mirror will pass after reflection, and it lies on this axis—but see below) = half the radius of curvature.

Let x = distance of object from focus,

y = " " " image " "

f = focal length of mirror,

then $f^2 = x \times y$.

If u = distance of object from pole (or centre of face of mirror),

v = " " " image " " " "

r = radius of curvature of mirror,

then $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{r}$,

linear dimensions of image = $\frac{v}{u} = \frac{f}{u - f}$.

linear dimensions of object

In all cases **distances** measured in a direction opposed to that of the incident light are to be taken as **positive** and *vice versa*. The reflected **image** will in all cases lie at the intersection of any two ways coming from the object.

* The angles being those made by the ray with the normal to the surface.

The image given by a concave mirror lies in front of it and is therefore "**real**" (*i. e.* can be received upon a screen); that from a convex mirror lies behind the mirror and is "**virtual**."

It is not accurately true, in the case of spherical mirrors, that all rays parallel to the axis meet at a point (the focus), and this irregularity (which is found in lenses also) is known as **spherical aberration**. If the "pencil" of rays is small, they may practically be supposed all to pass through the focus, but for wide pencils this is only so in the case of **Parabolic** mirrors—hence their use for projectors.

When light passes from one medium into another, it undergoes a change of direction at the surface, *i. e.* it is **refracted**. The ray remains, however, in the same plane, and on emerging again into the former medium it will be a second time refracted into a direction parallel to the first. For two given media, the ratio of the sines of the angles of incidence (ϕ) and of refraction (ϕ') will be constant (ϕ and ϕ' are the angles made with the normal to the surface), *i. e.* :—

$${}_b\mu_a = \frac{\sin \phi}{\sin \phi'};$$

${}_a\mu_b$ is called the **refractive index** from medium a to medium b .

It can be shown that ${}_a\mu_b = \frac{1}{{}_b\mu_a}$.

For a ray of light passing through any number of media (a to g say) ${}_a\mu_g = {}_a\mu_b \times {}_b\mu_c \times \dots f\mu_g$.

Further, ${}_a\mu_b = \frac{{}_x\mu_b}{{}_x\mu_a}$.

If a be denser than b , then $\phi < \phi'$, and hence, after a certain value of ϕ (known as the critical angle θ), ϕ' will be 90° , and if ϕ be further increased, the incident ray will be totally reflected— $\sin \theta = {}_a\mu_b$ when a is denser than b .

When only one medium is mentioned the other medium is assumed to be vacuum, as in the following table.

TABLE OF MEAN REFRACTIVE INDICES.

Diamond.....	2.6	Alcohol	1.37
Flint glass.....	1.57	Water... ..	1.34
Rock crystal	1.55	Chlorine	1.00078
Rock salt	1.54	Air.....	1.00029
Crown and plate glass ...	1.52	Oxygen	1.00027
Ice.....	1.31	Hydrogen	1.00014
Turpentine	1.47	Vacuum	1.00000
Sulphuric acid.....	1.43		

As a rough approximation the index from air to glass is $\frac{3}{2}$
and ,, ,, water = $\frac{4}{3}$

For prisms
$$\mu = \frac{\sin \frac{1}{2}(\alpha + d)}{\sin \frac{1}{2}\alpha}$$

or if α is small (*e. g.* $< 10^\circ$) $d = (\mu - 1)\alpha$,
when α = angle of prism and d = angle of deviation (*i. e.* the angle made by the emerging ray with the original direction on entering the prism).

For lenses
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{s} \right),$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{\text{linear dimensions of image}}{\text{linear dimensions of object}} = \frac{v}{u} = \frac{f}{u \times f},$$

where f is the focal length, v the distance of image, and u that of object, from lens, and when s = radius of curvature of far side and r = that of near side of lens. Distances are measured from the centre of the lens, and the same convention as to signs is adopted (see p. 49).

The focus is again at the intersection of any two rays, that of a convex lens being "real," and of a concave lens "virtual."

The magnifying power of a simple lens is $1 + \frac{D}{F}$, where F is its focal length and D is the user's "least distance of distinct vision."

The magnifying power of a telescope for a distant object = $\frac{f_1}{f_2}$.

When f_1 and f_2 are the focal lengths of object-glass and eye-piece respectively—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2},$$

where F = **focal length** of a single lens equivalent to **two lenses** of focal length f_1 and f_2 at a distance a apart.

The index of refraction is not the same for all colours (*i. e.* wave-lengths), being greater the shorter the wave-length. Hence the formation of the **spectrum**. If a narrow beam of white light be allowed to fall upon a prism, the component rays will be more or less refracted (*i. e.* bent out of the straight line) according to their respective wave-lengths, with the result that, if a screen be placed to receive them, a band of colour, gradually passing from red, at one end, to violet at the other, will be formed. In the case of white light the following is the series (starting with the shortest wave-lengths), though there is no abrupt change from one colour to the next—

Actinic Rays, Violet, Indigo, Blue, Green, Yellow, Orange, Red, Dark heat rays.

Both the Actinic and Dark heat rays are invisible.

From what has been said it will be apparent that Heat can be radiated in the same way, and follows the same laws, as Light in the matter of reflection, refraction, etc.

In measuring the intensity of heat rays the Thermopile takes the place of the Photometer. **Radiant Energy**, whether it be heat or light, on falling upon any surface is either (1) reflected (polished surfaces), (2) diffused (matt surfaces), (3) transmitted or (4) absorbed, and as a rule more than one of these processes takes place. Of these, diffusion takes place on rough surfaces, and is greatest in a direction perpendicular to the surface. Thus white paper which reflects some 80 % in a normal direction, only gives 45 % as the mean in all directions. (It is found to vary approximately as the cosine of the angle made with the normal.) The quantity of light so reflected (or, more strictly, diffused) is also greater the shorter the wave-length.

The quantity of radiation (q) entering a thickness (t) of a substance becomes qat on emerging, where a is a constant (<1), called the coefficient of transmission. The amount absorbed, however, varies with different wave-lengths, hence it follows that as the radiation travels through a substance it tends to become more and more constant in composition, and is capable of passing along the remainder of its path (through the same substance) with but little further loss. Rock salt is the most "**diathermanous**" substance known (it transmits 90 %); glass transmits 50 % of solar heat, but is almost "**adiathermanous**" (and alum is even more so) to a source at less than red heat. The rate of **cooling** (or heating) is approximately proportional to the excess of temperature of the one body over that of the other. Stefan found that emissive power was proportional to the 4th power of the absolute temperature.* Substances which absorb heat readily also radiate it well.

Emissive or absorptive powers at 100° C. (approximate).

Lamp-black	100	Shellac	72
White lead	100	Platinum (polished).....	10
Paper and wool	98	Copper foil	5
Glass	90	Silver (polished)	2.5
Indian Ink	85		

The following table † shows the relative reflection, absorption,

* See p. 38.

† W. E. Sumpner's paper before Physical Society.

etc. The reflection is that found in a direction normal to the surface. The mean reflection would be about 75 % of this,

	Reflection per cent.	Trans- mission per cent.	Absorb- tion per cent.
White blotting paper	82	9.2	13.8
White cartridge paper	80	11.2	12.2
Tracing cloth	35	54.4	15.0
Tracing paper	22	76.0	7.0
Newspaper	50 to 70	—	—
Yellow wall paper	20 to 40	—	—
Dirty deal.....	20	—	—
* Thin opal glass.....	—	—	15
* Thick opal glass	—	—	39
* Ground glass	—	—	42
Black cloth	1.2	—	—
Black velvet	.4	—	—

Ordinary **mirrors** reflect 80 % and very good mirrors 90 %.

* See also p. 208.

TABLE OF C. G. S. AND PRACTICAL UNITS.

1 Nature of Unit.	2 Derivation.	Name and Symbol			Pract. C.G.S. × k = Absolute C.G.S.	Pract. English × k' = Pract. C.G.S.
		3 Absolute C.G.S.	4 Practical C.G.S.	5 Practl. English	6 Value of k	7 Value of k'
Length	Fundamental (l)	centimetre (cm.)	metre (m)	foot (ft. or ')	100	·3047950
Mass	" (m)	gramme (g)	kilogramme. (kg.)	pound (lb.)	1000	·45357
Time	" (t)	second (s)	minute	minute	60	1
Surface	$A = l \times l$	square centimetre (cm. ²)	square metre (m ²)	square foot (sq. ft. or □')	10,000	·092898
Volume	$V = l \times l \times l$	cubic centimetre (cm. ³)	cubic metre (m ³)	cubic foot (cub. ft.)	1,000,000	·0283155
Velocity	$v = \frac{l}{t}$	cm. per sec.	m. per sec.	feet per min.	100	·0050799
Acceleration	$a = \frac{v}{t}$	cm. per sec. per sec.	m. per sec. per sec.	ft. per min. per min.	100	·000084665
Force (or weight) ...	$F = m \times a$	Dyne	kilogramme. (kg.)	pound (lb.)	981,000	·45357
Work	$W = F \times l$	Erg	kg.-metre (kgm.)	foot pound (ft.lb.)	100,000	·13825
Power	$P = \frac{W}{t}$	Erg per sec.	kgm. per sec.	{ horse - power (H.P. or HP.)	100,000	7·46
Pressure	$p = \frac{F}{A}$	Dyne per sq. cm.	kg. per sq. m.	lbs. per square inch.	98·1	·00000703
Moment of Inertia	$I = ml^2$			—	—	—
Angle	$\theta = \frac{\text{arc}}{\text{radius}}$	Radian	Degree (°)	Degree (°)	$\pi/180 = \cdot017453$	1
Angular Velocity ..	$\omega = \frac{v}{r}$	Radians per sec.	Revolutions per minute	Revolutions per minute	$\pi = \cdot1047197$ 30	1

PRACTICAL MECHANICS.

IN making use of the table of units on p. 54, the following points should be noted. In connection with the second column, "Derivation," attention must be paid to the conditions of each particular case. Thus, for example, the "Moment of Inertia" formula only holds good when the whole mass (m) can be regarded as being situated at the same distance (l) from the centre of revolution, and so on. From column 6 it will be seen that to reduce practical C.G.S. (that is metric) units (*e.g.* metres) to absolute C.G.S. units (*i.e.* centimetres), the former must be multiplied by k (*i.e.* 100), while, to reduce absolute to practical, the former must be divided by k . The same rule holds good for reducing English units to practical C.G.S. units and *vice versa* (see column 7).

If l = distance, t = time, m = mass, v = velocity, a = acceleration, F = force, W = work,

then with constant * velocity, $v = \frac{l}{t}$, with constant **accelera-**

tion, $a = \frac{v_2 - v_1}{t}$ (for acceleration means "gain of velocity per

unit of time"), and $v_2 = v_1 + at$, $l = (v_1 + \frac{1}{2}at)t = \frac{v_1 + v_2}{2}t$,

$$v_2^2 = v_1^2 + 2al.$$

The acceleration due to **gravity** varies slightly from place to place, but may be taken as approximately 32.2 (it is 32.1899 at London at sea-level) feet per second per second (unit has no name) or 9.81 metres per second per second. (For conversion of metric units to English and *vice versa*, see pp. 1, 2, 6, 7, and 8).

If the body, instead of moving in a straight line, be supposed to move round in a circle, the *linear* velocity of different points on it will vary, and it is therefore more convenient to speak of its "**angular velocity**," *i.e.* of the number of radians † swept out per second, or in practical language, of the number of **revolutions per minute** or second. Now since there are 2π radians in 360° (*i.e.* in one revolution), $\therefore \omega = 2\pi n$ and $v = 2\pi rn$,

$\omega = \frac{v}{r}$ and $v = \omega r$. Where ω = angular velocity in radians per second, n = revolutions per second, r = radius of path of the

* "Constant" usually refers to time and "uniform" to space. Thus a velocity is said to be constant during a certain time, while pressure is said to be uniform over a given area.

† A radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of that circle. This angle is independent of the size (*i.e.* radius) of the circle chosen, and is consequently a convenient unit.

$$\text{Its value is } \frac{\text{radius}}{\text{circumference}} \times 360^\circ = \frac{360^\circ}{2\pi} = 57.296^\circ.$$

body (regarded as a point only), and v = velocity of the body in feet per second.

Any two forces acting at a point can be replaced by a single force in the same plane which will be absolutely equivalent in its effect, to the original forces. This may be done in two ways—

(1) Geometrically, by the **Parallelogram of forces**. Let each of the forces be represented in magnitude and direction by a straight line. Complete the parallelogram, of which these lines form two adjacent sides. The diagonal of this parallelogram (passing through the *point of application*, as it is called) will represent in magnitude and direction the equivalent or **Resultant** force. This method can clearly be applied to any number of forces by taking them two at a time. It will be seen, however, that the same resultant can be obtained by drawing a triangle, of which two sides are equal and parallel to the given forces. This triangle must be drawn in such a way that in going *continuously* round it, the pen moves, not only parallel to, but also in the same sense* as, the original forces. It will be noticed that the resultant is equal in magnitude and opposite in sense to the third side of such a triangle, and that therefore “if three forces acting *at a point* can be represented in magnitude, sense and direction by the sides of a triangle, taken in order, the forces are in **equilibrium**.” This method can be extended to any number of forces, with this difference, that the words “closed polygon” must be substituted for “triangle” in the above statement. It is quite immaterial in what order the forces are taken, so long as the rule as to direction and sense be adhered to. From what has been said it will be seen that the converse is also true, *i. e.* that any force can be replaced by (or

Resolved into) two or more forces in any given directions. It is frequently of great value to be able so to resolve a force into two components in given directions. Thus, in the case of the earth's magnetic field it is only the horizontal component which tends to set a compass needle north and south. Fig. 8 shows how

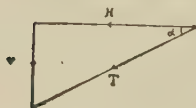


FIG. 8.

the total field, represented in magnitude and direction by T , can be resolved into a horizontal component H , and a vertical component V . It can also be done trigonometrically (see below). Thus $H = T \cos \alpha$ (where α is the “angle of dip”).

(2) **Trigonometrically**.—If the two forces be represented in magnitude by P and Q , and if θ be the angle between them, their resultant is given in magnitude by—

* Strictly two trains, one moving N.E. and the other S.W., are travelling in the same *direction* but in the opposite *sense*.

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

It will pass through the point of application, but its direction must be separately determined.

A force acting on a body may be regarded as acting at any point in its **line of action**, no matter whether this point be on the body or not. The above laws hold good also for velocities, accelerations, and all other "vector" quantities such as momentum.

It should be noted that the **relative velocity** of two points is not the rate at which one recedes from, or approaches, the other, but if T and H (Fig. 8) be the respective velocities of two points, V will be their relative velocity.

FRICTION.

Without **friction**, a weight resting on a horizontal plane will, if once started, continue in motion. Friction, however, renders the continued application of force necessary. The ratio of the translating force (f) to the perpendicular force (F) pressing the body upon the plane (each expressed in the same units, *e. g.* pounds) is known as the **coefficient of friction** (μ) or $\mu = \frac{f}{F}$. The work

done is $f \times l = \mu Fl$, $\therefore$ Power lost = $\frac{\text{work}}{t} = \frac{\mu Fl}{t} = \mu Fv$. The value of μ is found to be, within very wide limits, independent of the value of F and of the area of contact, also (except that it increases at very low speeds) of the velocity. The value of μ can be found roughly by placing one of the surfaces to be tested on an inclined plane of the other material and noting the angle θ at which it will, if once started, continue to slide down the plane; then $\mu = \tan \theta$.

The following coefficients must be taken as approximate only, since the laws governing sliding friction, and still more those governing rolling friction, are but little understood and are much complicated by questions of lubrication. Thus, for example, the coefficient of friction for two surfaces lubricated with sperm oil may be taken as .01 to .04, while for greased surfaces it will be perhaps double this value. On the whole the friction here given will be found higher, if anything, than that obtained in practice.

TABLE OF COEFFICIENTS OF FRICTION (*Marin and others*).

Nature of Materials.	Nature of Surface.	Form of Friction.	Coef. of Friction μ .	
			In motion.	At rest.
Oak on oak, fibres parallel to motion	Dry	Flat Friction	·48	} mean ·5
Oak on oak, fibres perpendicular to motion			·34	
Oak on oak, fibres endwise			·19	
Metals on oak, fibres parallel			·55	·6
Wrought iron on wrought iron	"	"	·18	} mean ·18
Cast iron on cast iron ...			·15	
Leather belting on cast iron pulley			·26	·54
Leather belting on oak pulley			·27	·47
W. I. on W. I. or C. I.; C. I. on C. I.; W. I. or C. I. on bell-metal	} With lubrication	Bearing friction	} ·15—·20	—
W. I. or C. I. on lignum vitæ				—
	"	"	·19	—

TABLE OF COMPARATIVE VALUE OF LUBRICANTS AND MAXIMUM ALLOWABLE PRESSURE (*Tower*).

Lubricant.	Comparative Friction.	Max. Press. per sq. in.
Bath of Sperm Oil	26	400 lbs.
" Rape Oil	29	500 "
" Mineral Oil	36	450 "
" Lard Oil	37	520 "
" Olive Oil	38	520 "
" Mineral Grease	57	600 "

In the case of syphon or similar lubricators the above frictional values must be multiplied by 4 or 5. The pressure is calculated per square inch of so-called "journal area" (= diameter of shaft $\times$ length of bearing). The maximum pressure per square inch (P) in shafting usually does not exceed 200 with ordinary lubrication, or 500 with an oil-bath. In reciprocating machinery the values of P given in the table may be greatly exceeded (owing to improved lubrication). Cast-iron bearings should not have $P > 100$ nor a velocity > 150 feet per minute at the surface. In the case of an ordinary axle—

Pull in pounds at a radius of 1 foot = $d \times F \times \mu$,

where d is the diameter of the shaft, F the load in pounds on bearing, and μ has the following values—

Method of Lubrication.	Values of μ .	
	Goodman.	Beauchamp Tower.
Oil-bath	·004	·003
Saturated oil-pad	·012	·01
Syphon lubricator.....	·020	·014

The coefficient of friction in lubricated bearings decreases with a rise of temperature. For example, with a lard oil-bath it had the following values—

Temp. F.	60°	70°	80°	90°	100°	120°
Relative friction	} 1·8	1·4	1·2	1·0	·85	·64

The ratio of **pull** to load in **vehicles** varies approximately inversely as the wheel diameter. For wheels 3 feet in diameter it may be taken as—

For bad roads	·072	} These figures apply to vehicles in motion. The starting pull will be almost double this. (See also p. 419.)
good roads.....	·027	
asphalt roads...	·035 to ·02	
tram lines	·011	
railway lines ...	·005	

BELTING.

In the case of **belting** the coefficient of friction will average about ·2, being rather less (say ·15) in new belts. In order that the belt may not slip, the required tension (t) on the slack side (which should be the upper side if possible) will be found to be from $\frac{1}{3}$ to $\frac{2}{3}$ (say $\frac{1}{2}$) of the tension (T) on the tight side. And since the H.P. transmitted = $\frac{(T - t)v}{33,000}$ (when v is the velocity in feet per minute and the tensions are measured in pounds), it follows that $\text{H.P.} = \frac{Tv}{2 \times 33,000}$. The maximum tension which single

belting of ordinary thickness (say $\frac{1}{4}$ inch) will stand in practice is about 90 lbs. per inch of width. Hence $H.P. = \frac{45v \times b}{33,000}$, or

$H.P. = \frac{vb^*}{750}$, say (b = width in inches), for single belts; double belts will transmit half as much again.

In the case of **rope-driving**, though the coefficient of friction on a smooth pulley is only about .07 (this applies also to cords and twine), the grooves on the pulleys usually employed so far increase the grip that T may be taken as $4t$ without fear of slip, and hence $H.P. = \frac{3Tv}{33,000}$ (where T is the safe working load on each rope $\times$ number of ropes);

or $H.P. = \frac{Tv}{44,000}$ say. T should be taken as about 200 lbs. per sq. inch and v from 4000 to 5000 feet per minute.

Ropes work best when either horizontal or at an angle of less than 45° . If at a greater angle the power transmitted will be less. The length of drive may be as great as 80 or 100 feet with large pulleys and a good sag. Cotton is the best material for driving ropes.

TRANSMISSION OF POWER BY VARIOUS MEANS.†

Toothed wheels (double flanged wheel one-third more).				Steel shafting.		Single belting.		One rope.	
Breadth of teeth.	Pitch of teeth.	Power that can be transmitted per 100 ft. of circumferential velocity per minute.		Diameter.	Power that can be transmitted per ten revolutions per min.	Breadth.	Power that can be transmitted for every 100 ft. per minute.	Diameter.	Power that can be transmitted for every 100 ft. per minute.
In.	In.	Spur. H.P.	Revel. H.P.	In.	H.P.	In.	H.P.	In.	H.P.
2	1	$\frac{3}{8}$	$\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	3	$\frac{1}{2}$	$1\frac{1}{8}$	$\frac{1}{2}$
3	$1\frac{1}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	2	$1\frac{1}{2}$	4	$\frac{3}{4}$	1	$\frac{3}{4}$
4	$1\frac{1}{2}$	1	$\frac{3}{4}$	$2\frac{1}{2}$	2	5	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{5}{8}$
5	$1\frac{3}{4}$	$1\frac{1}{2}$	1	3	$3\frac{1}{2}$	6	$\frac{3}{4}$	$1\frac{1}{2}$	$\frac{3}{4}$
6	2	$2\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	6	7	$\frac{7}{8}$	$1\frac{3}{4}$	$\frac{7}{8}$
7	$2\frac{1}{4}$	$4\frac{1}{2}$	$3\frac{1}{2}$	4	9	8	1	2	1
8	$2\frac{3}{4}$	6	$4\frac{3}{4}$	$4\frac{1}{2}$	13	9	$1\frac{1}{2}$	...	...
9	$2\frac{3}{4}$	$8\frac{1}{2}$	$6\frac{1}{2}$	5	18	10	$1\frac{1}{4}$	...	...
10	3	11	$8\frac{1}{2}$	$5\frac{1}{2}$	24	12	$1\frac{1}{2}$	...	...
11	$3\frac{1}{4}$	$14\frac{3}{4}$	11	6	31	15	$1\frac{3}{4}$	...	...
12	$3\frac{1}{2}$	19	14	...	...	18	$2\frac{1}{4}$	...	...

* $v \times b$ = watts is a convenient rule to remember.

† Harper.

For steel prime mover shafting or for wrought-iron line shafting take 70 % of the H.P. given in the table.

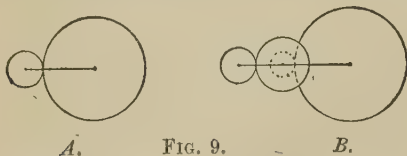
TABLE OF SUITABLE BEARING DISTANCES FOR LINE SHAFTING.

Diameter of shaft in inches.	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Centres of bearings apart.	7 0	7 6	8 0	8 6	9 0	9 6	10 0	11 0	12 0	13 0

5	$5\frac{1}{2}$	6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	9	10	Diameter of shaft in inches.
ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	Centres of bearings apart.
13 6	14 0	15 0	15 6	16 0	17 0	18 0	19 0	20 0	

GEAR WHEELS.

Toothed gearing is merely a special case of the lever. If two wheels gear into one another, they will revolve in opposite senses, but at the same peripheral speed. Hence the revolutions per minute made by each will be inversely proportional to their respective diameters (or numbers of teeth). If the two are geared together by an odd number of intermediate (idle) wheels, they will revolve in the same sense; if, on the other hand, by an even number, then in opposite senses. In either case the "**velocity ratio**" (*i. e.* ratio of revolutions per minute; also known as "mechanical advantage") will be unaffected by the number or size of the intermediate wheels. If, however, the intermediate wheels are not idle (that is if there are two wheels of different diameters on one spindle), then the velocity ratio will depend on the dimensions of these intermediate wheels. To obtain the ratio for the train, it is only necessary to multiply together the ratios for each pair.



In an **epicyclic train** (Fig. 9, of which *A* is a simple example and *B* a more complicated)—

Let a = revolutions per minute of arm,

b = „ „ first wheel of train,

c = „ „ last „ „

The revolutions per minute of first wheel, with respect to arm
 $= b - a,$

and revolutions per minute of last wheel, with respect to arm
 $= c - a,$

∴ velocity ratio of train $= \frac{c - a}{b - a} = r,$

$$b = \frac{c - ra - a}{r}, \text{ and } c = rb - ra + a,$$

will be positive or negative, according as the first and last wheels revolve in the same, or in opposite, senses. In calculating any complicated chain, one wheel (or arm) can if necessary be assumed to be at rest, and the revolutions made by each wheel during (say) one revolution of any other given wheel, can then be calculated.

It should be noticed that “leverage,” “velocity ratio,” and “mechanical advantage,” all convey the same notion. In the case of gear-wheels the expression “value of the train” is also often used in this sense. When a very great reduction of speed is required **worm-gearing** is usually employed. One revolution of the worm causes the wheel to move on through an angle equivalent to one tooth. As a rule the axes of driver and driven are at right angles. A steel worm engaging in, say, a phosphor-bronze wheel is found to work best, and should be run in oil. Great attention must be given to the thrust block, which, when possible, should work with ball or roller bearings.

PENDULUM.

A **simple pendulum** (*i. e.* a heavy “bob” of negligible size, supported by a rod or string of negligible weight) has a **time of swing**, or period of oscillation, $t = \pi \sqrt{\frac{l}{g}}$ when t = time in seconds from one end of path to the other,* l = length of pendulum in feet, $g = 32.191$ in London, $l = gt^2/\pi^2$, and $g = \frac{l\pi^2}{t^2}$ (this is the easiest way to determine g).

If the arc of swing be small compared with the length of

* There is often some confusion as to whether the single or double swing (*i. e.* back to same point) is meant. “One complete swing” means the latter, so does “period,” but “swing” usually denotes the former.

the pendulum, the time of swing will be independent of the length of arc (amplitude), and it is seen from the formula to be independent of the weight. The length of the "seconds pendulum" in London = 39.1393 inches.

SPRINGS.

TABLE SHOWING RELATIVE VALUE OF VARIOUS MATERIALS FOR SPRING-MAKING PURPOSES (*Perry*).

Material.	Subjected to bending.	Cylindrical spiral.
Cast steel (hardened).....	501	809
Cast steel (unhardened).....	107	186
Mild steel (hardened).....	83	126
Mild steel (unhardened).....	20	32
Phosphor-bronze.....	13.85	20
Wrought iron	10.0	19
Brass	2.62	4.00
Gun-metal.....	2.00	2.33

Spiral Springs—

For round wire
$$x = \frac{Wnr^3}{Nd^3} \times 64,$$

for rectangular wire
$$x = \frac{Wnr^3}{\left(\frac{b^3h^3}{b^2 + h^2}\right)N} \times 24,$$

for round wire
$$W_s = \frac{.196 Sd^3}{r},$$

for rectangular wire
$$W_s = \frac{.514 S \frac{b^2h^2}{\sqrt{b^2 + h^2}}}{r}$$

Maximum safe sudden load
$$= \frac{W}{2},$$

where

x = extension in inches,

W = load in lbs. (supposed to be less than W_s),

W_s = maximum safe load in lbs. (steady),

n = number of coils,

r = radius of coils from axis to centre of rim, in inches,

d = diameter of round wire in inches,

b = breadth of rectangular wire in inches,

h = depth of rectangular wire in inches,

N = modulus of transverse elasticity in lbs per sq. inch,

S = max. safe shearing stress in lbs. per sq. inch.

For steel and charcoal iron $\begin{cases} N = 10,500,000 \text{ to } 12,000,000, \\ S = 30,000. \end{cases}$

For phosphor-bronze $\begin{cases} N = 5,000,000 \text{ to } 6,000,000, \\ S = 14,500. \end{cases}$

For brass $\begin{cases} N = 3,000,000 \text{ to } 3,400,000, \\ S = 5,200. \end{cases}$

For spiral **clock springs**, if diameter of outer coil = diameter of inner coil $\times 3$.

$$M = \frac{Ebt^3\phi}{12l},$$

where M = moment in inch pounds,

E = Young's modulus of elasticity,

b = breadth of spring in inches,

t = thickness of spring in inches,

l = length of spring in inches,

ϕ = total angle wound through in radians (see p. 55).

For steel $E = 30,000,000$.

For phosphor-bronze $E = 14,000,000$.

For brass $E = 9,200,000$.

$b = 30t$ is a usual value.

b should not exceed $160t$.

Flat springs supported on knife-edges at each end—

$$\text{Deflection in inches} = \frac{W \times l^3}{4Ebt^3},$$

where W = load at centre in pounds,

l = length in inches,

b = breadth in inches,

t = thickness in inches,

E = Young's modulus (see above for values).

If fixed at one end and loaded at the other, the deflection will be 16 times that given by the above formula.

POWER TAKEN TO DRIVE VARIOUS MACHINES.*

Raworth gives—

Wood Planer (12 in. cutter)—

Watts.

Cutter running light 1,078

Cutter and bed running..... 1,188

Cutter taking 3 cuts to $\frac{1}{8}$ in. on 3 in. pine..... 1,738

* For further particulars and data see A. D. Williamson and A. Chaturm, *Jl. Inst. E. E.*, Aug. 1903, and J. A. Anderson, ditto, July 1904. For some further figures see Ker, *Proc. Liverpool Eng. Soc.*, 1901.

Circular Saw —10 in. diam.—	Watts.
Running light	1,007
Cutting 3 in. pine	2,290
Band Saw —wheel, 28 in. diam. ; saw, $\frac{3}{4}$ inch.—	
Running light	968
Cutting	1,738
Grindstone , 4 ft. diam., 7 in. wide	164
Lathe , 2 ft. 6 in. centres—	
Light	768
Working	1,200
Two 12 in. lathes—	
Light	318
Drill —1 $\frac{1}{4}$ in. spindle, 6 in. stroke—	
Light	404
Drilling 1 in. hole	818

Flather gives—

MACHINE.

	H.P.
Small screw-cutting lathe, 13 $\frac{1}{2}$ in. swing (back geared)...	0.41
Small screw-cutting lathe, 12 in. swing (back geared)...	0.330
Screw-cutting lathe, 17 $\frac{1}{2}$ in. (back geared)	0.867
Screw-cutting lathe, 20 in. (back geared)	0.47
Screw-cutting lathe, 26 in. (back geared)	0.462
Large facing and turning lathe, 80 in. face-plate will swing 108 in. (treble geared)	0.53
Large facing lathe, will swing 68 in. (treble geared)	0.91
Small shaper (stroke 4 in., work can traverse 11 in.) ...	0.16
Small shaper, Richards (capacity 9 $\frac{1}{2}$ in. stroke $\times$ 22 in. traverse)	0.24
Shaper (15 in. stroke)	0.63
Large shaper, Richards (capacity 29 in. stroke $\times$ 91 in. traverse)	1.14
Crank planer (capacity 23 in. $\times$ 27 in. $\times$ 28 $\frac{1}{2}$ in. stroke)	0.24
Planer (capacity 36 in. $\times$ 36 in. $\times$ 11 ft.)	0.84
Large planer (capacity 76 in. $\times$ 76 in. $\times$ 57 ft.)	1.47
Small drill press	0.62
Upright slot drilling machine (will drill slot 2 $\frac{1}{2}$ in. diameter)	0.41
Medium drill press	1.33
Large drill press	1.24
Radial drill (6 ft. swing)	0.53
Radial drill (8 $\frac{1}{2}$ ft. swing)	0.67
Radial drill press	1.08
Slotter (8 in. stroke)	0.28
Slotter (9 $\frac{1}{2}$ in. stroke)	0.44
Slotter (15 in. stroke)	0.95

MACHINE.	H.P.
Universal milling machine (Brown and Sharpe No. 1, without overhanging arm)	0·28
Milling machine (13 in. cutter-head, 12 cutters)	0·66
Small-head traversing milling machine (cutter-head 11 in. diameter, 16 cutters)	0·18
Gear cutter (will cut 20 in. diameter)	0·28
Horizontal boring machine for iron ($22\frac{1}{2}$ in. swing)	0·93
Hydraulic shearing machine	1·52
Large plate shears (knives 28 in. long, 3 in. stroke)	7·12
Large punch press (over-reach 28 in., 3 in. stroke, $1\frac{1}{2}$ in. stock can be punched)	4·41
Small punch and shear combined (knives $7\frac{1}{2}$ in. long, $1\frac{1}{2}$ in. stroke)	0·79
Circular saw for hot iron ($30\frac{1}{2}$ in. diameter of saw)	4·12
Plate bending rolls (diameter of rolls 13 in., length $9\frac{1}{2}$ ft.)	2·70
Wood planer, $13\frac{1}{2}$ in. (rotary knives, 2 horizontal and 2 vertical knives)	4·24
Wood planer, 24 in. (rotary knives)	3·03
Wood planer, $17\frac{1}{2}$ in. (rotary knives)	4·63
Wood planer, 28 in. (rotary knives)	5·00
Wood planer, 28 in.	3·20
Wood planer and matcher (capacity $14\frac{1}{2}$ in. $\times$ $4\frac{3}{4}$ in.) ...	6·91
Circular saw for wood (23 in. diameter of saw)	3·23
Circular saw for wood (35 in. diameter of saw)	5·64
Band saw for wood (34 in. band wheel)	0·96
Wood-mortising and boring machine	0·49
Horizontal wood-boring and mortising machine (drill 4 in. diameter, mortise $8\frac{1}{2}$ in. deep $\times$ $11\frac{1}{2}$ in. long) ...	3·68
Tenon and mortising machine	2·11
Tenon and mortising machine	2·73
Tenon and mortising machine	2·25
Edge moulder and shaper (vertical spindle)	2·00
Wood-moulding machine (capacity $7\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in. ; horizontal spindle)	2·45
Grindstone for tools, 31 in. diameter, 6 in. face (velocity 680 ft. per minute)	1·55
Grindstone for stock, 42 in. diameter, 12 in. face (velocity 1,680 ft. per minute)	3·11
Grindstone	1·08
Emery wheel, $11\frac{1}{2}$ in. diameter $\times$ $\frac{1}{4}$ in. (saw grinder)	0·56

SPEEDS OF SOME MACHINE TOOLS.

The following approximate data will be found useful in connection with motor driving—

Saws. —Circular saws	(wood)	9000 ft. per min. at rim.		
Band saws	„	4000 ft.	„	„
„	(hot iron and steel)	200 to 300 ft. per min.		
Grindstone ...	...	800 ft. per min. at rim.		
Emery wheels	...	5000 ft.	„	„
Drills	... (for wrought iron)	12 ft. per min. at outer edge.		
„	(„ cast „)	8 ft.	„	„
Milling cutters	(for brass)	120 ft.	„	„
„	(for cast iron)	60 ft.	„	„
„	(for wrought iron)	50 ft.	„	„
„	(for steel)	35 ft.	„	„
Screw-cutting	(gun-metal, etc.)	30 ft.	„	circumfer.
„	(steel)	8 ft.	„	„
Boring	(cast iron)	10 ft.	„	„
Sawing	(wood)	1,500 ft.	„	„
„	(brass)	70 ft.	„	„
„	(gun-metal)	30 ft.	„	„
„	(steel)	25 ft.	„	„
„	(wrought iron)	30 ft.	„	„
„	(cast iron)	20 ft.	„	„

Power wasted in Shafting.—This of course varies enormously with length, etc., but the following average estimates* will serve as a guide—

	Per cent.
Mr. Crompton...	32
Sir T. Richardson	25 to 70
Professor Kennedy	22
Messrs. Furness, Westgarth and Co.'s works	50
Mr. Benjamin	50 to 80
Bristol Wagon Works	22 to 57

The following **line-shafting speeds** will be found most suitable—

Cotton mills	350 revolutions per minute.
Wood shops	250 „ „
Machine shops	125 „ „

* W. Geipel, British Association, 1898 (Bristol). For some more data see Raworth, Manchester Association of Engineers, Oct. 29, 1899.

POWER REQUIRED FOR VENTILATION.

TABLE OF BLACKMAN'S AIR PROPELLER.

Dia- meter. ins.	Revolutions per minute.	With free inlet and exit.		Diameter of pulley. ins.	Width of belt. ins.	De- livery of sur- face. sq. ft.
		Cubic feet of air moved per minute.	Actual H.P. required.			
14	1,000 to 1,500	1,000 to 1,500	$\frac{1}{8}$ to $\frac{1}{4}$	2 $\frac{1}{4}$	1	1·0
18	700 " 1,200	2,000 " 3,000	$\frac{1}{6}$ " "	3	1 $\frac{1}{2}$	1·7
24	500 " 900	3,000 " 6,000	$\frac{1}{4}$ " "	4	2	3·1
30	450 " 750	5,000 " 9,000	$\frac{1}{3}$ " "	5	2 $\frac{1}{2}$	4·9
36	400 " 650	7,700 " 14,000	$\frac{1}{2}$ " "	6 and 7	3	7·0
42	350 " 600	11,500 " 20,000	$\frac{3}{8}$ " "	8	3 $\frac{1}{2}$	9·5
48	300 " 550	13,500 " 30,000	1 " "	9	4	12·5
54	300 " 450	17,000 " 36,000	1 $\frac{1}{4}$ " "	10	4 $\frac{1}{2}$	15·9
60	240 " 400	21,000 " 45,000	1 $\frac{1}{2}$ " "	12	5	19·6
72	200 " 350	30,000 " 60,000	2 " "	14	6	28·2
84	180 " 320	45,000 " 90,000	3 " "	16	8	38·0
96	150 " 280	54,000 " 120,000	4 " "	18	8	50·0
108	140 " 250	68,000 " 144,000	5 " "	20	8	63·6
120	120 " 220	82,000 " 180,000	6 " "	22	8	78·4

The following may be taken as a rough guide as to amount of ventilation required.

Type of room.	Time within which all air contained in room must be replaced.
Large public rooms.....	10 to 20 minutes.
Warehouses	10 to 15 ,,
Factories and work rooms.....	5 to 20 ,,
Shops, offices, halls, and churches	20 minutes.
Engine rooms	1 minute (if hot).
Laundries and drying rooms...	various.
Removing steam	down to 20 seconds.

Each person requires some 45 to 60 cubic feet of air per minute, and each gas-burner equals two or three men.

The following curve (Fig. 10), from tests by W. L. Smith,* of

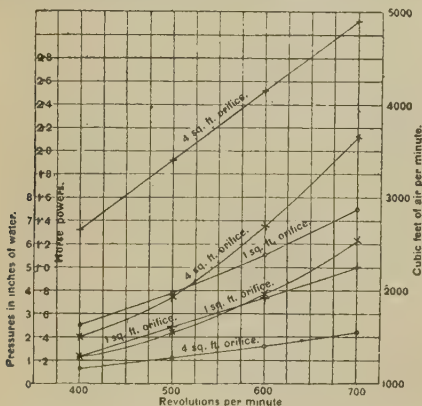


FIG. 10.

Air-Fan Results.

The points + show connection between volume of air and speed of fan.

„ „ × show connection between H.P. and speed of fan.

„ „ ○ air-pressure inside box and speed of fan.

* See *Electrical Review*, May 18, 1900 p. 827.

Boston, U.S.A., may be of interest as showing how the volume of air moved, the power required and the pressure produced vary with the speed of the fan. The fan was a 30-inch Blackman, driven by a $7\frac{1}{2}$ H.P. Thomson-Houston motor, which was first most carefully tested. Tests were made with an opening for discharge of 1 square foot and 4 square feet respectively, the fan being placed in a cubical box 10 feet each way.

CRANES AND LIFTS.

The following **data** relating to cranes and hoists in general will be found useful—

$$\text{Weight lifted} = \frac{P \times 2r \times n}{d} \text{ lbs.,}$$

where P = pull on handle of a hand-crane in pounds,

r = radius of handle in inches,

n = revolutions of handle to 1 of barrel,

d = diameter of barrel in inches,

r usually = 16 inches and d often = 9 inches.

With one man P = 15 to 20 lbs.

A **power-crane** may take 30 to 40 loads per hour, and will slew at 100 to 150 feet per minute.

Travelling and traversing speed, 30 to 60 feet per minute.

Lifting speeds—

Maximum load in tons $2\frac{1}{2}$ 5 10 15 20 30.

Speed in feet per minute at this load . 36 18 9 6 $4\frac{1}{2}$ 3.

Cost of lifting with a power-crane amounts to (say) $\frac{1}{4}d$. per ton lifted, assuming practically continuous use.

Strength of ropes and chains.—Breaking weight of rope in tons = kd^2 where d = diameter in inches, and k has following values—

4-strand tarred Russian hemp hawser laid .63

Iron wire shroud .88

Steel 4.3

Steel 9.7

Maximum working load ranges from $\frac{1}{4}$ to $\frac{1}{5}$ of the above values.

Test load should be 10 % to 25 % greater than maximum working load.

Admiralty **test loads** on chains are—

$18d^2$ tons for “stud link” chains,

$12d^2$ “ “ “open link” “

where d = diameter of iron in inches.

Horse-power required to lift a load of p lbs. at v feet per minute, neglecting friction $= \frac{p \times v}{33,000}$.

For **travelling-cranes**, electric power completely solves the difficulty of finding a satisfactory connection between power supply and crane. At first it was usual to fit a **single motor** to an ordinary crane, using the existing means of regulating the various motions, by means of belts, etc. This is still done, but the present practice seems to favour **three motors**, *i. e.* one to each motion. While, on the one hand, the first cost of the three-motor arrangement is greater, yet, on the other, the cost of repairs is less (owing to the gearing being reduced), and the efficiency is higher, as the motors work more nearly at full load. The question of efficiency is, however, of small importance compared with the greater reliability and ease of handling experienced with a three-motor crane. **Series motors** are usually employed. The **lubrication** should be automatic, and there should be no sparking at the brushes (carbon) from no load to a considerable overload in either direction of rotation. As the load is usually only being moved for a short time, the motors can be rated somewhat high. The speed reduction is usually effected by spur **gearing** and occasionally by worm gearing running in an oil-bath, the thrust being preferably taken up by roller bearings.

Great attention must be paid to the **starting and regulating gear**. Water resistances (see p. 230) have been much used, and have the advantage of being cheap; they require some attention, however. Controllers similar to those on tramcars have been used, but the most satisfactory result is to be expected from controllers designed specially for their work. An automatic switch is often employed for lifts, by means of which when once the circuit has been closed the resistance is cut out automatically as the speed rises, sometimes by means of a solenoid in the main circuit (Otis Co.), or the ordinary centrifugal governor (Siemens and Halske, Ravenshaw). The operation is reversed on switching off. The **controlling rope of a lift** should not have a travel of more than 3 or 4 feet, while the usual **speed** varies from 90 to 100 feet per min., and should not exceed 120 feet per min.

An efficient **brake** must be provided, and is usually of the magnetic type; besides which, in the case of lifts, an emergency brake should be added. Electrical brakes, in which the motor acts as generator, and either returns current to the line, or, better still, sends it through an independent resistance, work most smoothly and allow of the speed regulation being most exact. Many, however, prefer the ordinary brake controlled by the foot or by means of a rope.

The **friction** in a good lift should not exceed 5 %.

The following figures will be of interest—

Tests on Lifts AT LEEDS POST OFFICE.

(CARRIED OUT BY SIR W. H. PREECE.)

Description of Lift.	Weight lifted.	Load lbs.	Cost per trip in pence.	Speed in feet per min.	
				Up.	Down.
No. I. To carry 15 cwt.	12	0	0·01495	37·9	37·9
" " "	12	448	0·0170	42·3	42·3
" " "	12	896	0·0211	37·9	42·3
No. II. " 8 cwt.	51	0	0·0637	82·7	78·4
" " "	51	448	0·0528	80·5	80·5
" " "	51	896	0·0610	76·5	87·4
No. III. " "	51	0	0·0532	82·1	76·1
" " "	51	448	0·0528	82·1	82·1
" " "	51	896	0·0536	78·0	82·1

Messrs. Joseph Adamson & Co., Hyde, give the following figures relative to their electrical **cranes** each having a span of 45 feet—

Lifting capacity of crane		30 tons.	10 tons.	5 tons.
Approximate weight of crane		24 tons.	14 tons.	10 tons.
Hoisting.	{ Hoisting speed per minute, <i>light</i>	12 ft.	24 ft.	4·5 ft.
	{ Ditto, fully loaded	5 ft.	10 ft.	20 ft.
	{ Power absorbed at above speeds, <i>light</i>	7·7 H.P.	5 H.P.	4·4 H.P.
	{ Ditto, fully loaded	17 H.P.	10 H.P.	10 H.P.
Longitudinal travelling.	{ Travelling speed per minute, <i>light</i>	210 ft.	200 ft.	195 ft.
	{ Ditto, fully loaded	150 ft.	150 ft.	150 ft.
	{ Power absorbed at above speeds, <i>light</i>	8 H.P.	4 H.P.	3 H.P.
	{ Ditto, fully loaded	16 H.P.	7 H.P.	5 H.P.
Cross traversing.	{ Traversing speed per minute, <i>light</i>	125 ft.	100 ft.	108 ft.
	{ Ditto, fully loaded	60 ft.	60 ft.	60 ft.
	{ Power absorbed at above speeds, <i>light</i>	4·9 H.P.	1·4 H.P.	1·3 H.P.
	{ Ditto, fully loaded	10 H.P.	4 H.P.	3 H.P.
Hoisting efficiency at full load		58 %	62 %	68 %

As regards **maintenance** of lifts, Ravenshaw puts it down at 5 % per annum, while Crompton has given it as his opinion that the repairs required by an electric lift are less than those for one worked by hydraulic power.

FLY-WHEELS.

The necessary **Weight of Fly-wheel** = $\frac{43257 Pkr}{R^2 n^3}$ tons,

where P = H.P. of engine,

k = ratio of work stored in changing speed from n_1 to n_2
to work done by engine per revolution,

n = mean revolutions per minute.

n_1 = maximum „ „ „

n_2 = minimum „ „ „

$\frac{1}{r}$ = relative fluctuation = $\frac{n_1 - n_2}{n}$,

R = radius of gyration of wheel * in feet.

For single-cylinder condensing **steam-engines** k varies from .16 to .19, according to cut-off; and for non-condensing single-cylinder engines, from .16 to .23. For two-cylinder engines, with cranks at 90° , k will have one-fourth of above values. For ordinary calculation, R may be taken as .85 of external radius of wheel.

r may be taken as follows—

Pumps, punches, shearing machines, etc. 20 to 30.

Looms, paper machines, and machine tools 30 to 40.

Spinning machinery 50 to 100.

Dynamos (according to load) . . . 100 to 200.

In the case of **gas-engines** having an Otto cycle, $k = 1.5$ if no explosion is missed, or $k = 3.5$ if one explosion is missed. For ordinary purposes $r = 20$ for gas-engines. The average **rim velocity** may be taken as 4,200 feet per minute, and should not exceed 5000 for cast-iron rims, or 3000 for built-up rims. In those cases where very great and sudden changes of load are probable, these figures should be reduced.

$$* \text{ "Radius of gyration"} = \sqrt{\frac{\text{moment of inertia}}{\text{mass}}}$$

CONSTANTS AND ELECTRICAL PROPERTIES OF VARIOUS SUBSTANCES.

	Ultimate Tensile Strength, Lbs. per sq. in.	Specific Gravity.	Weight of One Cubic Inch. Lbs.	Specific Resistance at 0° Centigrade (Mil- icrohms per cm. Cube).	Microhms per Inch Cube at 0° Centigrade.	Resistance of Wire, 1 ft. long and .001 dia. Ohms at 0° Centigrade.	Per Cent. Increase of Resistance per Degree Centigrade.	Melting Point, Degrees Centigrade.	Specific Heat, Mean.
Pure Electrolytic (annealed) Silver. Dewar & Fleming.....		10.5	.379	1.47	.579	8.82	.400	950°	.056
Electrolytic Copper (annealed). Dewar & Fleming.....		8.91	.322	1.56	.614	9.35	.428	1050°	.093
Copper (annealed). Matthiessen.....		8.9	.321	1.59	.625	9.54	.388	1050°	.093
Gold (annealed). Matthiessen.....		19.3	.695	2.04	.803	12.3	.365	1100°	.032
Gold, 99.9% (pure). Dewar & Fleming.....		19.3	.695	2.20	.865	13.2	.377	1200°	.032
Aluminum (Neuhausen) 99% Al. Dewar & Fleming.....		2.6	.094	2.56	1.01	15.4	.423	600°	.21
Aluminum (commercial) 97.5% Al. Dewar & Fleming.....		2.6	.094	2.67	1.05	16.0	.435	600°	.21
Magnesium. Dewar & Fleming.....		1.74	.063	4.36	1.72	26.2	.381		.25
Cast Copper.....				4.65	1.83	27.9	.406	415°	.095
Zinc (very pure). Dewar & Fleming.....		7.1	.256	5.75	2.26	34.5	.365	415°	.095
Zinc (compressed). Matthiessen.....		7.1	.256	5.80	2.28	34.8	.247	1775°	.032
Platinum (annealed). Matthiessen.....		1.2	.765	8.98	3.53	53.9	.625		.113
Iron (very pure). Dewar & Fleming.....		7.8	.282	9.07	3.57	54.5	.419		
Cadmium (pure). Dewar & Fleming.....		8.60	.310	10.0	3.93	60.0	.354		
Palladium (pure). Dewar & Fleming.....		21.2	.765	11.0	4.02	61.1	.35	1775°	.032
Platinum (pure) Wire .0259 cm. Diam. Dewar & Fleming.....				10.2	4.34	66.0	.62	1500°	.109
Nickel. Dewar & Fleming.....		8.9	.321	12.3	4.85	73.7	.50	1500°	.109
Nickel (annealed). Matthiessen.....		8.9	.321	12.4	4.89	74.4	.440	230°	.056
Tin (pure). Dewar & Fleming.....		7.3	.264	13.1	5.16	78.5	.365	230°	.056
Tin (compressed). Matthiessen.....		7.3	.264	13.1	5.16	78.5			.113
Wrought-Iron (annealed). Hopkinson.....		7.8	.282	13.8	5.44	82.8	.398		
Thallium (pure). Dewar & Fleming.....				17.6	6.94	108°	.387	330°	.032
Lead (compressed). Matthiessen.....		11.4	.410	19.5	7.68	117°	.411	330°	.032
Lead (pure). Dewar & Fleming.....		11.4	.410	20.4	8.04	123°	.389	440°	.049
Antimony (compressed). Matthiessen.....		6.7	.242	35.2	13.9	211°	.072		.032
Mercury. (Matthiessen).....		13.6	.490	94.3	37.1	566°	.354	260°	.030
Bismuth (compressed). Matthiessen.....		9.8	.354	130°	51.2	780°			
Phosphor-Bronze, Copper, Tin and Phos. Hospitalier.....	64000°	8.9	.321	1.6	.630	9.6	.394		
Phosphor-Bronze, Copper, Tin and Phos. Hospitalier.....	117000°	8.9	.321	5.6	2.20	33.6	.394		
Phosphor-Bronze with 8% Phosphorus. Abbott.....				32.5	12.8	195°			

Ultimate Tensile Strength. Lbs. per sq. in.	Specific Gravity.	Weight of One Cubic Inch. Lbs.	Specific Resistance at 0° Centigrade (Mils. per cm. Cube).	Microhms per Inch Cube at 0° Centigrade.	Resistance of Wire, 1 ft. long and .001 dia. Ohms at 0° Centgrade.	Per Cent. Increase of Resistance per Degree Centigrade.	Melting Point. Degrees Centigrade.	Specific Heat. Mean.
64000.	8.9	.321	1.87	.657	10.0	.152	...	...
93000.	8.9	.321	2.69	1.06	16.2	...	...	...
107000.	8.9	.321	7.80	3.07	34.6	...	...	...
143000.	8.9	.321	8.84	3.48	46.8	.080	...	...
...	...	...	...	...	...	...	...	...
Copper 97% Aluminum 3% Dewar & Fleming.	...	...	...	...	...	...	...	...
Copper 80.5% Manganese 16.5% Nickel 3% Manganite tests by G. E. Co. U.S.A.	8.9	.321	49.0	19.3	53.0	.0	...	...
Copper 65.8% Zinc 34.2% R. Haas.	...	...	6.39	2.48	37.8	.158	...	...
Iron with 25% Mn. and .01% S. Dewar & Fleming.	7.8	.282	10.5	4.14	63.0	.544	...	.113
Bessemer Soft Steel. C. .045; Mn. .200; S. .030; Si. 0; P. .040. Hopkinson.	7.8	.282	10.5	4.14	63.0	...	...	.117
Whitworth Soft Steel (annealed). C. .080; Mn. .153; S. .016; Si. 0; P. .042. Hopkinson.	7.8	.282	10.8	4.25	64.8	...	...	.117
Nickel Steel (Hadfield). 4.35% Nickel. Dewar & Fleming.	...	...	29.5	11.6	177	.201	...	...
Manganese Steel (annealed). C. .674; Mn. .473; S. .023; Si. .608; P. .078. Hopkinson.	7.8	.282	39.3	15.5	236	...	1200	...
Silicon Steel (annealed). C. .685; Mn. .694; S. .024; Si. 3.44; P. .133. Hopkinson.	...	...	61.9	24.3	372	...	...	...
Manganese Steel (Hadfield's "Hecla" Foundry). C. 1.001; Mn. 11.40; P. .068. Tests by G. E. Co. U.S.A.	7.8	.282	69.0	27.1	414	.135	1200	...
Grey Cast-iron. C. 3.46; Graphite 2.06; Mn. .173; S. .042; Si. .204; P. .151. Hopkinson.	7.20	.260	114	44.9	684	...	1250	...
White Cast-iron. C. .204; Graphite 0; Mn. .386; S. .467; Si. .764; P. .488. Hopkinson.	7.20	.260	56.6	22.3	340	...	1120	...
German Silver. Cu. (60); Zn. (25); Ni. (15). Feussener & Lindeck.	...	...	30.0	11.8	180	.036	...	...
Silver 65%; Platinum 33% Dewar & Fleming.	...	...	31.6	12.4	190	.0243	...	...
Platinoid. Martino.	...	...	33.0	13.0	198	.024	...	...
Platinoid. Dewar & Fleming.	8.8	.318	41.7	16.4	251	.031	...	...

STANDARD, BIRMINGHAM, AND AMERICAN (BROWN & SHARP) WIRE GAUGES.

S. W. G.	B. W. G.	B. & S. Wire Gauge.	Diam. Inches.	Diam. Millimetres.	S. W. G.	B. W. G.	B. & S. Wire Gauge.	Diam. Inches.	Diam. Millimetres.
7/0	...	...	·500	12·699	15	15	13	·072	1·828
6 0	...	...	·464	11·785	...	16	...	·065	1·650
...	...	0000	·460	11·683	16	...	14	·064	1·625
...	0000	...	·454	11·531	...	17	...	·058	1·472
5/0	...	...	·432	10·972	...	...	15	·057	1·447
...	000	...	·425	10·794	17	...	...	·056	1·421
...	...	000	·409	10·388	...	...	16	·050	1·270
0000	...	...	·400	10·159	...	18	...	·049	1·244
...	00	...	·380	9·651	18	...	...	·048	1·218
000	...	...	·372	9·448	...	...	17	·045	1·142
...	...	00	·365	9·271	...	19	...	·042	1·066
00	...	...	·348	8·839	19	...	18	·040	1·016
...	0	...	·340	8·635	20	...	19	·036	·9140
...	...	0	·325	8·254	...	20	...	·035	·8826
0	...	...	·324	8·229	21	21	20	·032	·8124
1	1	...	·300	7·620	...	22	...	·030	·7617
...	...	1	·289	7·340	...	...	21	·0284	·7213
...	2	...	·284	7·213	22	...	...	·028	·7109
2	...	...	·276	7·010	...	...	22	·0253	·6126
...	3	...	·259	6·578	...	23	...	·025	·6347
...	...	2	·257	6·527	23	...	...	·024	·6093
3	...	...	·252	6·400	24	24	23	·022	·5585
...	4	...	·238	6·045	25	25	24	·020	·5078
4	...	...	·232	5·892	26	26	25	·018	·4570
...	...	3	·229	5·816	27	27	26	·016	·4062
...	5	...	·220	5·588	28	28	27	·014	·3555
5	...	...	·212	5·384	29	29	...	·013	·3300
...	...	4	·204	5·181	...	...	28	·0122	·3100
...	6	...	·203	5·156	30	30	...	·012	·3046
6	...	...	·192	4·876	31	...	29	·011	·2800
...	...	5	·182	4·622	32	...	...	·0108	·2743
...	7	...	·180	4·571	33	31	30	·010	·2539
7	...	...	·176	4·470	34	32	31	·009	·2300
...	8	...	·165	4·191	35	33	32	·008	·2031
...	...	6	·162	4·114	36	34	33	·007	·1777
8	...	...	·160	4·064	37	...	...	·0068	·1727
...	9	...	·148	3·759	38	...	34	·006	·1523
9	...	7	·144	3·657	...	...	35	·0056	·1422
...	10	...	·134	3·403	39	35	36	·005	·1269
10	...	8	·128	3·251	40	...	...	·0048	·1219
...	11	...	·120	3·047	41	...	37	·0044	·1118
11	...	...	·116	2·946	42	36	38	·004	·1015
...	...	9	·114	2·895	43	...	39	·0036	·0914
...	12	...	·109	2·768	44	...	40	·0032	·0813
12	...	...	·104	2·641	45	...	...	·0028	·0713
...	...	10	·102	2·590	46	...	...	·0024	·0610
...	13	...	·095	2·412	47	...	...	·002	·0507
13	...	...	·092	2·336	48	...	...	·0016	·0406
...	...	11	·090	2·286	49	...	...	·0012	·0305
...	14	...	·083	2·108	50	...	...	·001	·0253
14	...	12	·080	2·032	...	...	...	...	...

PARTICULARS OF COPPER WIRES, B. W. G. (see note, p. 84).
(Messrs. Glover and Co.)

Dimensions, B. W. G.					Length, Weight, and Resistance.				
Birm- ham Wire Gauge.	Diameter.		Area.		Stand- ard pounds pr. yard.	Stand- ard pounds pr. mile.	Stand- ard ohms pr. yard	Birm- ham Wire Gauge.	
	Ins.	Milli- metres.	Sq. ins. = $d^2 \times .7854$.	Sq. Milli- metres.					
4/0	.454	11.53	.1619	104.4	1.872	3295.	.0001485	4/0	
3/0	.425	10.80	.1419	91.54	1.641	2888.	.0001694	3/0	
2/0	.380	9.652	.1134	73.15	1.311	2308.	.0002120	2/0	
1/0	.340	8.636	.09079	58.57	1.050	1848.	.0002648	1/0	
1	.300	7.620	.07069	45.60	.8176	1439.	.0003401	1	
2	.284	7.214	.06335	40.87	.7324	1289.	.0003795	2	
3	.259	6.579	.05269	33.99	.6091	1072.	.0004563	3	
4	.238	6.045	.04449	28.70	.5144	905.4	.0005403	4	
5	.220	5.588	.03801	24.52	.4395	773.5	.0006325	5	
6	.203	5.156	.03237	20.88	.3743	658.7	.0007427	6	
7	.180	4.572	.02545	16.42	.2943	517.9	.0009446	7	
8	.165	4.191	.02138	13.79	.2472	435.1	.001124	8	
9	.148	3.759	.01720	11.10	.1989	350.0	.001398	9	
10	.134	3.404	.01410	9.096	.1630	286.9	.001705	10	
11	.120	3.048	.01131	7.296	.1308	230.2	.002126	11	
12	.109	2.769	.009331	6.019	.1079	189.9	.002576	12	
13	.095	2.413	.007088	4.572	.08193	144.2	.003392	13	
14	.083	2.108	.005411	3.491	.06256	110.1	.004443	14	
15	.072	1.829	.004072	2.627	.04709	82.87	.005904	15	
16	.065	1.651	.003318	2.140	.03836	67.52	.007245	16	
17	.058	1.473	.002642	1.704	.03055	53.76	.009099	17	
18	.049	1.245	.001886	1.217	.01897	33.38	.01275	18	
19	.042	1.067	.001385	.8935	.01601	28.18	.01736	19	
20	.035	.8890	.0009621	.6207	.01113	19.58	.02499	20	
21	.032	.8128	.0008042	.5188	.009301	16.37	.02989	21	
22	.028	.7112	.0006158	.3973	.007119	12.53	.03904	22	
23	.025	.6350	.0004909	.3167	.005676	9.990	.04897	23	
24	.022	.5588	.0003801	.2452	.004395	7.735	.06325	24	
25	.020	.5080	.0003142	.2027	.003633	6.394	.07651	25	
26	.018	.4572	.0002545	.1642	.002943	5.179	.09446	26	
27	.016	.4064	.0002011	.1297	.002325	4.092	.1195	27	
28	.014	.3556	.0001539	.09928	.001780	3.132	.1562	28	
29	.013	.3302	.0001327	.08560	.001534	2.700	.1812	29	
30	.012	.3048	.0001131	.07296	.001308	2.302	.2126	30	

PARTICULARS OF COPPER WIRES. S. W. G. (see note, p. 84). (Messrs. Glover and Co.)

Dimensions, S. W. G.				Length and Weight.			Length and Resistance.					
Legal Stand-ard Gauge.	Diameter.		Area.		Amperes at 1000 per sq. in. loss is 2½ volts. per 100 yards.	Amperes at I. E. F. Stand-ard Current = 2.6 × (Area) ^{1/2}	Pounds per yard.	Pounds per mile.	Pounds per kilometre.	Ohms per yard.	Ohms per mile.	Ohms per kilo-metre.
	Ins.	Milli-metres.	Square ins. = d ² × .7854.	Square Millimetres.								
1 sq. in.	1.128	28.65	1 sq. inch	645.1	1000.	749.9	11.56	20350.	12650.	.00002405	.04232	.02630
7/0	.500	12.70	.1963	126.6	196.3	197.3	2.270	3995.	2482.	.0001225	.2156	.1340
6/0	.464	11.79	.1691	109.1	169.1	174.6	1.955	3441.	2138.	.0001432	.2503	.1555
5/0	.432	10.97	.1466	94.57	146.6	155.3	1.695	2983.	1854.	.0001640	.2887	.1794
4/0	.400	10.16	.1257	81.09	125.7	136.9	1.453	2558.	1590.	.0001913	.3367	.2092
3/0	.372	9.449	.1087	70.12	108.7	121.5	1.257	2212.	1375.	.0002212	.3893	.2419
2/0	.348	8.839	.09511	61.36	95.11	108.9	1.099	1935.	1202.	.0002523	.4450	.2765
1/0	.324	8.230	.08245	53.19	82.45	96.90	.9534	1678.	1043.	.0002917	.5133	.3193
1	.300	7.620	.07069	45.60	70.69	85.41	.8176	1439.	894.2	.0003402	.5987	.3720
2	.276	7.010	.05983	38.60	59.83	74.49	.6921	1218.	756.9	.0004019	.7073	.4395
3	.252	6.401	.04988	32.13	49.88	64.17	.5767	1015.	630.7	.0004321	.8484	.5272
4	.232	5.893	.04227	27.27	42.27	56.02	.4888	860.2	534.5	.0005688	1.001	.6220
5	.212	5.385	.03530	22.77	35.30	48.33	.4082	718.4	446.4	.0006313	1.199	.7451
6	.192	4.877	.02895	18.68	28.95	41.07	.3347	589.1	366.1	.0008307	1.462	.9085
7	.176	4.470	.02433	15.70	24.33	35.61	.2813	495.1	307.7	.0009381	1.739	1.081
8	.160	4.064	.02011	12.97	20.11	30.46	.2325	409.2	254.3	.001195	2.104	1.307
9	.144	3.658	.01629	10.51	16.29	25.63	.1884	331.5	206.0	.001476	2.598	1.614
10	.128	3.251	.01287	8.302	12.87	21.13	.1488	261.9	162.7	.001868	3.288	2.043
11	.116	2.946	.01057	6.819	10.57	17.98	.1222	215.1	133.7	.002275	4.004	2.488
12	.104	2.642	.008495	5.480	8.495	15.03	.09824	172.9	107.4	.002831	4.982	3.096
13	.092	2.337	.006648	4.289	6.648	12.29	.07688	135.3	84.08	.003617	6.366	3.956
14	.080	2.032	.005027	3.243	5.027	9.775	.05813	102.3	63.57	.004784	8.419	5.232
15	.072	1.829	.004072	2.627	4.072	8.223	.04709	82.87	51.50	.005904	10.89	6.456
16	.064	1.626	.003217	2.075	3.217	6.779	.03720	65.47	40.68	.007478	13.16	8.178
17	.056	1.422	.002463	1.589	2.463	5.445	.02848	50.12	31.14	.009762	17.13	10.68

18	1.219	0.01810	1.168	1.810	4.230	0.2093	36.83	22.89	0.1228	23.38	14.53
19	1.016	0.01257	8109	1.257	3.137	0.1458	25.58	15.90	0.0913	33.67	20.92
20	0.86	0.01018	6567	1.018	2.638	0.1177	20.72	12.88	0.07302	41.57	25.83
21	0.828	0.008042	5188	0.8042	2.142	0.09301	16.37	10.17	0.05990	52.62	32.70
22	0.712	0.006158	3973	0.618	1.747	0.07120	12.53	7.786	0.04905	68.72	42.70
23	0.606	0.004524	2918	0.4524		0.05231	9.206	5.721	0.03905	93.55	58.13
24	0.558	0.003801	2452	0.3801		0.04395	7.735	4.807	0.03224	111.3	69.16
25	0.508	0.003142	2027	0.3142		0.03633	6.394	3.973	0.02654	134.7	83.70
26	0.4572	0.002545	1642	0.2545		0.02943	5.179	3.218	0.02194	166.3	103.3
27	0.4166	0.002112	1362	0.2112		0.02442	4.298	2.671	0.189	200.4	124.5
28	0.3759	0.001720	1110	0.1720		0.01989	3.500	2.175	0.158	246.0	152.9
29	0.3454	0.001453	09373	0.1453		0.01680	2.957	1.837	0.132	291.3	181.0
30	0.3150	0.001208	07793	0.1208		0.01397	2.458	1.527	0.109	350.3	217.7
31	0.2946	0.001057	06819	0.1057		0.01222	2.151	1.337	0.090	400.4	248.8
32	0.2743	0.0009161	05910	0.09161		0.01059	1.864	1.158	0.080	462.0	287.1
33	0.2540	0.0007854	05067	0.07854		0.009080	1.598	0.930	0.0620	538.8	334.8
34	0.2337	0.0006648	04289	0.06648		0.007688	1.353	0.808	0.0501	636.6	395.6
35	0.2134	0.0005542	03575	0.05542		0.006409	1.128	0.709	0.0439	763.6	474.5
36	0.1980	0.0004536	02926	0.04536		0.005245	0.9231	0.5736	0.0362	933.0	579.8
37	0.1727	0.0003632	02343	0.03632		0.004200	0.7391	0.4593	0.0306	1165.	723.9
38	0.1524	0.0002827	01824	0.02827		0.003269	0.5753	0.3575	0.0258	1497.	930.2
39	0.1321	0.0002124	01370	0.02124		0.002456	0.4322	0.2686	0.0206	1992.	1238.
40	0.1219	0.0001810	01188	0.01810		0.002093	0.3683	0.2289	0.0177	2338.	1453.
41	0.1118	0.0001521	009812	0.01521		0.001759	0.3095	0.1923	0.0153	2782.	1720.
42	0.1016	0.0001257	008109	0.01257		0.001453	0.2558	0.1590	0.0130	3367.	2092.
43	0.9144	0.0001018	006567	0.01018		0.001177	0.2072	0.1288	0.0113	4157.	2583.
44	0.8128	0.00008042	005188	0.008042		0.0009301	0.1637	0.1017	0.0090	5262.	3270.
45	0.7112	0.00006158	003973	0.006158		0.0007120	0.1253	0.0786	0.0073	6872.	4270.
46	0.6096	0.00004524	002918	0.004524		0.0005231	0.09206	0.05721	0.0059	9355.	5813.
47	0.5080	0.00003141	002026	0.003141		0.0003632	0.06392	0.03972	0.0043	13470.	8370.
48	0.4064	0.00002011	001297	0.002011		0.0002325	0.04092	0.02543	0.0036	21040.	13070.
49	0.3048	0.00001181	0007296	0.001181		0.0001308	0.02302	0.01430	0.0026	37420.	23250.
50	0.2540	0.000007854	0005067	0.0007854		0.00009080	0.01598	0.00930	0.0015	53880.	33480.
1 sq. cm.	0.4442	0.1550	100.0	155.0	162.6	1.792	315.4	1960.	0.001551	2730	1694

PARTICULARS OF MAIN CABLES FOR TOWN LIGHTING, ETC. (Messrs. Glover and Co.)

Effective Area in square inches.	Number of Strands and Size.	DIAMETER				AREA OF EACH WIRE		Effective Area of the Strand in square milli-metres.	AMPERES.		RESISTANCE OF STRAND IN OHMS.		WEIGHT OF STRAND IN POUNDS.	
		Of each Wire		Of the Strand		In square inches.	In square milli-metres.		At 1000 per sq. inch, loss = 2½ volts per 100 yds.	At I. E. E. standard current = 2½ (Area) .82	Per mile.	Per 1000 yards.	Per mile.	Per 1000 yards.
				In inches.	In milli-metres.	In inches.	In milli-metres.							
.05	19/.05822	.05822	1.479	.2911	7.394	.002662	1.717	32.25	50.00	64.29	.8464	.4960	1041.	591.5
.075	19/.07130	.07130	1.811	.3565	9.055	.003992	2.575	48.38	75.00	89.64	.5643	.3206	1561.	886.9
.1	19/.08234	.08234	2.091	.4117	10.46	.005324	3.435	64.51	100.0	113.5	.4232	.2405	2082.	1183.
.15	37/.07230	.07230	1.836	.5061	12.85	.004105	2.648	96.76	150.0	158.3	.2821	.1603	3128.	1777.
.2	37/.08344	.08344	2.119	.5841	14.84	.005470	3.529	129.0	200.0	200.4	.2116	.1202	4168.	2368.
.25	37/.09330	.09330	2.370	.6531	16.59	.006837	4.411	161.3	250.0	240.6	.1693	.09619	5209.	2960.
.3	37/.1022	.1022	2.595	.7154	18.17	.008203	5.292	193.5	300.0	279.4	.1411	.08017	6250.	3551.
.35	61/.08598	.08598	2.184	.7738	19.65	.005807	3.746	225.8	350.0	317.1	.1209	.06869	7294.	4144.
.4	61/.09193	.09193	2.335	.8274	21.02	.006636	4.281	258.0	400.0	353.8	.1058	.06011	8335.	4736.
.45	61/.09750	.09750	2.476	.8775	22.29	.007466	4.816	290.3	450.0	389.6	.09404	.05343	9377.	5328.
.5	61/.1028	.1028	2.611	.9252	23.50	.008295	5.351	326.6	500.0	424.8	.08464	.04809	10420.	5920.
.6	91/.09218	.09218	2.341	1.014	25.76	.006672	4.304	387.1	600.0	493.3	.07053	.04007	12500.	7102.
.7	91/.09957	.09957	2.529	1.095	27.81	.007784	5.021	451.6	700.0	559.7	.06046	.03435	14590.	8290.
.75	91/.1030	.1030	2.616	1.133	28.78	.008339	5.379	483.8	750.0	592.3	.05643	.03206	15630.	8881.
.8	91/.1064	.1064	2.703	1.170	29.72	.008896	5.739	516.1	800.0	624.5	.05290	.03006	16670.	9472.
.9	91/.1129	.1129	2.868	1.242	31.55	.01001	6.457	580.6	900.0	687.8	.04702	.02672	18760.	10660.
1.0	91/.1190	.1190	3.023	1.309	33.25	.01112	7.174	645.1	1000.	749.9	.04282	.02405	20840.	11840.

No. of Strands and Size. S. W. G.	Amperes at 1000 per sq. in. of solid conductr. Loss 2½ volts per 100 yards.	Amperes at I.E.E. stand- dard, viz. 26 × area of strand, equiv.	Dmtr. of each wire. Ins.	Diameter of the Strand.		Effective Area, viz. area of solid wire having same conductivity.		Standard Resistance at 60° F. Standard Ohms.			Standard Weight.		
				Ins.	Mllms.	Sq. Ins.	Sq. Mllm.	Pr. 1000 Yards.	Per Mile.	Per Kilom.	Pr. 1000 Yards. In lbs.	Pr. Mile. In lbs.	Pr. Klm. In lbs.
3/25	9311	2.452	.020	.043	1.092	.0009311	.6007	25.82	45.45	28.24	11.04	19.42	12.07
3/24	1.127	2.868	.022	.047	1.194	.001127	.7271	21.33	37.55	23.33	13.35	23.49	14.60
3/23	1.341	3.307	.024	.052	1.321	.001341	.8651	17.94	31.57	19.62	15.89	27.96	17.37
3/22	1.825	4.258	.028	.060	1.524	.001825	1.177	13.18	23.19	14.41	21.62	38.05	23.64
3/21	2.384	5.301	.032	.069	1.753	.002384	1.538	10.09	17.75	11.03	28.24	49.71	30.89
3/20	3.016	6.444	.036	.078	1.981	.003016	1.946	7.972	14.03	8.718	35.75	62.92	39.10
3/19	3.725	7.644	.040	.086	2.184	.003725	2.403	6.455	11.36	7.059	44.14	77.68	48.27
3/18	5.364	10.31	.048	.104	2.616	.005364	3.461	4.432	7.889	4.902	63.52	111.8	69.47
7/25	2.177	4.921	.020	.060	1.524	.002177	1.404	11.05	19.44	12.08	25.70	45.23	28.11
7/24	2.633	5.751	.022	.066	1.676	.002633	1.699	9.131	16.07	9.986	31.08	54.71	34.00
7/23	3.135	6.636	.024	.072	1.829	.003135	2.023	7.670	13.50	8.389	37.00	65.12	40.47
7/22	4.266	8.543	.028	.084	2.134	.004266	2.752	5.636	9.920	6.164	50.36	88.63	55.07
7/21½	4.896	9.565	.030	.090	2.286	.004896	3.159	4.910	8.643	5.371	57.84	101.8	61.40
7/21	5.571	10.63	.032	.096	2.438	.005571	3.594	4.316	7.596	4.720	65.79	115.8	62.27
7/20½	5.925	11.19	.033	.099	2.515	.005925	3.822	4.059	7.143	4.439	70.00	123.2	70.39
7/20	7.052	12.90	.036	.108	2.743	.007052	4.550	3.410	6.001	3.729	88.30	146.6	91.10
7/19	8.708	15.34	.040	.120	3.048	.008708	5.618	2.761	4.860	3.020	102.8	180.9	112.4
7/18	12.54	20.68	.048	.144	3.658	.01254	8.090	1.918	3.375	2.097	148.0	260.5	161.9
7/17	17.06	26.62	.056	.168	4.267	.01706	11.01	1.410	2.480	1.541	201.4	354.5	220.3
7/16	22.27	33.12	.064	.192	4.877	.02227	14.37	1.080	1.900	1.181	263.1	463.1	287.8
7/15	28.22	40.22	.072	.216	5.486	.02822	18.21	.8523	1.500	.9321	333.1	586.2	364.3
7/14	34.83	47.80	.080	.240	6.096	.03483	22.47	.6903	1.215	.7550	411.1	723.6	449.6
7/13	46.05	60.10	.092	.276	7.010	.04605	29.71	.5222	.9190	.5711	543.8	957.1	594.7
7/12	58.84	73.47	.104	.312	7.925	.05884	37.96	.4086	.7192	.4469	694.9	1223.	759.9
7/11	73.22	87.90	.116	.348	8.839	.07322	47.24	.3284	.5780	.3592	864.8	1522.	945.7
7/10	89.17	103.3	.128	.384	9.754	.08917	57.53	.2637	.4746	.2950	1053.	1853.	1151.
7/9	112.9	125.4	.144	.432	10.97	.1129	72.84	.2131	.3750	.2330	1332.	2345.	1457.

PARTICULARS OF COPPER CABLES (see note, p. 84). (Messrs. Glover and Co.), continued.

No. of Strands and Size. S.W.G.	Amperes at 1000 per sq. in. of solid conductr. Loss 2½ volts per 100 yards.	Amperes at I.E.K. standard, viz. 26 X area sq. in. of solid conductr.	Dmr. of each Wire. Ins.	Diameter of the Strand.		Effective Area, viz. area of solid wire having same conductivity.	Standard Resistance at 60° F. Standard Ohms.				Standard Weight.		
				Ins.	Mllms.		Pr. 1000 Yards.	Per Mile.	Per Kilom.	Pr. 1000 Yards. In lbs.	Pr. Mile. In lbs.	Pr. Klm. In lbs.	
7/8	139.3	149.0	.160	.480	12.19	.1393	89.87	.1726	.3037	.1887	1644.	2894.	1798.
7/6	200.6	200.9	.192	.576	14.63	.2006	129.4	.1199	.2110	.1311	2368.	4167.	2589.
19/24	7.141	13.03	.022	.110	2.794	.007141	4.607	3.367	.5926	3.682	84.49	148.7	92.40
19/23	8.946	15.68	.024	.120	3.048	.008946	5.771	2.830	4.981	3.095	100.5	176.9	109.9
19/22	11.57	19.36	.028	.140	3.556	.01157	7.464	2.079	3.659	2.274	136.8	240.8	149.6
19/21	15.10	24.09	.032	.160	4.064	.01510	9.742	1.592	2.802	1.741	178.7	314.6	195.5
19/20	19.12	29.23	.036	.180	4.572	.01912	12.34	1.257	2.213	1.375	226.3	398.3	247.5
19/19	23.60	34.74	.040	.200	5.080	.02360	15.23	1.019	1.793	1.114	279.4	491.7	305.5
19/18	33.99	46.85	.048	.240	6.096	.03399	21.93	.7074	1.245	.7786	402.2	707.9	439.9
19/17	46.27	60.33	.056	.280	7.112	.04627	29.85	.5197	.9147	.5684	547.3	963.3	598.6
19/16	60.39	75.06	.064	.320	8.128	.06039	38.96	.3981	.7007	.4354	714.8	1258.	781.7
19/15	76.50	91.12	.072	.360	9.144	.07650	49.35	.3143	.5532	.3437	905.1	1593.	989.9
19/14	94.42	108.3	.080	.400	10.16	.09442	60.91	.2547	.4482	.2785	1117.	1966.	1222.
19/13	124.9	136.2	.092	.460	11.68	.1249	80.58	.1926	.3389	.2106	1478.	2601.	1616.
19/12	159.5	166.4	.104	.520	13.21	.1595	102.9	.1507	.2653	.1649	1888.	3323.	2065.
19/11	198.5	199.2	.116	.580	14.73	.1985	128.1	.1211	.2132	.1325	2349.	4134.	2569.
19/10	241.7	234.0	.128	.640	16.26	.2417	155.9	.09949	.1751	.1088	2860.	5034.	3128.
19/9	306.0	284.0	.144	.720	18.29	.3060	197.4	.07854	.1383	.08594	3620.	6372.	3960.
19/8	377.9	337.6	.160	.800	20.32	.3779	243.8	.06364	.1120	.06959	4469.	7865.	4887.
19/7	457.1	394.6	.176	.880	22.35	.4571	294.9	.05261	.09259	.05753	5407.	9516.	5913.
37/24	13.83	22.48	.022	.154	3.912	.01388	8.955	1.729	3.044	1.891	164.5	289.6	180.0
37/23	16.54	25.95	.024	.168	4.267	.01654	10.67	1.454	2.559	1.590	195.9	344.7	214.2
37/22	22.52	33.43	.028	.196	4.978	.02252	14.53	1.063	1.879	1.168	266.5	469.1	291.5
37/21	29.41	41.61	.032	.224	5.690	.02941	18.97	.8176	1.439	.8941	348.2	612.9	380.9
37/20	37.22	50.47	.036	.252	6.401	.03722	24.01	.6460	1.137	.7065	440.8	775.8	482.1
37/19	45.96	61.07	.040	.280	7.112	.04596	29.65	.5232	.9203	.5722	544.3	957.9	595.2
37/18	66.19	80.91	.048	.336	8.534	.06619	42.70	.3633	.6394	.3973	733.5	1379.	856.9
37/17	90.06	104.2	.056	.392	9.957	.09006	58.10	.2670	.4699	.2920	1066.	1877.	1166.

37/16	117.6	129.6	.064	.448	11.38	.1176	75.87	.2045	.3599	.2236	1393.	2451.	1523.
37/15	148.9	157.3	.072	.504	12.80	.1489	96.06	.1615	.2842	.1766	1763.	3103.	1928.
37/14	183.8	187.0	.080	.560	14.22	.1838	118.6	.1809	.2303	.1431	2176.	3830.	2380.
37/13	243.1	235.2	.092	.644	16.36	.2431	156.8	.09892	.1741	.1082	2878.	5066.	3148.
37/12	310.5	287.4	.104	.728	18.49	.3105	200.3	.07744	.1363	.08469	3678.	6474.	4023.
37/11	386.5	343.9	.116	.812	20.62	.3865	249.3	.06222	.1095	.06804	4570.	8044.	4399.
37/10	470.6	404.2	.128	.896	22.76	.4706	303.6	.05109	.08992	.05587	5566.	9796.	6087.
37/9	595.5	490.2	.144	1.008	25.60	.5955	384.2	.04037	.07105	.04415	7047.	12402.	7707.
37/8	735.5	582.9	.160	1.120	28.44	.7355	474.5	.03269	.05154	.03575	8699.	15311.	9514.
61/24	22.91	33.89	.022	.198	5.029	.02291	14.78	1.049	1.847	1.148	271.3	477.5	296.7
61/23	27.27	39.11	.024	.216	5.486	.02727	17.59	.8818	1.552	.9644	322.9	563.8	353.1
61/22	37.12	50.36	.028	.252	6.401	.03712	23.95	.6477	1.140	.7084	439.5	773.5	480.7
61/21	48.48	62.69	.032	.288	7.315	.04848	31.28	.5426	.8730	.5425	574.4	1011.	628.2
61/20	61.36	76.05	.036	.324	8.230	.06136	39.59	.3919	.6897	.4286	726.7	1279.	794.8
61/19	75.76	90.40	.040	.360	9.144	.07576	48.88	.3174	.5.86	.3471	897.2	1579.	981.2
61/18	109.1	121.9	.048	.432	10.97	.1091	70.38	.2204	.3879	.2410	1292.	2274.	1413.
61/17	148.5	157.0	.056	.504	12.80	.1485	95.80	.1619	.2850	.1770	1758.	3094.	1923.
61/16	193.9	195.4	.064	.576	14.63	.1939	125.1	.1240	.2183	.1356	2297.	4042.	2512.
61/15	245.5	237.0	.072	.648	16.46	.2455	158.4	.09795	.1724	.1071	2907.	5116.	3055.
61/14	302.9	281.6	.080	.720	18.29	.3029	195.4	.07937	.1397	.08681	3389.	6316.	3925.
61/13	400.8	354.3	.092	.828	21.03	.4003	258.6	.06000	.1056	.05562	4746.	8353.	5191.
61/12	512.0	433.1	.104	.936	23.77	.5120	330.3	.04697	.08266	.05136	6065.	1067.4	6633.
61/11	637.1	518.2	.116	1.044	26.52	.6371	411.0	.03774	.06543	.04128	7545.	13279.	8252.
61/10	775.8	609.0	.128	1.152	29.26	.7758	500.5	.03099	.05455	.03390	9187.	16169.	10050.
91/18	162.8	169.3	.048	.528	13.41	.1628	105.0	.1477	.2600	.1616	1927.	3392.	2108.
91/17	221.5	217.9	.056	.616	15.65	.2215	142.9	.1055	.1911	.1187	2623.	4616.	2868.
91/16	289.1	271.0	.064	.704	17.83	.2891	186.5	.08318	.1464	.09097	3426.	6080.	3747.
91/15	366.1	329.0	.072	.792	20.12	.3661	236.2	.06568	.1156	.07183	4337.	7683.	4743.
91/14	451.9	391.0	.080	.880	22.35	.4519	291.5	.05320	.09364	.05319	5333.	9422.	5855.
91/13	597.7	491.7	.092	1.012	25.07	.5977	385.6	.04023	.07080	.03400	7081.	12462.	7744.
91/12	763.8	600.1	.104	1.144	29.06	.7638	492.8	.03148	.05541	.03443	9048.	15925.	9896.
91/11	950.4	719.8	.116	1.276	32.41	.9504	613.1	.02530	.04453	.02767	11253.	19811.	12310.
1 sq. in.	1000	749.9	1.128	1.128	28.65	1 sq. in.	645.1	.02405	.04232	.02680	11560.	20850.	12950.
1 sq. cent.	155.0	162.6	.4442	.4442	11.28	.1550	100.0	.1551	.2730	.1694	1792.	3154.	1960.

NOTE ON THE FOREGOING TABLES.

The above tables have been calculated from the formulæ adopted by the Committee on Copper Conductors in May 1899. This Committee consisted of representatives from the General Post Office, the Institution of Electrical Engineers, and the eight principal manufacturers of Rubber Insulated Cables in the United Kingdom.

SOLID WIRES.

The formulæ are as follow —

Copper weighs 555 lbs. per cubic foot at 60° F. Specific gravity = 8·912.

The resistance of annealed high-conductivity commercial copper is—

Resistance per cubic inch ...	...	·00000066788	standard ohms.
Resistance per cubic cm. ...	...	·00000169639	" "
Resistance of 100' weighing 100 grains		·150158	" "

Resistance per mile ...	...	=	·042317
			Area in square inches.

Resistance per yard ...	...	=	·0000244044
			Area in square inches.

Resistance per mil. foot ...	...	=	10·2044 standard ohms.
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The resistance of hard-drawn high-conductivity commercial copper is—

Resistance per cubic inch ...	=	·000000681327	standard ohms.
Resistance per cubic cm. ...	=	·00000173054	" "
Resistance of 100' weighing 100 grains	=	·153181	" "

Resistance per mile ...	...	=	·0431689
			Area in square inches.

Resistance per yard ...	...	=	·0000245277
			Area in square inches.

Resistance per mil. foot ...	...	=	10·4099 standard ohms.
------------------------------	-----	---	------------------------

Weight per mile = 20350 × area in square inches.

Weight per yard = 11·5625 × area in square inches.

CABLES.

A lay of twenty times the pitch diameter is adopted as a standard, and the resistance in parallel of the wires is taken as the resistance of the cable.

Resistance of three-strand cable = 3·3742 × resistance of each wire.

" four "	·253065	" "
" seven "	·1443557	" "
" twelve "	·085355	" "
" nineteen "	·0532424	" "
" thirty-seven "	·0273493	" "
" sixty-one "	·0165911	" "
" ninety-one "	·0111222	" "

Weight of three-strand cable = 3·03678 × weight of each wire.

" four "	4·04904	" "
" seven "	7·07356	" "
" twelve "	12·1471	" "
" nineteen "	19·2207	" "
" thirty-seven "	37·4414	" "
" sixty-one "	61·7356	" "
" ninety-one "	92·1034	" "

The formulæ have been carried to six significant figures for reference, but the tables have been calculated by using four significant figures, as being sufficiently accurate for practical use.

The above formulæ give the standards, but a variation of 2% in resistance or weight is allowed for losses in manufacture.

TINNED WIRE.

An allowance of 1% increased resistance is made on tinned copper wires between Nos. 22 and 12 inclusive.

TEMPERATURE COEFFICIENT.

The determination of Messrs. Clark, Forde & Taylor, given in their pamphlet dated February 20, 1899, is adopted as a standard, and the average coefficient of .00238 per degree Fahrenheit, or .00428 per degree Centigrade, is taken for commercial use.

The effective area of a stranded cable is the area of a solid wire of equal resistance.

One mil.	...	...	...	= .001 inch.
Area in square inches	...	...	...	= $d^2 \times .7854$.
Circular mil.	...	...	...	= d^2 (in mils.).
One knot or nautical mile per hour	* = 1.1528 statute miles per hour.			

* See p. 5.

LAMP-CARRYING CAPACITY, ETC., OF SOME ELECTRIC
LIGHT CABLES.

Size of wire or strand. S.W.G.	Current at 1000 amp. per sq. in.	Number of 8 c.p. 30 watt lamps at 100 volts. ¹	Number of 16 c.p. 60 watt lamps at 100 volts. ¹	Number of 16 c.p. 60 watt lamps at 220 volts. ²	Width of wood casing.	Width of grooves.
					INCH.	INCH.
22	·6	2	1	2	1	$\frac{3}{16}$
20	1·0	4	2	4	"	"
18	1·8	6	3	7	$1\frac{1}{4}$	"
3/22	1·9	6	3	7	"	"
16	3·2	10	5	11	$1\frac{1}{2}$	$\frac{1}{4}$
7/22	4·4	16	8	18	"	"
14	5·0	18	9	20	$1\frac{3}{4}$	$\frac{5}{16}$
3/18	5·6	20	10	22	"	"
7/20	7·2	24	12	26	"	"
12	8·5	28	14	30	"	"
7/18	12·8	42	21	44	2	$\frac{3}{8}$
19/20	19·7	66	33	70	$2\frac{1}{2}$	$\frac{1}{2}$
7/16	22·9	76	38	80	"	"
19/18	35·0	116	58	120	3	$\frac{5}{8}$
7/14	35·8	120	60	125	"	"
7/12	60·6	196	98	200	"	"
19/16	62·4	200	100	205	$3\frac{1}{2}$	$\frac{3}{4}$
19/14	97·5	320	160	326	4	$\frac{7}{8}$
37/16	122·0	400	200	408	$4\frac{1}{2}$	"
19/12	164·7	540	270	550	5	1
37/14	190·9	620	310	630	"	"
61/16	201·3	660	330	670	$5\frac{1}{2}$	$1\frac{1}{4}$
37/12	322·0	1000	500	1050	"	"
61/12	531·6	1750	875	1800	6	$1\frac{1}{2}$

RISE OF TEMPERATURE OF WIRES DUE TO SUDDEN CURRENT
RUSH.

When radiation can be neglected, rise of temperature per second—

$$\theta = k \left(\frac{C}{A} \right)^2$$

when C = current in amperes, A = sectional area in sq. mms., and k has the following values—

Copper	...	...	...	·005
Iron	...	...	...	·0158
German silver	...	...	...	·08

¹ 50 volt circuits will take half the number of lamps given for 100 volts.
² 110 volt circuits will take half the number of lamps given for 220 volts.

TABLE OF SPECIFIC INDUCTIVE CAPACITIES.

Material.	Specific inductive capacity.	Observer.
Flint glass, light, density 3.2	6.72	J. Hopkinson
„ dense, density 3.66	7.38	J. Hopkinson
„ ex. dense, density 4.5	9.90	J. Hopkinson
Crown glass, hard, density 2.485	6.96	J. Hopkinson
Plate glass	8.45	J. Hopkinson
White mirror glass.....	5.83	Schiller
Paraffin wax	1.977	{ Gibson and Barclay Wüllner Boltzman Schiller
„	1.96	
„	2.32	
„	1.63 to 1.92	
„	2.19 to 2.34	
Indiarubber, pure	2.12	Schiller
„ vulcanized	2.69	Schiller
Resin	2.55	Boltzman
Ebonite	2.21 to 2.76	Schiller
„	3.15	Boltzman
Sulphur	2.88 to 3.21	Wüllner
„	3.84	Boltzman
Shellac	3.15	Boltzman
Gutta-percha	4.2	Faraday
„	3.84	Boltzman
Mica	5.	{ Faraday and Boltzman
Pitch	1.8	
Petroleum, oil, Field's	2.07	Faraday
„ „ common	2.10	J. Hopkinson
Turpentine, commercial	2.23	J. Hopkinson
„ oil of, at 17.1° C.	1.94	Quincke
Castor oil.....	4.78	J. Hopkinson
Sperm oil.....	3.02	J. Hopkinson
Ozokerite oil	2.16	J. Hopkinson
Benzine	2.20	Silow
Bisulphide of carbon at about 11° C.	1.97 to 2.22	Quincke
Water at 14° C.	83.8	Tereschin
„ 25° C.	75.7	E. B. Rosa
Air at about 0.001 mm. pressure	0.94	Ayrton
Air at about 5 mm. pressure...	0.9985	Ayrton
„ „	0.9994	Boltzman
Hydrogen at about 760 mm. pressure	0.9997	Boltzman

TABLE OF SPECIFIC INDUCTIVE CAPACITIES (*continued*).

Material.	Specific inductive capacity.	Observer.
Carbon dioxide at about 760 mm. pressure	1·0004	Boltzman
Olefiant gas at about 760 mm. pressure	1·0007	Boltzman
Sulphur dioxide at about 760 mm. pressure	1·0037	Ayrton

INSULATION RESISTANCES AFTER SOME MINUTES' ELECTRIFICATION (FROM VARIOUS SOURCES).¹

Material.	Megs. per c.c. ÷ 10 ⁶ .
Benzine	·0014
Distilled water (ordinary)	7
Mica	4
Ordinary paper.....	·05
Parchment	·03
Glass flint.....	20
„ ordinary.....	9
Paraffin wax, solid	3,000
„ liquid.....	110
Shellac	1,500
Vulcanized fibre, black	68
„ „ red	10
„ „ white	14
Ebonite	4,000
Micanite	2,490
„ cloth	310
„ paper.....	1,240
Gutta-percha.....	450
Wood, untreated	50
„ paraffined	300
Rubber, ordinary	800
„ high insn.	1,400
Crude ozokerite	450
Asbestos (v. variable)	·16
“Press-spahn”.....	·011
Atmospheric air (with low voltage).....	Inf.

¹ For breaking down voltages, see p. 96.

NOTE ON PRECEDING TABLE.

The insulation resistance of ordinary paper depends much on temperature, but still more on damp. The same applies to asbestos (insulation in one case rising from 400 to 160,000 after being heated to redness). Slate will stand 5000 volts per inch of thickness, and is much improved (as is also wood) by being boiled in paraffin wax. To thoroughly impregnate wood or slate the paraffin should be absorbed *in vacuo*. Vulcanized fibre will stand 10,000 volts per inch of thickness, but in this case, as with all such materials, the insulation cannot be taken as proportional to the thickness, (1) on account of possible irregularities, and (2) owing to the difficulty of drying the interior. Hard rubber (ebonite) is a good insulator, but softens at 70° C. In the case of gutta-percha—

$$R_t = (.875)^{t-15} \times R_{15}$$

R_t = resistance at t° C., and R_{15} = resistance at 15° C.

FUSES.

Preece gives the formula—

Fusing amperes = $\sqrt{d^3} \times k$, when d = diameter of wire and k has the following values according as diameter is measured in inches or millimetres.

	Values of k .	
	Diameter in inches.	Diameter in mm.
Copper	10,244	80
Aluminium	7,585	59.2
German silver (platinum and platinoid are practically the same)	5,230	40.8
Iron	3,148	24.6
Tin	1,642	12.8
Lead	1,379	10.8
Alloy of lead and tin (2:1)	1,318	10.3

While Preece's figures and table (p. 93) may be taken as a rough guide, careful experiments made by subsequent observers have shown that so many variables enter into the question that for accurate results considerable modification is necessary.

In a paper read before the Institution of Electrical Engineers¹ Professor Schwartz and Mr. James record a careful series of tests

¹ *Journ. of Inst. E. E.*, vol. xxxv. No. 174, p. 364, of July 1905.

on various fuses, and may be said to arrive at the following **conclusions**.

No formula of the form "fusing amps = $\sqrt{d^3} \times k$ " is possible for any considerable range of fusing currents. They recommend the formula—

$$\text{Fusing amperes} = Kd^n *$$

where K and n have the following values (d being the diameter of the fuse wire in inches)—

			Size S.W.G.	K.	n.
Copper	1-10	amps.	47-33	2,501	1.125
"	10-100	"	33-18	6,537	1.403
Silver	7-70	"	35-18	3,210	1.287
Aluminium	3-50	"	42-22	8,539	1.461
Tin	1-10	"	43-20	421	1.131
"	10-30	"	20-7	819	1.332

The above values hold good for **lengths** exceeding 2 in. up to 10 amps. and exceeding 6 in. up to 100 amps. (If these lengths are halved add 30%.) The wires are **horizontal** (if vertical deduct 6% from the fusing current).

The **minimum "no arc lengths"** for various fuses, *i.e.* the least lengths at which no damage is done to the terminals of the fuse-holder, are shown in Fig. 11. The length of wire necessary to prevent the formation of an arc on an open fuse may be considerably reduced by utilizing the leads to the fuse in such a way as to form a magnetic blow out.

The effect of heavy, as compared with light, terminals is small, say $\pm 5\%$.

The effect of **mechanical injury** is much less than usually assumed. Thus, five blows with a hammer in a length of $3\frac{1}{2}$ in. of No. 33 S.W.G. copper wire only reduced fusing current 6%, and it did not fuse at one of the injured points. A similar wire cut half through only fused at a 20% lower current. In the case of a lead wire fusing at 35 amps. the effect was still less.

In the case of a horizontal copper **strip** not less than $5\frac{1}{2}$ in. long,

$$\text{Fusing amperes} = 40,550b(t + .00355).$$

Where t = thickness and b = breadth of strip in inches a vertical strip fuses at 10% less. For a length of $2\frac{3}{4}$ in. add 11% to the fusing current as found above.

For lead the formula is fusing amperes = $2,060 t^{.74}(b + 0.016)$.

* To work out values with a formula of this form logarithms should be used. See p. 13.

The **normal working current** of a fuse should be taken as (say) 50 % of the fusing current. On this basis for copper wire the **voltage drop** in fuses will vary from $\frac{1}{2}$ volt for a 4-ampere fuse to $\frac{1}{10}$ volt for a 150-ampere fuse—·15 volt may be taken as a fair average for copper or silver and ·06 for tin or lead. The **watts lost** in copper fuse are, on this basis, rather more than double those in tin or lead.

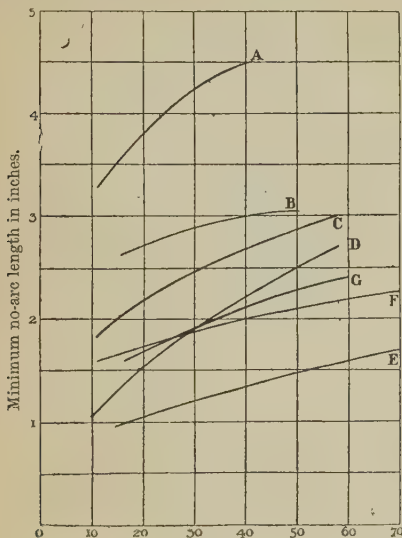


FIG. 11.

Fusing current in amperes.

- A Copper wire free in air.
- B Zinc strip free in air.
- C Copper wire in tubular holder.
- D Copper wire passing through asbestos partition at centre.
- E Copper wire enclosed with Calais sand filling.
- F Copper wire enclosed on inductive circuit.
- G Zinc strip enclosed on non-inductive circuit.

All the above on 220-volt circuit. A to E, non-inductive; F, inductive (0·023 henry).

A copper wire fusing in air at 4 amperes was found to fuse at 18 amperes **under oil**.

Probably apart from the watt consumption copper is the **best metal** to use for fuses, although in Germany silver is extensively used for small fuses.

For open fuses on circuits up to 220 volts a clear **length** of $2\frac{1}{2}$ inches should be allowed up to 20 amperes, and an additional $\frac{1}{2}$ inch for each 15 amperes up to 100.

Enclosed and semi-enclosed fuses are coming more and more into use. The former usually consist of a glass or fibre tube filled completely or partially with incombustible material such as limestone, fire-clay, emery or French chalk.*

In **enclosed fuses** a metal of low melting-point, such as tin or zinc, is usually employed.

Enclosing a fuse may increase the fusing current of the wire or strip by an amount not exceeding 50 % of the normal fusing current in air, the exact increase depending chiefly upon the heat conductivity of the packing employed.

On inductive 220-volt circuits a pressure rise (on breaking the current) of 700 volts is frequently obtained, especially if the filling of the fuse is solidified plaster of Paris.

Even in a lighting circuit a pressure rise may be obtained owing to the inductance of the leads, etc. ; this, however, is usually of short duration and small amount.

A considerable **voltage rise** usually takes place even with direct current, owing to the self-induction of the circuit. Oelschläger† has found this to amount to eight times the supply voltage with a 40-ampere fuse. The circuit opened in .006 secs. and the voltage returned to its normal value in .01 secs.

The tighter the packing in an enclosed fuse, the more quickly will the circuit be ruptured.

The external temperature of an enclosed fuse is about twice as high for zinc as for tin under similar conditions.

Of open and enclosed fuses having equal normal fusing currents, the open fuse will operate the quicker for small overloads, and the enclosed the quicker for large overloads (exceeding, say, 20 % overload).

The current carried by a fuse on a dead **short circuit** usually amounts to at least twenty times the normal fusing current.

* See Sachs, *Trans. Am. Inst. Elec. Eng.*, vol. 17, p. 131, 1900.

† *Elektrotechnische Zeitschrift*, vol. 25, Sept. 1904.

APPROXIMATE FUSING CURRENTS (Sir W. H. Preece).

Current in amperes.	Tin wire.		Lead wire.		Copper wire.		Iron wire.	
	Diameter inches.	Approx. S.W.G.	Diameter inches.	Approx. S.W.G.	Diameter inches.	Approx. S.W.G.	Diameter inches.	Approx. S.W.G.
1	0·0072	36	0·0081	35	0·0021	47	0·0047	40
2	0·0113	31	0·0128	30	0·0034	43	0·0074	36
3	0·0149	28	0·0168	27	0·0044	41	0·0097	33
4	0·0181	26	0·0203	25	0·0053	39	0·0117	31
5	0·0210	25	0·0236	23	0·0062	38	0·0136	29
10	0·0334	21	0·0375	20	0·0098	33	0·0216	24
15	0·0437	19	0·0491	18	0·0129	30	0·0283	22
20	0·0529	17	0·0595	17	0·0156	28	0·0343	20·5
25	0·0614	16	0·0690	15	0·0181	26	0·0398	19
30	0·0694	15	0·0779	14	0·0205	25	0·0450	18·5
35	0·0769	14·5	0·0864	13·5	0·0227	24	0·0498	18
40	0·0840	13·5	0·0944	13	0·0248	23	0·0545	17
45	0·0909	13	0·1021	12	0·0268	22	0·0589	16·5
50	0·0975	12·5	0·1095	11·5	0·0288	22	0·0632	16
60	0·1101	11	0·1237	10	0·0325	21	0·0714	15
70	0·1220	10	0·1371	9·5	0·0360	20	0·0791	14
80	0·1334	9·5	0·1499	8·5	0·0394	19	0·0864	13·5
90	0·1443	9	0·1621	8	0·0426	18·5	0·0935	13
100	0·1548	8·5	0·1739	7	0·0457	18	0·1003	12
120	0·1748	7	0·1964	6	0·0516	17·5	0·1133	11
140	0·1937	6	0·2176	5	0·0572	17	0·1255	10
160	0·2118	5	0·2379	4	0·0625	16	0·1372	9·5
180	0·2291	0	0·2573	3	0·0676	16	0·1484	9
200	0·2457	3·5	0·2760	2	0·0725	15	0·1592	8
250	0·2851	1·5	0·3203	0	0·0841	13·5	0·1848	6·5

FUSE TABLE FOR LEAD STRIP † (Siemens and Halske).

Fusing Amperes.	Breadth of strip. mm.	Length. mm.	Fusing Amperes.	Breadth of strip. mm.	Length. mm.
6	1.0	45	84	14.0	65
9	1.5	"	90	15.0	"
12	2.0	"	96	16.0	"
15	2.5	"	102	17.0	"
18	3.0	"	108	18.0	"
21	3.5	"	114	19.0	"
24	4.0	"	120	20.0	"
27	4.5	"	130	2 of 11.0	"
30	5.0	"	140	2 " 12.0	"
33	5.5	"	150	2 " 13.0	80
36	6.0	"	160	2 " 14.0	"
39	6.5	"	170	2 " 14.5	"
42	7.0	"	180	3 " 10.0	"
45	7.5	65	190	3 " 11.0	"
48	8.0	"	200	3 " 11.5	"
51	8.5	"	210	3 " 12.0	"
54	9.0	"	220	3 " 12.5	"
57	9.5	"	230	3 " 13.0	"
60	10.0	"	240	3 " 13.5	"
66	11.0	"	250	3 " 14.0	"
72	12.0	"	260	3 " 14.5	"
78	13.0	"	270	1 " 15.0	"

RESISTANCE OF ELECTROLYTES.

In the following table the first column shows the amount of salt, etc. dissolved in water, expressed as a percentage.

Under "Ohms" will be found the resistance of a column of liquid 1 decimetre long and 1 square decimetre in area.

Under "Temperature Coefficient" is given the percentage decrease in resistance per 1° C. rise of temperature.

The * shows approximately the point of maximum conductivity, when such exists.

To obtain the corresponding resistance for a length of 1 metre and 1 square millimetre area multiply by 10⁵. Or for 1 inch long and 1 square inch section multiply by 3.94. At one ampere per square inch the drop in volts per inch of length between the electrodes equals the resistance given multiplied by 3.94. (See also p. 232, and Fig. 67.)

† Thickness of strip, 22 to 23 S.W.G.

TABLE OF RESISTANCE OF ELECTROLYTES.¹

Composition %	Nitric Acid HNO ₃ .		Hydrochloric Acid HCl.		Sulphuric Acid H ₂ SO ₄ .		Silver Nitrate AgNO ₃ .		Caustic Potash KOH.		Zinc Sulphate ZnSO ₄ .	
	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.
5	0.390	1.5 %	0.255	1.6 %	0.482	1.2 %	3.93	2.2 %	0.584	1.9 %	5.23	2.2 %
10	0.218	1.4	0.159	1.6	0.257	1.3	2.14	2.2	0.319	1.9	3.14	2.3
15	0.164	1.4	0.135	1.6	0.185	1.4	1.47	2.2	0.236	1.9	2.41	2.3
20	0.141	1.4	0.132*	1.5	0.154	1.5	1.16	2.1	0.201	2.0	2.19	2.4
25	0.131	1.4	0.139	1.5	0.140	1.5	0.950	2.1	0.186	2.1	2.14*	2.6
30	0.128*	1.4	0.152	1.5	0.136*	1.6	0.811	2.1	0.185*	2.3	2.29	3.0
35	0.131	1.4	0.170	1.5	0.139	1.7	0.718	2.1	0.197	2.4	2.85	4.0
40	0.137	1.5	0.195	1.5	0.148	1.8	0.644	2.1	0.223	2.7	—	—
50	0.159	1.6	—	—	0.187	1.9	0.544	2.1	—	—	—	—
60	0.196	1.6	—	—	0.270	2.1	0.480	2.1	—	—	—	—
70	0.254	1.5	—	—	0.466	2.6	—	—	—	—	—	—
80	0.376	1.3	—	—	0.913	3.5	—	—	—	—	—	—
Composition %	Copper Sulphate CuSO ₄ .		Magnesium Sulphate MgSO ₄ .		Sodium Sulphate Na ₂ SO ₄ .		Alum KAl(SO ₄) ₂ .		Sodium Chloride NaCl.		Salammoniac HyNCl.	
	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.	Ohms.	Temp. Coef.
5	5.23	2.2 %	3.93	2.3 %	2.48	2.4 %	3.93	2.0 %	1.49	2.2 %	1.09	2.0 %
10	3.14	2.3	2.41	2.4	1.47	2.5	—	—	0.833	2.1	0.567	1.0
15	2.41	2.3	2.09*	2.5	1.13	2.6	—	—	0.615	2.1	0.389	1.7
20	—	—	2.09	2.7	—	—	—	—	0.514	2.2	0.298	1.6
25	—	—	2.41	2.9	—	—	—	—	0.470	2.3	0.250	1.5

¹ Kohlrausch (*Wiedemanns Annalen*, i. 406, 1833). (See also p. 233.)

SPARK LENGTH AND DIELECTRIC STRENGTH.

By D. K. MORRIS, PH.D.

THE distance depends upon the maximum P.D., and is therefore about the same for an alternating pressure (effective value) as for a continuous voltage 1·41 times as great.

Porcelain insulators for h.t. transmission lines (see p. 396) are generally tested in parallel with a spark gap which serves, either directly or indirectly, to measure the testing voltage. In the first case the spark length is measured and the voltage may then be estimated by the figures below. In the second case (Watson's method *) the gap is shunted by two condensers in series whose outer electrodes are joined through a sensitive galvanometer which indicates the rate of charging, and consequently the voltage if the interval between successive sparks is noted.

TABLE OF STRIKING DISTANCES IN AIR FOR ALTERNATING AND CONTINUOUS PRESSURES.

(Approx. figures : point and plane.)

ALTERNATING.			CONTINUOUS.		
Alternating pressure.	Striking Distance.		Continuous pressure.	Striking Distance.	
Effective volts.	Inches.	Mm.	Volts.	Inches.	Mm.
2,000	·016	·4	2,000	·010	·25
4,000	·050	1·26	4,000	·030	·75
6,000	·105	2·66	6,000	·057	1·45
8,000	·161	4·08	8,000	·092	2·33
10,000	·228	5·78	10,000	·131	3·33
12,000	·309	7·60	12,000	·175	4·44
14,000	{ ·369†	{ 9·37† }	14,000	·223	5·67
	{ ·389‡	{ 9·88‡ }	16,000	·275	7·00
16,000	·462	11·73	18,000	·325	8·30
18,000	·520	13·20	20,000	·380	9·60
20,000	·62	15·7	25,000	·55	14·00
25,000	1·0	25·00	30,000	·75	19·00
30,000	1·6	40·00	50,000	2·05	52·00
50,000	4·0	100·00	70,000	4·0	100·00

* *Jour. Inst. E. E.*, 1906, vol. xxxvii. p. 295.

† See also *Elektrotechnische Zeitschrift*, Sept. 22, 1904, p. 841.

‡ *Jour. Inst. E. E.*, 1892, vol. xxi. p. 180.

The striking distances for alternating pressure up to 14,000 volts are derived from experiments by A. Siemens,* using a point and plane, which arrangement gave the greatest sparking distance in moist air at 12° C. The remaining figures are mostly from results by Steinmetz,† who obtains slightly higher values than Siemens for pressures below 14,000.

The continuous pressure figures are from Siemens' results, which are in agreement with those of De la Rue.

For effect of different electrodes see Watson's paper in 1909, from which also the following figures are derived—

D.C. VOLTAGE TO GIVE 1 MM. SPARK IN
COMPRESSED AIR.

Pressure (absolute).		Volts for 1 mm. spark.
Lbs. per sq. in.	Atmos.	
Atmospheric.	1	4,000
50	$3\frac{1}{3}$	10,000
100	$6\frac{2}{3}$	19,000
150	10	25,500
200	$13\frac{1}{3}$	34,000

The striking distance is much less (from $\frac{1}{5}$ to $\frac{1}{20}$) through rubber insulation, and much less again through good mica. Steinmetz gives ‡ the striking distances through mica as about $\frac{1}{150}$ th part of that in air between the limits of 5000 and 20,000 volts. Walter's§ value is $\frac{1}{50}$ th between 20 and 30,000 volts.

T. Gray|| finds the dielectric strength of certain insulating materials to be as follows—

* *Elektrotechnische Zeitschrift*, 1893, p. 249.

† Dielectric Strength of Compressed Air, *Jour. I. E. E.*, 1909.

‡ *Loc. cit.*

§ *E. T. Z.*, 1903, p. 796; *Sci. Abstr.*, No. 2004 A., 1904.

|| T. Gray, *Physical Review*, vol. vii. 1898, p. 199.

DIELECTRIC STRENGTH IN VOLTS PER MM. OF SPECIMENS
1 MM. THICK (UNLESS OTHERWISE STATED).

	Volts continuous.	Effective volts alternating.
	per mm. thickness.	
Mica, $\frac{1}{16}$ mm. thick ...	200,000	141,000
„ 1 mm. thick ...	61,000	43,000
Micanite ...	40,000	28,000
Hard rubber or ebonite ...	53,000	37,500
Pure rubber ...	47,000	33,000
Window glass, 2 mm. ...	16,000	11,300
Linseed oil ...	8,300	5,900
Air, 1 mm. gap ...	4,360	3,080
„ 2 mm. gap ...	5,740	4,060
„ 1 cm. gap ...	2,980	2,100

The breaking-down strength per unit of thickness is always greater as the thickness decreases. This is especially so in the case of mica, good sheets of which, when only $\frac{1}{16}$ mm. thick, will stand a pressure of 14,000 volts alternating.

The striking distance is slightly greater through the insulation of small wires than through that of large ones. This arises from the greater dielectric stress just near the wire. In this connection see O'Gorman's paper in 1901 on the great advantage of "grading" insulation of High Tension Cables,* so as to obtain specially low stress and high dielectric strength where it is wanted near the wire.

Dielectric break-down occurs more easily (in some cases with only $\frac{1}{2}$ the voltage) if the surface discharge is localized to a point by any means, even though the surface remain undamaged. Walter† gives the following *minimum* dielectric strengths—defined as the air-spark-length (in cms.) of that voltage which will pierce sample, divided by the thickness of the sample in cms.—

Ebonite, ordinary	20 to 39
„ special	54 to 77
Soft rubber	about 37
Glass	11 to 16
Porcelain	15 to 21
Marble	about 13
Paraffin wax	about 23

* "Insulation of Cables," *Jour. Inst. El. Engrs.*, vol. xxx. p. 608 et seq.
See also same paper, pp. 660-670, for collected data on dielectric strength.

† E.T.Z., 8203, p. 796, Sci. Abstr., No. 2004A., 1904.

The figures obtained in this way are nearly independent of the thickness. Mica, being laminated, cannot be readily pierced by localized discharge.

The sparking distance through heavy resin oil is about $\frac{1}{30}$ that of air. In linseed and lubricating oils about $\frac{1}{3}$ to $\frac{1}{5}$ that of air.

The **oil insulation** of transformers must be protected from falling dust, and its circulation at the surface should be very slight. Transformers for high tension work have repeatedly broken down through particles of dust or globules of moisture circulated by the oil.

The oil used by the General Electric Co. of America for transformers is understood to withstand 10,000 volts across $\frac{1}{20}$ of an inch, and 15,000 across $\frac{1}{15}$ of an inch, and to have a flash-point of 516° F.*

INCREASED RESISTANCE WITH ALTERNATING CURRENTS.

(SKIN EFFECT.)

By D. K. MORRIS, PH.D.

ALTERNATING currents, especially of high frequency, are by their inductive action to some extent confined to the surface of solid conductors.† The increase of ohmic resistance due to this cause is separate from the *apparent* increase due to impedance. With ordinary frequencies the effect is negligible in conductors less than $\frac{1}{2}$ inch in diameter of non-magnetic material.

The increase of resistance per cent. is given in the table below, for copper and aluminium conductors; the specific resistance being taken at 1,700 and 2,800 C.G.S. units respectively.

* See *Electrician*, Oct. 13 and Oct. 20, 1899.

† Lord Kelvin, *Jour. Inst. E. E.*, 1889. Presidential Address; appendix. W. M. Mordey, *ibid.* p. 603, "Alternate Current Working." See also *Electrician*, January 19, 1900, p. 430.

SKIN EFFECT IN COPPER AND ALUMINIUM.

PERCENTAGE INCREASE OF RESISTANCE.

Diameter.			Frequency.				
Cm.	in.	Nearest S.W.G.	50	83	100	133	500
Increase per cent.: Copper.							
·5	·20	6	0	0	0	0	0·5
1·0	·39	0000	0	0	0	0·5	15
1·5	·59	...	0·2	2	3·5	6·5	53
2·0	·79	...	2·7	8	10	17	98
2·5	·98	...	7	16	23	35	140
4·0	1·57	...	33	64	79	104	267
10·0	3·94	...	194	270	305	364	781
Increase per cent.: Aluminium.							
·5	·20	6	0	0	0	0	0
1·0	·39	0000	0	0	0	0	6·5
1·5	·59	...	0	0·1	0·5	2·3	26
2·0	·79	...	0·5	2·8	4·5	7·5	58
2·5	·98	...	2·6	7	9	15	94
4·0	1·57	...	15	34	44	63	192
10·0	3·94	...	140	196	223	266	593

When the increase is above 25 % it may be expressed within 2 or 3 % by the formulæ—

$$\text{Increase per cent.} = 401 d'' \sqrt{\frac{\text{frequency}}{\rho}} - 73\cdot4,$$

$$= 9\cdot75 d'' \sqrt{\text{frequency}} - 73\cdot4, \text{ for copper,}$$

$$= 7\cdot60 d'' \sqrt{\text{frequency}} - 73\cdot4, \text{ for aluminium,}$$

where d = diameter of conductor in decimals of an inch,

ρ = specific resistance in C.G.S. units.

The above table and formulæ are applicable to any large conductors (iron excepted, see below), solid or stranded.

The skin effect is the same in conductors of different material and area but of the same conducting power and sectional configuration. It is less in flat strips and tubes than in round solid conductors of the same cross sectional area.

At **very high frequencies** the conductivity of stout wires is proportional to the diameter, and not to the square of diameters.

With **iron wires** the skin effect may be much greater, especially if the current is such as to cause circular magnetization at a flux density for which the permeability is high. Merritt* finds that this increase of resistance in the case of an iron wire .144 in. diameter was small both with a current of $\frac{1}{4}$ amp. and of 15 amps. (at 130 ~); while the intermediate current of 6 amps. experienced a rise of resistance 230 per cent.

Important tests have been published on the magnitude of the effect on **steel rails**. Mordey and Jenkin† give a table from figures due to Blathy which is summarized below. It relates to a single track laid with rails weighing 90 lbs. per yard, and having about nine times the specific resistance of copper (p. 436).

SKIN EFFECT IN STEEL RAILS.‡

Current amps.	Amps. per sq. in.	Volts "drop" per mile.		
		D.C.	15 ~	~
1000	56.5	21	120	210
700	39.5	14	66	112
500	28.2	11	39	67
300	16.9	6	21	34

* E. Merritt, *Physical Review*, Dec. 1899. See also B. Hopps, *Electrician*, Oct. 1900, p. 920.

† *Electrical Traction on Railways*, Proc. Inst. C.E., 1902, vol. 149, p. 62.

‡ Methods of avoiding this "drop" in volts, see p. 434.

MAGNETISM AND ELECTRICITY.

By R. C. CLINKER and E. BASIL WEDMORE.

MAGNETISM.

THE **force at any point** due to a given magnetic pole varies inversely as the square of the distance of the point from that pole.

The **moment** of a magnet is the product of its pole strength into the distance between the poles, and is a measure of the magnet's tendency to turn when in a magnetic field.

Field in the neighbourhood of a magnet—

Let f = field strength (or magnetic force),

M = moment of the magnet,

r = distance of a point p from the centre of the magnet.

Along the axis, at any point p

$$f = \frac{2M}{r^3}.$$

Along a line normal to the magnet, and bisecting it, at any distant point p

$$f = \frac{M}{r^3}.$$

At any other point find f from the law of inverse squares and the parallelogram of forces.

Substances such as iron, nickel, and cobalt, which are attracted by a magnet, are called "**paramagnetic**," or simply "**magnetic**."

Substances such as bismuth, which are repelled by a magnet, are called "**diamagnetic**."

Different substances have different degrees of magnetic **susceptibility**, the paramagnetic being by far the most susceptible.

All kinds of iron and steel are very **permeable** to magnetic flux. If we apply a given magnetizing force to (say) a column of air, we shall get a certain magnetic flux in the air (a certain strength of field, or, as it is sometimes called, a certain magnetic "flux density"). This, in lines per square centimetre, is represented by the letter H . If we now substitute a column of iron for the column of air, and apply the same magnetizing force to it, we shall find that there is a far greater flux density in the iron. Call the increased flux density B , then B/H is a measure

of the **permeability**, and is generally represented by the Greek letter μ .

$$\mu = \frac{B}{H}.$$

If the magnetizing force applied be small, it will be found that the permeability is comparatively small, and that as the force increases, the permeability increases up to a certain point, beyond which it decreases. If the magnetic force is now

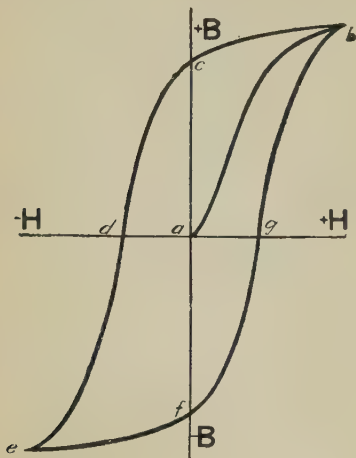


FIG. 12.

diminished it will be found that while B decreases it will not reach the same values as it had during the previous change, and, in fact, when the magnetic force reaches zero B may still have a considerable value. These properties may be represented graphically by curves.

Thus, in Fig. 12, the curve ab represents the first change described above, and bc the second. If H be decreased still further, and eventually given a negative value, B will reach a zero value, as at d , and then reverse.

The residual value ac when $H = 0$ is called the **remanence**, and the value ad of the magnetizing force required to bring B to zero is called the **coercive force**.

If H be continually reversed, B will follow a complete cycle of values as indicated by the curve $bcdefg$. It may be shown that there is a waste of energy accompanying this lagging of B behind H , the phenomena being called **hysteresis**, and that the waste of energy per cycle is proportional to the area enclosed by the curve.

The energy dissipated takes the form of heat, and the temperature of the iron rises. The heating of transformer and dynamo cores is partly due to this cause.

Steinmetz found that the energy dissipated per cycle of magnetic force and induction was closely proportional to the 1·6th power of the induction density, B . Hence the loss may be expressed by the formula $E = \eta B^{1.6}$,

where E = hysteresis loss in ergs per cub. cm. per cycle, and η = a constant, usually called the Steinmetz coefficient.

Note that, as $2^{1.6} = 3$ nearly, the effect of doubling the induction is to treble the hysteresis loss.

The value of η in commercial magnetic iron varies from ·002 in iron of poor quality down to ·0009 in the very best transformer iron. For cast iron and steel η is much higher, but this is not usually of much consequence, as these materials are used only in such parts of a machine as carry a flux fairly constant in direction and value.

In commercial work, hysteresis loss is usually measured in watts per lb. or per cubic inch at a certain frequency. To convert from ergs per cubic cm. per cycle to watts per lb. at 100 cycles per second, multiply by ·00059.

A standard of reference for hysteresis loss sometimes used in this country is to express it as so many watts per lb. at an induction density of 4000 C.G.S. lines and a frequency of 100 cycles per second. This standard is used with the Ewing tester (see p. 140). Another standard which is coming into more general use is to express it in watts per lb. at 50 cycles per second and 10,000 lines per sq. cm. This standard is usually applied to the *total* iron loss (*i. e.* including that due to eddy currents).

Curves showing the relation between B and H for various qualities of iron and steel will be found on p. 250.

Note that if H is in lines per sq. cm., then ampere turns per inch length = $2H$ nearly.

Magnetic Circuit.—Corresponding to the law of the electric circuit we have the law—

$$\text{Magnetic Flux} = \frac{\text{Magneto-motive force}}{\text{Reluctance}}.$$

Let a = sectional area of magnetic path in sq. cm.,

l = length of magnetic path in cm.,

N = turns of wire,

C = current in amperes,

then,

$$\text{Magneto-motive force} = \frac{4\pi NC}{10},$$

$$\text{Magnetic flux} = Ba,$$

$$\text{Reluctance} = \frac{l}{a\mu}.$$

Note that the expression for reluctance corresponds to the expression for resistance, $\frac{1}{\mu}$ corresponding to specific resistance.

The formula therefore stands—

$$\frac{4\pi NC}{10} = Ba \times \frac{l}{a\mu} = \frac{Bl}{\mu}.$$

If there are several reluctances in parallel, treat them as resistances.

Corresponding to the formula for the resistances in series—

$\frac{E}{C} = \left(\frac{l_1 \rho_1}{a_1} + \frac{l_2 \rho_2}{a_2} + \text{etc.} \right)$ where ρ = specific resistance, we have—

$$\frac{4\pi NC}{10} = Ba \left(\frac{l_1}{a_1 \mu_1} + \frac{l_2}{a_2 \mu_2} + \text{etc.} \right)$$

Letting F stand for flux ($= Ba$), we have—

$$1.257 NC = F \left(\frac{l_1}{a_1 \mu_1} + \frac{l_2}{a_2 \mu_2} + \text{etc.} \right)$$

$$\text{Note that } \frac{Bl}{\mu} = Hl, \therefore 1.257 NC = Hl.$$

If l is measured in inches, then—

$$\text{Amp-turns per inch} = 2.02H.$$

Pull of a Magnet.

If P = pull in dynes (see p. 54),

A = area of magnet pole in square centimetres,

$$P = \frac{B^2 A}{8\pi}.$$

If P = pull in lbs., and A is in square inches, then

$$P = 577 B^2 A \times 10^{-9}.$$

NOTE.—If the area of a magnet pole be decreased, B will tend to increase in direct proportion, so that $B^2 A$, or the pull, will be increased.

ELECTRICITY.

Resistance, Current, and Electromotive force.

Units.—Ohm, Ampere, and Volt.

Symbols.— R = resistance.

C = current.

E = E.M.F.

b = battery resistance.

g = galvanometer resistance.

s = resistance of shunt.

Ohm's Law.—If the units used are the ampere, volt, and ohm, then—

$$C = \frac{E}{R} \quad \therefore E = CR \quad \therefore R = \frac{E}{C}$$

Let r_1 , r_2 , and r_3 , etc., be any number of **resistances in parallel**. It is required to find the value R of the combined resistances.

It can be shown that—

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3},$$

$\therefore R = \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1}$, and so on for any number of resistances. $\frac{1}{R}$ is a measure of the conductivity, and it will be seen

by the first formula that the combined conductivity is the sum of the conductivities of the parallel paths.

Kirchhoff's Laws.

1. The algebraical sum of the current strengths in all the wires which meet at a point = 0.

2. The algebraical sum of all the products of resistances and current strengths in all the wires forming a closed figure equals the algebraical sum of all the electromotive forces in the circuit.

Galvanometer Shunts.

If a **galvanometer** be **shunted** with a resistance s , and a current C be flowing through the circuit, then there will be a difference of potential E between the galvanometer terminals such that $E = C \times R$ where R = the combined resistance of the galvanometer and shunt and = $\frac{gs}{g+s}$. (See "resistances in parallel," p. 106.)

Now the galvanometer current $c = \frac{E}{g}$, but the main current

$$C = \frac{E}{R} = \frac{E}{\frac{gs}{g+s}}$$

$$\therefore C : c :: g : \frac{gs}{g+s},$$

$$\therefore C = c \frac{g+s}{s},$$

i. e. the main current = the galvanometer current $\times \frac{g+s}{s}$.

$\frac{g+s}{s}$ is called the **multiplying power** of the shunt.

If the resistance of the shunt be $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, or $\frac{1}{n}$ of that of the galvanometer, then the multiplying power will be 10, 100, 1000, or $(n+1)$ respectively.

If it is desired to shunt a galvanometer without altering the total resistance in circuit, a compensating resistance must be placed in series with the shunted galvanometer.

Required resistance = resistance of galvanometer unshunted
– resistance of shunted galvanometer

$$= g - \frac{gs}{g+s} = g \left(\frac{n}{n+1} \right).$$

Ayrton and Mather Universal Shunt.

If a galvanometer be permanently shunted with any resistance r (Fig. 13) the current taken by the galvanometer will be $\frac{r}{g+r}$ times the total current flowing. The terminal connections are those marked “*a*” and “*c*.”

If now the current be led out at a point *b* instead of *c*, so as to enclose a part x of the resistance r as a shunt, then the galvanometer current will be $\frac{x}{g+x}$ times the main current, *i. e.*

the new galvanometer current is $\frac{x}{r}$ times the old current. Note

that g need not be known, and that $\frac{x}{r}$ is independent of g .

Whatever g may be, provided r is constant, then the multiplying power of the combination will be proportional to $\frac{1}{x}$.

Electromagnetic Units.

Unit **Current** flowing through a conductor of unit length bent to unit radius will produce unit magnetic field at the centre of curvature.

Unit **Quantity** is conveyed by unit current in unit time.

Unit **E.M.F.** is produced in a conductor cutting unit flux per second.

Unit **Resistance** is that through which unit E.M.F. will send unit current.

Unit **Capacity** is that which will hold unit quantity at unit potential difference.

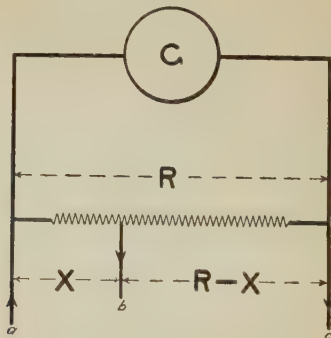


FIG. 13.

Relation between Electrostatic and Electromagnetic Units.

1 Electromagnetic unit of current = v electrostatic units.

1 " " " quantity = v " "

1 " " " capacity = v^2 " "

1 " " " E.M.F. = $1/v$ " "

1 " " " resistance = $1/v^2$ " "

where v = velocity of light, cms. per second = $3 \cdot 009 \times 10^{10}$.

Practical Units.

Current ... - Ampere = 10^{-1} C.G.S. electromagnetic units.

Quantity ... - Coulomb = 10^{-1} " " "

E.M.F. ... - Volt = 10^8 " " "

Resistance ... - Ohm = 10^9 " " "

Capacity ... - Farad = 10^{-9} " " "

Self-induction ... - Henry = 10^9 " " "

Power ... - Watt = 10^7 " " "

1 Megohm = 1,000,000 (Ω).

1 Microhm = $\frac{1}{1,000,000}$ ohm.

1 Microfarad = $\frac{1}{1,000,000}$ Farad. (Mf.)

1 Millihenry = $\frac{1}{1,000}$ henry (Mh.)

1 Kilowatt = 1,000 watts (Kw.)

International Units.

The **Ohm** is represented by the resistance of a column of mercury 14·4521 grammes in mass, of constant cross section, and having a length of 106·3 centimetres at 0° C.

The **Ampere** is represented by the steady current which, when passed through a neutral solution of pure silver nitrate in water, deposits silver at the rate of ·001118 gramme per second.

The **Volt** is represented by that E.M.F. which bears a ratio of 1,000 : 1434 to that of a Clark cell at 15° C.

Note.—E.M.F. of Clark cell = $1·4342\{1 - ·00077(t - 15)\}$
(See also p. 152.)

Condensers and Capacity.

If a number of condensers be connected **in parallel**, the resultant capacity will be the sum of the capacities of the separate condensers.

If a number of condensers of capacity K_1, K_2, K_3 , etc., be connected **in series**, the resultant capacity K will be such that

$$\frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}, \text{ etc.}$$

In practice the distribution of the quantity of electricity amongst the condensers will depend not only on the relative capacity but also on the relative leakiness, the more leaky condensers letting through current which will charge up the others. Hence in continuous current work the distribution of potential between condensers in series depends more on their insulation resistances. In alternating current work, the distribution depends upon their relative capacities. This has an important bearing on the strength of insulations.

The **capacity of a condenser** is proportional to the area of the plates, inversely proportional to the distance between the plates, and varies with the dielectric between the plates. For a given material, therefore, the insulation resistance between the plates varies inversely as the capacity.

The proportional rate of loss of charge with time should therefore be independent of capacity and be constant for each material. In practice, however, surface leakage interferes with this. (See also p. 148, Fig. 40.)

Let K = capacity in microfarads,

k = specific inductive capacity (see p. 87),

d = distance between plates,

r = radius of circular plates,

a = area of plates of any shape,

then

$$K = kr^2/4d \div 900,000,$$

also

$$K = ka/4\pi d \div 900,000.$$

Electrolytic Condensers.—If metal plates be immersed in an electrolyte and a potential difference be applied between them which is not sufficient to produce decomposition, the ions which would be liberated with a larger E.M.F. will be drawn towards their respective electrodes and will receive a charge of electricity. The quantity of electricity which may be accumulated in this way is very considerable. Platinum plates in dilute acid have an electrolytic capacity equivalent to approximately 63 microfarads per square centimetre when the E.M.F. = 1 volt.

Condensers of large capacity are made commercially on this principle for motor-starting, iron plates being used and solutions of salts of soda and potash.

(See also motor-starting devices, p. 447.)

Inductance.—The **inductance** or **coefficient of self-induction** of a circuit may be defined as the total number of magnetic lines linking with the circuit when unit current is flowing in it.

Thus a circuit having 10 turns which are linked with 1000 lines due to unit current, has a total linkage or inductance of 10,000. It can be shown that the dimension of inductance is a length. The practical unit (corresponding to the volt, ampere, and ohm) is called the henry, and is equal to 10^9 C.G.S. units.*

The inductance of a coil is proportional to the square of the number of turns, other things being constant. For if we increase the turns n times, we increase the flux per unit current also n times, so that the interlinkage is now equal to n^2 times its original value.

HEATING EFFECTS OF CURRENT.

When a current C passes through a resistance R the rate of dissipation of energy as heat = C^2R watts, where C = amperes and R = ohms.

1 watt = .24 calorie per second.

This heat will be partly absorbed, causing a rise of temperature and partly dissipated by radiation. The relation between **temperature rise** and watts assuming no radiation is given by the following formula—

$$\text{watts} = WS \frac{\theta}{t} \times 0.524,$$

where W = lbs., S = specific heat., θ = temperature rise in °C., t = time in hours.

Thus for copper at 1000 amperes per square inch the rise of temperature without radiation will be about 0.77° C. per minute.

* See Fleming, *The Alternate Current Transformer*, vol. i. p. 54.

Apparatus which has to carry current for a short time only (such as motor starting gear) can be run at a high current density, the rise of temperature in a given time being calculated by assuming that all the heat produced is taken up in the copper or iron.

The heat dissipated by **radiation** depends upon the following—

(1) The nature of the radiating surface, represented by a constant k .

(2) The area of the surface, represented by A .

(3) The temperature difference between the radiating surface and the surrounding space, represented by θ .

$$\text{watts dissipated} = kA\theta.$$

The following is given by Esson for shellacked cotton **bobbins**—

$$\text{watts dissipated} = \frac{\text{diff. temp. } ^\circ \text{Fahr.} \times \text{surface in sq. in.}}{100}$$

The average temperature throughout a bobbin wound 2 or 3 inches deep may be 20 % to 33 % higher than the surface temperature.

The best way to measure the temperature rise of such a bobbin is by the increase in resistance of the wire. Take the resistance at the beginning and end of the temperature test. Find the percentage increase, and divide by 0.4. This will give the average temperature rise of the wire in $^\circ \text{C}$. sufficiently near for practical purposes,

Fuses of a given material will have a constant radiation coefficient and a constant melting-point. The watts dissipated at the fusing-point will therefore be proportional to the area.

Now, in a given length of fuse the watts dissipated $\propto \frac{C^2}{d^2}$, where

C = current, d = diameter, while the area will be $\propto d$, $\therefore \frac{C^2}{d^2} \propto d$, $\therefore C \propto \sqrt[2]{d^3}$, *i. e.* the current carrying capacity is $\propto \sqrt[2]{d^3}$.

In practice the terminals conduct away some heat, and the above only holds for long fuses of relatively small section.

For Fuse Tables see p. 89.

THERMO-ELECTRIC PHENOMENA.

If a junction between two dissimilar metals be heated or cooled, electricity will tend to flow across the junction. If a current be passed through such a junction, a change of temperature will be produced. The former phenomenon is known as the Seebeck effect, and the latter as the Peltier effect.

For a given pair of metals, the thermo-electromotive force is approximately proportional to the excess of temperature of one junction over the other. If, however, one of the metals be iron, an inversion of polarity will take place at a temperature of something like 300° C.

The following table (Matthieson) gives the value of the Thermo-E.M.F. for various metals at a mean temperature of 20° C.

METAL.	E.M.F.	METAL.	E.M.F.
Bismuth of commerce		Antimony, pure, in Wire	- 3.8
in Wire	+ 97.0	Silver, " "	- 3.0
Bismuth pure in Wire	+ 89.0	Zinc, " "	- 3.7
" crystallized		Copper, electrolytic ...	- 3.8
along axis	+ 65.0	Antimony of commerce	
Bismuth crystallized		in Wire	- 6.0
normal to axis	+ 45.0	Arsenic	- 13.56
Cobalt	+ 22.0	Iron, piano wire	- 17.50
German Silver	+ 11.75	Antimony, crystallized	
Mercury	+ 0.418	along axis	- 22.60
Lead	0	Antimony, normal to	
Tin	- 0.1	axis	- 26.40
Copper of commerce ...	- 0.1	Phosphorus (red)	- 29.70
Platinum	- 0.9	Tellurium	- 502.00
Gold	- 1.2	Selenium	- 807.00

In the above table lead is taken as the starting-point.

To obtain the E.M.F. of any combination multiply the sum of the E.M.F.s by the difference of temperature available. The result will be in micro-volts ($\frac{1}{1000000}$). A couple of "Constantan" (Cu 60 %, Ni 40 %) and copper has a very high thermo-E.M.F., namely, 37 micro-volts per degree Centigrade, and is often used for the measurement of moderate temperature (up to 800° C.). Above that temperature a platinum and platinum-rhodium couple is generally employed (see p. 167).

ELECTRO-CHEMICAL PHENOMENA.*

If a current be passed through a liquid conductor other than an element, either in the form of a solution or a fused salt, decomposition takes place.

The quantity of material decomposed depends only on the quantity of electricity, and is proportional to the same, *i. e.* the quantity decomposed per second is proportional to the current.

* See also p. 465 *et seq.*

The conductors conveying the current to and from the liquid are known respectively as the anode and the kathode. The current flows in the liquid from the anode to the kathode. When the decomposition takes place, the metals and hydrogen flow as it were with the current, being deposited on the kathode. The other elements are liberated at the anode.

One ampere liberates '000010384 gramme of hydrogen per second. This figure is known as the **electro-chemical equivalent** of hydrogen. The chemical equivalent of hydrogen is unity. The weight of any other element liberated per ampere second will be found by multiplying the electro-chemical equivalent of hydrogen by the chemical equivalent of the element in question. Thus the chemical equivalent of silver being 107'67, the electro-chemical = $107'67 \times '000010384 = '0011181$.

MAGNETIC EFFECTS OF THE ELECTRIC CURRENT.

By definition unit current flowing through unit length of conductor bent to unit radius will produce unit magnetic field at the centre of curvature.

The **magnetic force** due to any element of current varies inversely as the square of the distance of that element. The direction of the force is at right angles to that of the current.

From the above it follows that the **field at the centre of a circular coil** will be inversely proportional to the radius, for the effect of each element of conductor will be inversely as the square of the radius, but the total length of effective conductor will be proportioned to the radius, $\therefore$ the total effect of the total length of conductor will be inversely proportional to the radius. If there is more than one turn, the length of conductor is proportionally increased. The whole expression for the field at the centre of the coil is therefore

$$f = \frac{2\pi nCr}{10r^2} = \frac{2\pi nC}{10r}$$

Where C = current in amperes,

$\frac{C}{10}$ = current in electro-magnetic units

r = radius of coil,

n = number of turns,

$2\pi rn$ = total length of conductor.

If current flows clockwise, the clock-face is the south pole.

Field along the axis of circular coil.—At any point p along the axis of a circular coil of radius r and at a distance x from the centre of the coil the field strength will be proportional to

$$\frac{1}{(\sqrt{x^2 + r^2})^3}$$

If l be the distance of the point from the conductor itself, the field is proportional to $1/l^3$. The value at the centre where $l = r$ is given by the formula above, and from the formula $1/l^3$ the absolute value at any other point may be obtained.

The magnetic **moment of a circular coil** carrying a current

$$= \frac{NCA}{10}$$

where N = number of turns, C = amperes, A = area of the coil.

Field due to a straight wire of infinite length.

d = distance of point considered, measured from axis of wire,

r = radius of the wire,

c = current, absolute units, $\therefore c$ = amperes $\div 10$.

Outside the wire

$$\text{field} = 2c/d.$$

Inside the wire

$$\text{field} = 2cd/r^2.$$

The field appears right-handed when the current flows away from you.

Inductance of straight wire.—The inductance of a length of straight conductor of diameter d cm., and from which the parallel return conductor is D cm. distant is given by

$$L = \left(2 \log_e \frac{D}{d} + \frac{1}{2} \right) 10^{-9} \text{ henries per cm. length.}$$

The inductance per cm. of *circuit* is double this.

Field in a long solenoid.—Provided the length of the solenoid is large in comparison with the diameter, the field = $\frac{4\pi nc}{l}$, where n = number of turns, l = length in centimetres, and c = current in C.G.S. units.

The field decreases rapidly towards the end of the solenoid. At the ends the strength is exactly half that at the middle.

Inductance of solenoid.—The inductance of a solenoid whose length is large compared with its diameter is equal to

$$\frac{L^2}{l} \cdot 10^{-9} \text{ henries,}$$

where

L = total length of wire in cm.

l = length of solenoid.

Force on conductor in magnetic field.—If a conductor be carrying current in a magnetic field at right angles to the lines of force, the force acting on it will = cHl , where c = current, H = strength of field, l = length of conductor.

If the conductor makes an angle θ with the lines of force, the force on it will be = $cHl \sin \theta$.

If current is in amperes, H in lines per sq. in., l in inches,
then the force in lbs. $= 8.85 \times 10^{-8} CHI \sin \theta$.

E.M.F. generated.—If a conductor moving in a magnetic field or a magnetic field moving across a conductor causes n lines per second to cut the conductor, then n units of E.M.F. are generated in the conductor (in C.G.S. units).

Let l = length of conductor,

H = strength of field,

v = velocity of wire in cms. per second at right angles to field.

$$\text{E.M.F.} = Hlv.$$

or

$$\text{E.M.F. in volts} = Hlv \times 10^{-8}.$$

ALTERNATING AND INTERMITTENT CURRENTS.

If a current in a circuit varies, the magnetic field produced by that current will vary. As the field decreases or increases the lines of force will cut the conductor and induce an E.M.F. in the circuit. This effect is called **self-induction**.

Again, as the current varies, the potential of different parts of the circuit will also vary. Owing to the capacity of the conductor, the transference of a certain quantity of electricity necessarily accompanies these variations of potential. These quantities do not depend on the current flowing, but on the variations of the current. The effects produced are called *capacity effects*.

The self-induction in henries is measured by the volts induced when the current changes at the rate of an ampere per second.

The self-induction of a given coil will be proportional to the *square of the number of turns* (see p. 110).

As the current changes, the E.M.F. set up is in such a direction as to tend to oppose the change of current.

It will readily be seen from the above that if two or more coils are near each other, so that the magnetic field produced by each coil links with the other coils, then each coil will induce E.M.F.s in the others. This effect is known as **mutual induction**.

Self-induction, considered as a property of electricity, is analogous to the property of matter known as inertia. In the same way as inertia may be defined as that property of matter which causes it to resist any change of velocity, so self-induction may be defined as that property of electricity which causes it to resist any change of its rate of flow.

If a mechanical force of P units be applied to a mass of

M lbs., the rate of change of velocity will be P/M units. Similarly, if an E.M.F. of V volts be applied to a circuit of inductance L henries, the initial rate of change of current will be equal to V/L amperes per second.

Skin Effect.*—In a thick conductor the total field linking with the portions near the axis is greater than the field linking with the outer portions. Hence the opposing E.M.F. due to self-induction is greater near the axis of the conductor than in the outer layers, and an alternating current will tend to keep more to the outer layers of the conductor, thus apparently increasing its resistance. This effect becomes of practical importance only in the case of very thick copper conductors, and higher frequencies than are commonly used. In an iron conductor it is far more marked for ordinary frequencies than in copper, owing to the greater field inside the wire. But with exceedingly high frequencies, such as obtain in Leyden jar or lightning discharges, according to Lodge there is not much to choose between iron and copper.

CIRCUITS WITH RESISTANCE, SELF-INDUCTION, AND CAPACITY.

If an alternating E.M.F. be applied to a circuit having resistance only, the current at any instant in that circuit will

be given by Ohm's law $C = \frac{V}{R}$. This current varies with time

exactly as the E.M.F. varies from instant to instant. The current is therefore said to be **in phase** with the E.M.F.

If an alternating E.M.F. be applied to a circuit having self-induction as well as resistance, the current will not be in phase with the E.M.F., for when the current is rising rapidly from zero there will be a large back E.M.F. generated due to self-induction, while when the current has reached its maximum value, and is for the moment constant, there will be no back E.M.F. generated. It is necessary, therefore, that the E.M.F. applied shall already have a positive value (to overcome the self-induced E.M.F.) just when the current is beginning to rise from zero in a positive direction. The current, therefore, rises later than the E.M.F. The reverse effect takes place when the current is decreasing, the current falling later than the E.M.F. The current, therefore, lags behind the E.M.F. in time, and is called a **lagging current**.

If an alternating E.M.F. be applied to a circuit having capacity as well as resistance, the current will be a **leading** one,

For actual values, see p. 99.

as may be seen thus: As the E.M.F. passes zero value, and is rising rapidly in a positive direction, it is changing rapidly. A large capacity current must therefore be flowing already. The current in the circuit, therefore, has attained a positive direction, while the E.M.F. is only just passing through zero.

An alternating current or E.M.F., whether it vary according to the sine law or not, passes through a number of complete cycles every second, which number is called the **frequency** or

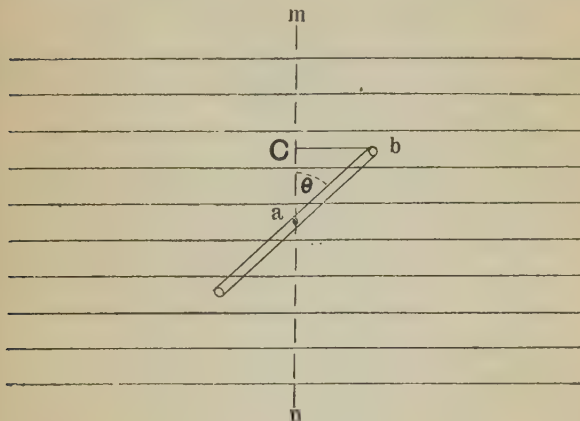


FIG. 14.

periodicity. Thus, if the value of the E.M.F. or current vary according to any given law, the time taken between two corresponding points is the time of one complete cycle, the same curve being described again and again.

For simplicity in calculation, it is often assumed that the E.M.F. waves have a peculiar form, having certain useful properties. This form is one which often occurs in practice, and it can be shown that the other forms which are obtained may be mathematically analyzed into a number of curves having the particular form here being considered, and that the effects produced by the various wave forms may be calculated by summing the effects produced by each component having this peculiar form.* The form here alluded to is that known as a

* See p. 135.

sine curve, and is the form which represents a simple harmonic vibration. Consider a loop of wire revolving at a uniform rate, in a uniform magnetic field. As the loop revolves, the number of lines of force enclosed will alter. Let Fig. 14 represent a section of the coil normal to its axis. Draw a line mn normal to the axis and to the field. If at any moment we project a line bC on to mn , then Ca will be proportional to the number of lines of force enclosed by the coil. But Ca is proportional to $\cos \theta$, where θ = the angle between the coil and mn . Therefore the number of lines of force enclosed at any moment is proportional to $\cos \theta$.

We may represent the number of lines of force at any moment by points on a curve of cosines, the scale of $\cos \theta$ being proportional to the number of lines of force, and the scale of degrees being proportional to the time.

Now the E.M.F. generated in the revolving coil is at any instant proportional to the rate of change of lines of force. *i. e.* to the rate of cutting of lines of force, but this is proportional to the line bC in Fig. 14, which is proportional to $\sin \theta$. The E.M.F. at any moment is therefore proportional to $\sin \theta$, and a curve of sines represents the variation of E.M.F. with time. (See p. 21.)

The sine and cosine curves are of exactly the same form, the only difference being one of phase, the cosine curve being 90° in advance of the sine curve in point of time.

Properties of a Sine Wave.—We are now in a position to understand the advantages of this curve.

(1) E.M.F. is proportional to rate of change of flux. Capacity current is proportional to rate of change of E.M.F. Often we find one quantity proportional to the rate of change of another. If, therefore, one of the quantities varies according to a sine law, the other quantities may also be represented by sine waves differing from the original wave in phase only. The method of expressing phase differences by angles will now be clear.

(2) The sum of two sine waves is itself a sine wave. If, therefore, we have a current dividing between an inductive circuit and a non-inductive circuit, the component of the current in each circuit may vary according to a sine law, and the main current will do the same, notwithstanding that the currents in the two branches differ in phase. If oa (Fig. 15) be revolved uniformly about the centre o , the projection ob will vary according to a sine law, as we have already seen, and may therefore represent from moment to moment the value of a current or an E.M.F. varying according to the same law. If oa is a current in an inductive resistanceless circuit, oc at right angles to it might represent a current in a non-inductive circuit

having resistance, the projection of oc reaching its maximum and minimum values 90° out of phase with oa . One complete revolution will represent one complete period; oc is therefore $\frac{1}{4}$ period ahead of oa . Completing the parallelogram $oadc$, the

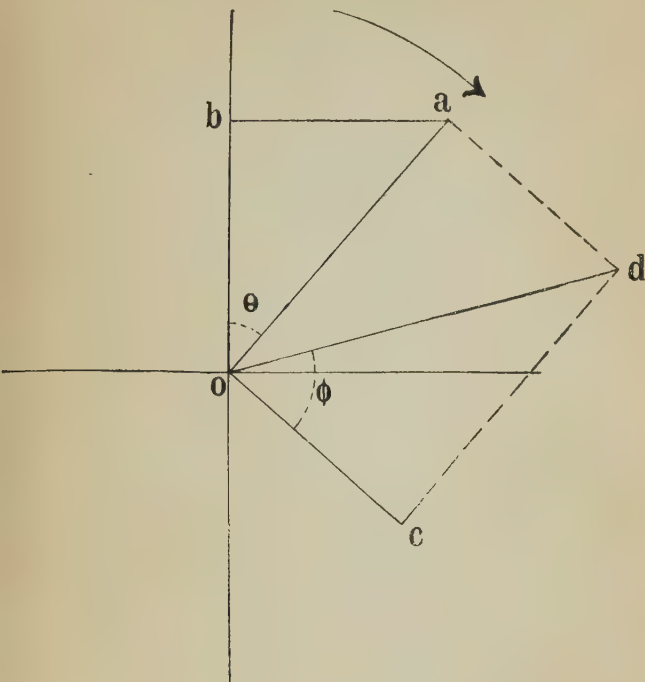


FIG. 15.

projection of the resultant od will at any moment represent the sum of the projections of oa and oc . Note that $oc = od \cos \phi$. oc will be in phase with the applied E.M.F.

(3) A similar diagram to the above may be used to represent E.M.F.s in combination. Thus, if we have a circuit with both resistance and self-induction, an E.M.F. oc will be required to

overcome the self-induction, and another oa , 90° behind oc , to overcome the resistance, od being the resultant E.M.F.

In fact, in any circuit we may subdivide either the E.M.F. or the current into two components at right angles. If od represents a current in a circuit with resistance and self-induction, making an angle ϕ with the E.M.F., then oc will represent that component of the current which is in phase with the E.M.F.

When the strength of an alternating current is stated to be 10 amperes, by this is meant, not that the **average** value of the current is 10 amperes, but that the **square root of the mean of the squares** of the instantaneous values is equal to 10 amperes. The reason for this is as follows: Suppose an alternating current is flowing through a resistance. The power dissipated by the current at any moment is proportional to the square of its value at that moment. Therefore the mean power is proportional to the mean square of the current strength. Hence the effective current strength is equal to the square root of the mean square. This is called the **virtual** or **effective current**, and is equivalent in heating or power-carrying effect to the same number of direct-current amperes. For a sine wave, the effective value = max. value $\div \sqrt{2}$, while the true mean value = max. value $\times \frac{2}{\pi}$.

(4) The average value of the product of two sine waves 90° out of phase = 0. The work done in a circuit will be at any moment equal to the product of the current and the E.M.F. We have seen that the current (od) may be subdivided into two portions, one in phase with the E.M.F. (oc) and equal to the whole current multiplied by $\cos \phi$, the cosine of the angle of lag of the current behind the E.M.F., and one 90° out of phase with the E.M.F. The average value of the product of the E.M.F. with this latter component will equal zero; the whole power is, therefore, represented by the average value of the product of the other component with the E.M.F. If C = current, V = volts, ϕ = angle of lag, then

$$\text{watts (average)} = VC \cos \phi.$$

$\cos \phi$ is called the **power factor**.*

$\sin \phi$ is sometimes called the **wattless factor** or **inductance factor**.

If P = power factor and W = wattless factor, we have $P^2 + W^2 = 1$.

(5) The same construction may be applied to circuits having resistance and capacity, the capacity currents *leading* by 90° .

* For instruments to measure this, see p. 162.

(6) From the above the following constructions will be obvious—

ABC is a right-angled triangle (Fig. 16).

(a) If AC represents the E.M.F. required to send the current through the resistance in a circuit, and AB represents the E.M.F. required to send the current against the self-induction of the circuit, then CB will represent the total E.M.F. which must be applied, and θ will be the angle of lag of the current behind the E.M.F.

(b) The sides AC , AB , and BC represent E.M.F.s. If, therefore, we divide them by the current we shall get values corresponding to resistances. There being but one current, this necessitates only an alteration of scale. Thus: If AC represents to scale the volts required to send 10 amperes through the circuit, then AC represents ohms if measured on a scale 10 times as large. On the scale on which AC represents ohms, AB represents *reactance* and CB *impedance*.

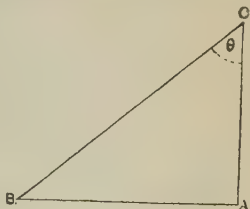


FIG. 16.

The term **form factor** as applied to the wave form of an alternating E.M.F. was introduced by Fleming. It denotes the ratio of the root of the mean square of equispaced ordinates of

the wave to their true mean value, or $= \frac{\text{R.M.S.}}{\text{T.M.}}$. If the wave

be peaky, the R.M.S. value will be greater in proportion than the mean value, and if the wave be flat-topped the converse will be true. Hence the form factor is a sort of "coefficient of

peakiness" of the wave. Its value for a sine wave is $1.11 - \frac{\pi}{2\sqrt{2}}$

as explained above. If there be a coil of T turns, and an effective voltage V be applied to its terminals having a frequency N and a form factor f , then we have (neglecting resistance)—

$V = 4fFNT \cdot 10^{-8}$, where F = total flux linked by the T turns. Now, if f increases, then, V and N being constant, F will decrease. Conversely, a decrease of f will increase F .

If there be iron in the coil, it is obvious that the hysteresis-loss (which depends upon the magnetic density) will depend upon the value of f . Hence, in a transformer the core-loss depends upon the form-factor of the voltage wave upon which it is run. A peaked wave will cause less core-loss than a flat-

topped wave for the same effective voltage. The eddy-loss is unaffected by variation of form factor.

Calculation of Reactance and Impedance.—The current flowing through an inductive circuit of negligible resistance is equal to $\frac{V}{2\pi nL}$, where V is the effective impressed voltage across the ends of the coil, n is the frequency, and L is the coefficient of self-induction of the coil. This is derived as follows—

Let C = effective current (amps.),

V = „ voltage,

T = no. of turns in coil,

F = max. magnetic flux threading coil.

By definition of L we have

$$F = \frac{L}{T} \sqrt{2} C$$

where $\sqrt{2}C$ = max. value of C .

Now the average E.M.F. developed in the coil during one cycle is equal to the total change of induction $\times$ no. of turns $\div$ time of one cycle—

$$= 4FT \div \frac{1}{n} = 4FTn$$

$$= 4FTn \times 10^{-8} \text{ volts.}$$

But for a sine wave of E.M.F.—

effective value
average value $= f = \frac{\pi}{2\sqrt{2}}$. (See Form-factor on preceding page.)

Therefore $V = \frac{\pi}{2\sqrt{2}} FTn \times 10^{-8} \text{ volts.}$

But $F = \frac{L}{T} \cdot \sqrt{2} C$,

$$\begin{aligned} \therefore V &= 4 \cdot \frac{\pi}{2\sqrt{2}} \cdot \frac{L\sqrt{2}C}{T} \cdot Tn \\ &= 2\pi nLC \text{ (} L \text{ being in henries),} \end{aligned}$$

$$\therefore C = \frac{V}{2\pi nL}$$

In the above calculation we have assumed a sine wave of E.M.F., and a uniform value of L for all currents. In iron these conditions do not hold; hence C will not be proportional to V , and the current wave will not be a sine wave, distortion being produced by hysteresis. This distortion is due to the introduction of higher harmonics into the wave, the chief of

which are the third and fifth. The values of these increase rapidly with the saturation of the iron (see p. 135).

The expression $2\pi nL$ is a measure of the *reactance* of the circuit, and is expressed in ohms. As shown on p. 121, reactance and resistance may be represented by two sides of a right-angled triangle, the third side representing the impedance. Hence the impedance is equal to $\sqrt{\text{resistance}^2 + \text{reactance}^2} = \sqrt{R^2 + 4\pi^2 n^2 L^2}$, and is expressed in ohms. If, therefore, we have a voltage V , at n cycles per second, applied to a circuit

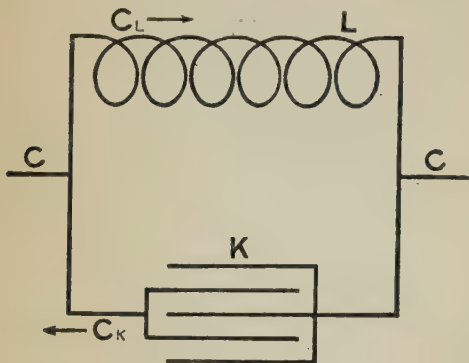


FIG. 17.

possessing resistance R and self-induction L , the current is equal to

$$\frac{\text{voltage}}{\text{impedance}} = \frac{V}{\sqrt{R^2 + p^2 L^2}}$$

where $p = 2\pi n$.

As seen above, the current through an inductive circuit of negligible resistance lags 90° behind the impressed E.M.F. Also the current in a condenser circuit *leads* the E.M.F. by 90° . It is therefore obvious that in a case like that shown in Fig. 17, where we have an inductance L , in parallel with a condenser K , the current C_L will be in exact opposition at every instant to the current C_K , their phase difference being 180° . The total current C will therefore be the difference of C_L and C_K . If $C_L = C_K$, then $C = 0$. This is an impossible case in practice, however, as the circuit must have resistance, however small,

and some current must therefore enter the system to supply the watts dissipated.

The value of C_K is obtained from V , n , and K as follows—

The maximum value of V is $\sqrt{2}V$. Hence the total quantity of electricity in the condenser at the point of maximum voltage = $\sqrt{2}VK$. In one cycle this quantity flows first in, then out again, then in again in a negative direction, and then out again. This is a total change of quantity during one cycle (*i. e.* $\frac{1}{n}$ seconds) of $4\sqrt{2}VK$. The *mean* current, therefore,

$$= \frac{\text{quantity}}{\text{time}} = \frac{4\sqrt{2}VK}{\frac{1}{n}} = 4\sqrt{2}nVK.$$

The current following a sine law, its form factor is $\frac{\pi}{2\sqrt{2}}$.

Therefore the *effective* current

$$= C = \frac{\pi}{2\sqrt{2}} \cdot 4\sqrt{2}nVK = 2\pi nVK.$$

K is measured in farads.

$\frac{1}{2\pi nK}$ is sometimes called the capacity reactance. This quantity is expressed in ohms, and, having the same dimensions as reactance, the two may be added algebraically, the capacity reactance being considered as opposite in sign to the inductive reactance. To obtain **capacity current** divide the voltage by the capacity reactance.

We have seen (p. 119) how a clock diagram may be used to represent alternating E.M.F.s or currents. Let this diagram be applied to the case of a **circuit having resistance, self-induction, and capacity**. These three may have various relations to the impressed voltage. They may be (i) all in series, so that the same current traverses all three, or (ii) all in parallel, so that all three are subjected to the same E.M.F., or (iii) in various combinations of series and parallel. One of the most usual cases is that in which we have a coil having reactance and resistance, shunted by a condenser.

Let the inductance of the coil be L , and its resistance R . Let the capacity of the condenser be K , and let V be the voltage impressed, C being the resultant current.

Draw a line (Fig. 18) OC_{LR} to represent by its length, on any convenient scale, the current through the coil ($= V \div \text{impedance of coil}$). Draw OV_L 90° in advance of OC_{LR} to

represent on any convenient scale the voltage required to send the current against the *reactance* of the coil, and set off OV_{RI} on the same scale in phase with OC_{LR} to represent the *resistance* component of the voltage. The resultant OV of OV_L and OV_{RI} will then be the terminal voltage. Now set off OC_K 90° in advance of OV to represent the condenser current. The resultant OC of OC_{LR} and OC_K will be the total resultant current of the combination. It will be seen that with the relative values of R , L , and K shown in Fig. 18, the resultant current OC *leads* the E.M.F. in phase by an angle ϕ . The condenser has here the preponderating influence. If OC_K had been somewhat smaller, OC would have been exactly in phase with OV . With still less capacity OC would lag behind OV , the self-induction preponderating. Hence the action of capacity in an alternating circuit is exactly opposite to that of self-induction, and one can be used to neutralize the other.

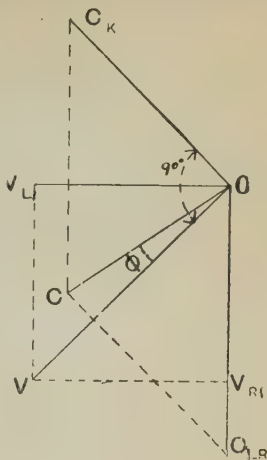


FIG. 18.

For example, let R be a non-inductive resistance placed across an alternating supply. It will take a certain current. Now place a choking coil L in series with it. The effect of this will of course be to diminish the current. Now, add further a condenser K in series with the other two (Fig. 19). The effect of this will be to *increase* the current in the circuit, if K be above a certain value. By adjusting K we shall find a certain capacity where L and K exactly neutralize one another, the current in the circuit now being the same as with R alone. V_L is now exactly equal to V_K . These voltages may be very many times greater than V , according to the value of R . If $R = 0$ (an impossible case in practice) there is no limit to V_L and V_K . The phenomenon accompanying these conditions is known as electric "**resonance**" or "syntony," the true explanation of the condition being that a combination of inductance and capacity (with small resistance) has a definite rate of electrical oscillation or frequency of its own, and if L and K have such a relation that this natural frequency coincides

with that of the impressed E.M.F., the current and voltages rise to a high value, as we have seen.

Now

$$\begin{aligned} V_L &= V_K \\ \therefore pLC &= \frac{C}{pK} \\ \therefore pL &= \frac{1}{pK}, \text{ or } p^2 LK = 1. \end{aligned}$$

This last equation gives the condition for the neutralization of the self-induction by the capacity. As stated above, this takes place when the frequency of supply is equal to the

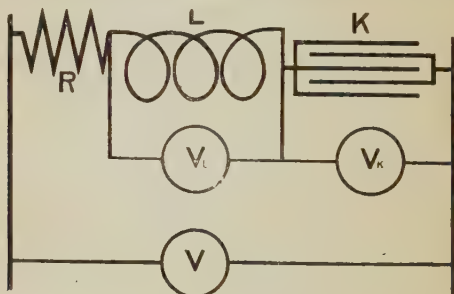


FIG. 19.

natural frequency of the combination. But $p = 2\pi n$, where n is the frequency of supply.

So we have from the above

$$4\pi^2 n^2 LK = 1.$$

Let t = time of one oscillation

$$= \frac{1}{n},$$

$$\therefore \frac{4\pi^2}{t^2} \cdot LK = 1,$$

$$\text{or } t = 2\pi \sqrt{LK}.$$

By hypothesis this is true only when the resistance is negligible.

In applying high potential tests to apparatus possessing sensible capacity, such as a long cable, care should always be taken to bring the pressure up gradually, and to avoid the use of choking coils in series with the cable. The cable should either be connected to the generator when running at full

speed, and then the field increased till the maximum voltage is attained, or, if this cannot be done, a non-inductive resistance should be inserted and gradually cut out.

As a practical example, we may assign values to R , L , and K in Fig. 19.

$$\begin{aligned}\text{Let } R &= 1 \text{ ohm,} \\ L &= .25 \text{ henry,} \\ K &= 10 \text{ microfarads,} \\ V &= 100 \text{ volts,} \\ n &= 100 \text{ cycles per sec.}\end{aligned}$$

The values of L and K have been chosen so that $p^2 LK = 1$.

The current through the circuit is therefore $= \frac{V}{R} = 100$ amps.

The voltage across the condenser is $= \frac{C}{pK} = 16,000$ volts. That across L is also 16,000. This example illustrates how easily a cable might be damaged in this way.*

MEASUREMENT OF POWER IN ALTERNATING CURRENT CIRCUITS.

In measuring the power in an alternating circuit transmitted by a current of C amperes at a voltage V , it is not sufficient to multiply V by C as in the case of direct current. Account has to be taken of the phase difference between current and voltage. As shown on p. 120, the power transmitted by C amps. at V volts, and with a difference of phase of θ° , is equal to $VC \cos \theta$.

Three-voltmeter Method.—This method, which is due to Ayrton and Sumpner, is sometimes useful where three voltmeters and a non-inductive resistance are procurable.

M (in Fig. 20) is the apparatus which is absorbing the power to be measured. It is placed in series with a non-inductive resistance, which should preferably drop about the same number of volts as M . V_1 , V_2 , and V_3 are three voltmeters connected as shown, and A is an ammeter measuring the main current. The power absorbed by M is given by

$$W = \frac{A}{2V_1} \left(V_3^2 - V_1^2 - V_2^2 \right) \text{ watts.}$$

The power factor is equal to

$$\frac{V_3^2 - V_1^2 - V_2^2}{2V_1V_2}$$

and may be obtained without knowing the current.

* See also paper by Duddell, *Journal I. E. E.*, 1901.

Care must be taken that the voltmeter currents are small compared with the main current.

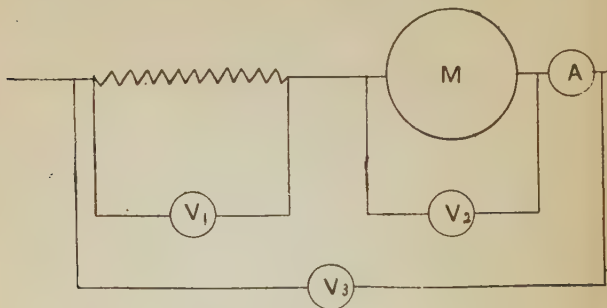


FIG. 20.

Three-ammeter Method.—This method, due to Fleming, consists in placing a non-inductive resistance in parallel with

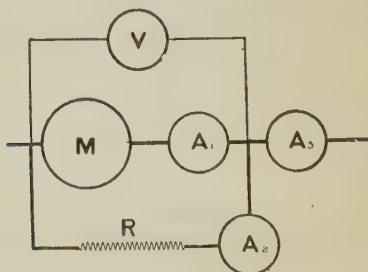


FIG. 21.

the load, three ammeters and a voltmeter being connected as shown. (Fig. 21.) Then

$$W = -\frac{V}{A_2} \left(A_3^2 - A_2^2 - A_1^2 \right).$$

One drawback to both these methods is that the result depends on the difference of the squares of observed readings,

and hence any small error in the readings is magnified in the final result.

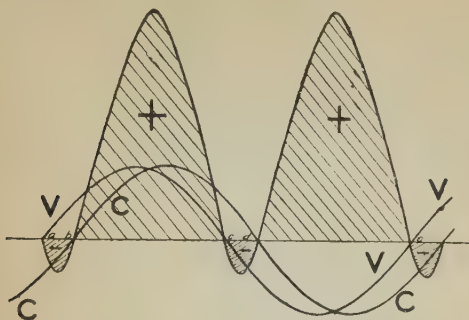


FIG. 22.

Wattmeter Method.—The easiest and best way of measuring alternating current-power is by means of a wattmeter. (See p. 162.)

To understand the way in which a wattmeter measures the mean power given to an alternating circuit, consider Fig. 22. C represents a wave of current lagging behind the impressed voltage V . Now the power at any moment is equal to the

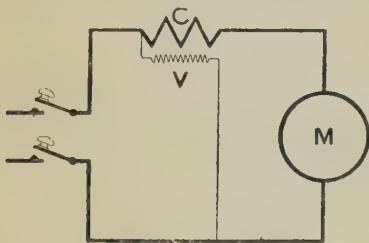


FIG. 23.

product of V and C at that moment. If we draw a power-curve, it will take the form shown by the curve enclosing the shaded areas.

Between b and c both V and C are $+$; hence power is being

given by the source to the circuit. Between c and d , however, V is $-$, while C is still $+$; hence their product is $-$, and power is being given by the circuit to the source. Between d and e again the power is $+$. The total power, therefore, given to the circuit between a and e is the algebraic sum of the two small shaded areas (reckoned negative), and the two large areas (reckoned $+$). Now in the wattmeter we have two coils, one traversed by a current proportional to V , and the other traversed by C . Hence the force exerted between the coils will be represented by the curve enclosing the shaded areas. The movable coil will be urged first to the left and then to the right, accord-

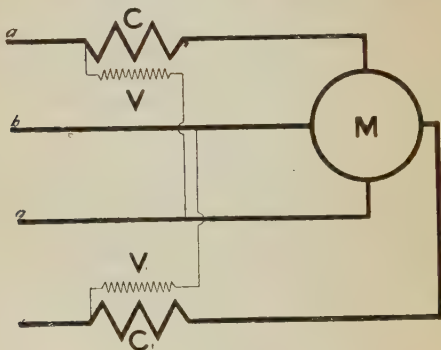


FIG. 24.

ing as the force is $-$ or $+$, and the needle will take up a position such that the mean force on it is balanced by the spring.

Inductive Error in Wattmeters.—The chief source of error in using a wattmeter arises from the self-induction of its potential circuit, which causes the current in that circuit to be smaller than it is assumed to be, and to lag slightly behind the voltage. This has the effect of decreasing the reading on non-inductive or capacity loads, and increasing it on an inductive load. The error, which may be negligible for non-inductive loads, increases rapidly as the power factor of the circuit approaches zero, and at a lag of 90° becomes infinitely large. In using a wattmeter on circuits with low power factors, therefore, a correction factor should be used when extreme accuracy is required.* This factor

* In many cases capacity in the volt circuit has to be reckoned with. For further particulars see C. V. Drysdale, D.Sc., *Electrician*, p. 61, vol. xlvii, 1901.

can be found when the self-induction, *i. e.* the angle of lag of current in shunt coil, is known.

The power carried by a current of C amperes at V volts, lagging by an angle θ , is equal to

$$VC \cos \theta \text{ watts.}$$

But the shunt current, instead of being $\frac{V}{R}$ amps., is $\frac{V}{R} \cos \alpha$ amps., where R is the resistance and α the lag in the potential circuit. This current is out of phase with the main current by an angle equal to $(\theta - \alpha)$, so that the wattmeter reading is equal to

$$VC \cos \alpha \cos (\theta - \alpha).$$

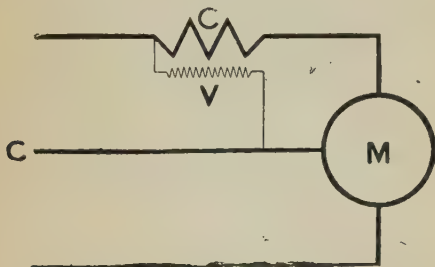


FIG. 25.

Hence the correction factor is equal to

$$\frac{\cos \theta}{\cos \alpha (\cos \theta - \alpha)}.$$

As α is always a very small angle, $\cos \alpha =$ approximately unity. Hence the above becomes

$$\frac{\cos \theta}{\cos (\theta - \alpha)}.$$

It will be seen that for the two cases in which $\alpha = 0$, and $\alpha = \theta$, the wattmeter reads correctly. The latter condition is when a balance exists between the tendency of the shunt lag to *increase* the reading, and the tendency of the inductance to *decrease* the reading by decreasing the shunt current.

Wattmeter in Single-phase Circuit.—Connect as in Fig. 23, where C is the current coil and V the potential coil, and M is the load. The watts indicated by the wattmeter include the loss in current coil, C . This is usually small if the current

does not exceed the rated current-capacity of the coil ; but in any case a correction can be made by subtracting C^2R watts, where C is the current and R the resistance of the coil.*

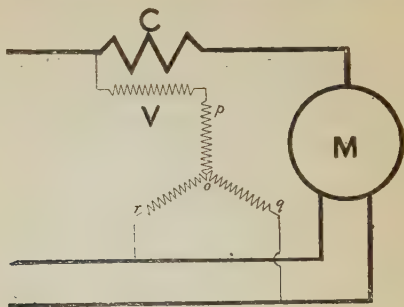


FIG. 26.

Wattmeter in Two-phase Circuit.—In Fig. 24 *aa* represents one phase and *bb* the other. If the load is balanced, *i. e.*

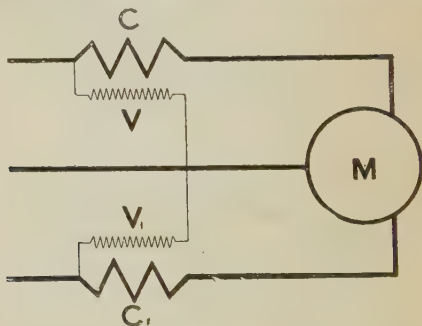


FIG. 27.

equal on each phase, as is approximately the case with a motor-load, connect C in one circuit, and V across the same circuit.

* In Weston wattmeters of small range this correction is unnecessary, as the volt coil is connected to the load side of the current coil, and a number of turns (equal in number to the current coil turns) are put in series with the volt coil and act in the opposite direction to the current winding.

Multiply the reading by 2 to get total power. If load is unbalanced, use another wattmeter V_1C_1 in the other phase. When measuring two-phase power with only **3 wires**, connect a wattmeter as in Fig. 25, C being the middle wire. Multiply the reading by 2. For **unbalanced load**, use another wattmeter in the other phase.

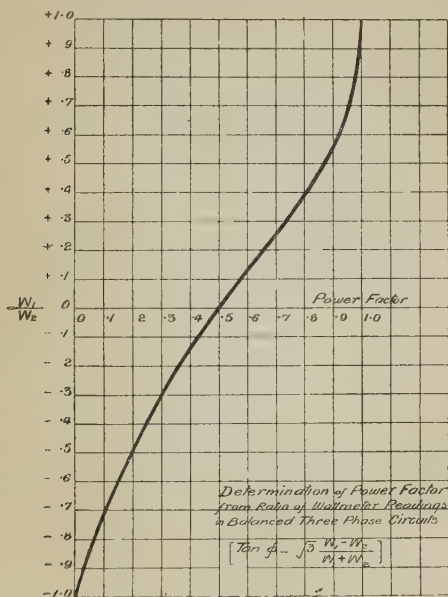


FIG. 28.

Wattmeter in Three-phase Circuit.—(i) When the load is balanced, connect the current coil, C , in one of the three lines, and use a Υ resistance* in connection with the potential coil as shown (Fig. 26). The resistance of op + resistance of V must equal that of og , which is also equal to or .

Multiply the reading by 3, to get the total power.

* When the "neutral point" can be got at, the coil V may be connected to it through a resistance, and the Υ resistance can be dispensed with.

(ii) When the load is unbalanced use two wattmeters, CV and C_1V_1 (Fig. 27), connected as shown. The total power will be given by the algebraical sum of the readings.*

With this method, if $\cos \theta = 0.5$ (*i. e.* if $\theta = 60^\circ$), one wattmeter will indicate zero, the other one giving the whole power. For power factors less than 0.5, one wattmeter will give a negative reading, and its connections must be reversed.

The ratio of the readings of two wattmeters so connected, on a balanced load, gives an indication of the **power factor** of the circuit.† The curve in Fig. 28 is very convenient when dealing with three-phase power measurement, as the power factor can be read off when the ratio of the readings is obtained. The negative sign indicates that one of the meters is reading negative power, *i. e.* that its connections have been reversed.

The curve holds strictly only for sine waves, and in many cases will be found not to agree exactly with the power factor obtained by dividing total watts by $\sqrt{3}$ times the volt-amperes.

The **two-wattmeter method** of measuring three-phase power is only a particular case of the three-wattmeter method, connections of which are shown in Fig. 29. A current coil is connected in each line, and the potential coils joined to a common

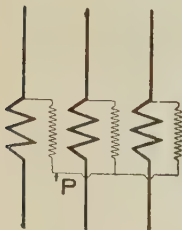


FIG. 29.

point P . It can be shown that whatever the potential of P may be, *i. e.* whatever resistances the potential coils may have, the sum of the wattmeter readings is a measure of the true power. Make the resistance of one potential coil = 0 (*i. e.* remove one meter), and the other two wattmeters together indicate the power. The only assumption involved in the proof of this is that the sum of the three currents at every instant is equal to zero. This being true for any and every system of transmission using three wires, the two-wattmeter method is applicable to any three-wire system, whether continuous or alternating.‡

In general the power carried by a system of n wires can be measured by $n - 1$ wattmeters.

* If the current coil of a wattmeter be inserted in one line of a balanced three-phased system, and the shunt coil connected across the other two lines, the instrument will read the numerical product of volts and wattless current, *i. e.* $V C \sin \theta$, thus giving an indication of the wattless current in the circuit.

† See also Weeks, *Elec. Engr.*, June-July 1904.

‡ See *Electrician*, vol. li. p. 743, August 1903.

Distorted Wave-forms.—It frequently happens in practice that the wave-forms of alternating current and voltage are far from following the sine law. It can be shown mathematically that any periodic wave, however irregular in shape, can be analyzed into a number of simple sine waves of various amplitudes and frequencies. In electrical work (as in acoustics), the sine component having the full frequency is called the fundamental, and the other components of higher frequency are called the harmonics.

The following points relating to non-sinoidal waves of current and E.M.F. are worth noting—

(1) Since the wave is symmetrical on either side of the zero, all the harmonics must be of *odd* frequency (considering the fundamental as unity). An even harmonic would produce dissimilarity between the + and - halves of the waves.

(2) Since the average power carried by an E.M.F. wave and a current wave whose frequencies differ is zero, it follows that with a sine-shaped E.M.F. all harmonics in the current wave are idle currents and carry no useful power. For instance, the 3rd and 5th harmonics in the magnetizing current of a transformer (supplied with a sinoidal E.M.F. wave) are idle components.

(3) The R.M.S. or effective value of a current or E.M.F. wave is equal to the square root of the sum of the squares of the effective values of all the components, irrespective of phase difference and frequency.

(4) If a non-sinoidal E.M.F. wave be applied to a non-inductive circuit, the resulting current wave will be of the same shape as the E.M.F. wave, and each component of the latter will produce a current of its own frequency and will carry power.

(5) If a non-sinoidal E.M.F. wave be applied to an inductive circuit, the resulting current wave will be more nearly a sine wave than the E.M.F., because the reactance of the

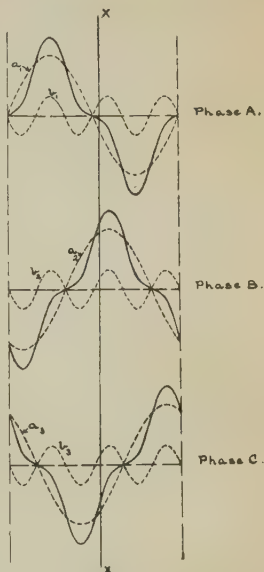


FIG. 30.

circuit will choke back the harmonics more than the fundamental on account of their higher frequencies.

(6) If a non-sinoidal E.M.F. wave be applied to a circuit having considerable electrostatic capacity, the resulting current wave will be farther still from a sine-shape, because the magnitudes of the currents due to the harmonics will be directly proportional to their frequency.

(7) In a balanced 3-phase 3-wire circuit the wave shape of the E.M.F. between lines can contain no triple harmonics. Also the current wave in the lines can contain no triple harmonics. Such harmonics can only exist as E.M.F.s between the neutral point and the lines, or as currents in closed "meshes" in the system.

To illustrate the truth of the statement in the last paragraph, Fig. 30 has been drawn. The full line curves may be taken to represent the three voltages induced in the windings of a 3-phase star-connected generator, and which contain a 3rd harmonic. The dotted curves show the components; a_1, a_2 , and a_3 being the fundamentals, and b_1, b_2 , b_3 the 3rd harmonics. The

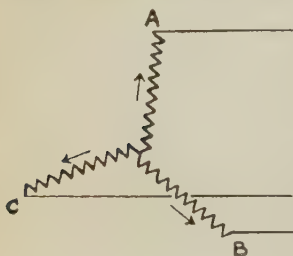


FIG. 31.

curves are so placed (120° apart) that the intersections with the curves of any line, such as $\times \times$, give the instantaneous values of the three E.M.F.s at any moment. It may readily be seen that whereas the sum of the instantaneous values of the fundamental components is zero, the values of the 3rd harmonics are the same in the 3-phases, and are acting in the same direction at every instant, *i. e.* either towards or away from the neutral point. With an insulated neutral, therefore, these harmonics can produce no currents in the lines or load, and the voltage wave between lines contains no 3rd harmonic, and, in the case illustrated, has a simple sine shape. The arrows in Fig. 31 show the directions of the 3rd harmonics at a certain instant, and it is clear that they have no closed circuit.

With a "mesh" connected machine, however, the triple harmonics have a closed circuit within the mesh, and current therefore flows round the windings to an amount depending on the triple frequency voltage and the value of the reactance (Fig. 32). But in this case also no triple current or voltage reaches the external circuit, the induced voltage harmonic being absorbed by the internal resistance and reactance.

A further illustration of the action of triple harmonics in a 3-phase system is to be found in the magnetizing currents of a 3-phase or 3 single-phase transformers, supplied from a 3-phase line. The triple harmonic required in the magnetizing current of a closed iron circuit to produce a sine wave of induction cannot be supplied by a balanced 3-phase circuit. If, therefore, the transformer windings are "star" connected, the current harmonic cannot flow, and the results are a flux harmonic and a distorted wave shape of voltage between lines and neutral. With a "mesh" connection, however (on either primary or secondary), a current harmonic flows internally, and reduces the flux harmonic practically to zero.*

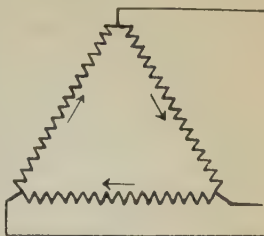


FIG. 32.

All that has been stated above with regard to 3rd harmonics applies equally to any harmonics whose frequency is a multiple of 3 times the fundamental frequency.

Testing of Iron for Hysteresis and Permeability.†

Magnetometer Method.—The specimen must be in the form of a rod, and in length should be at least 300 to 400 times its diameter. It is fixed in a vertical position, and magnetized by a solenoid carrying a current C and having N turns per cm. length. A small magnetic needle is suspended by a thread of silk so that it can turn in a horizontal plane, the lower end of the rod being situated in this same plane. The needle must be mounted due east or west of the specimen, so that when no current is flowing it points in a direction normal to a line joining it to the lower end of the rod. Let a and b be the distances of the needle from the lower and upper ends of the rod

respectively, then the force on the needle = $\left(\frac{m}{a^2} - \frac{mb}{b^3} \right)$, where

m = the total number of magnetic lines due to each pole $\div 4\pi$. Some of these lines are due to the magnetizing solenoid, and the effect of these on the magnetometer is usually balanced by a coil connected in series with the solenoid, and placed at such a

* See *The Electrician*, Nov. 10, 1905, p. 135. Also Jan. 5, 1906, p. 463.

† For a full discussion of this subject see Ewing's *Magnetic Induction in Iron and other Metals*.

distance from the needle, and with its axis in line with solenoid and needle, that when the specimen is removed, and current is allowed to flow, no effect is produced on it. The deflecting force on the needle during test, therefore, is due to the specimen alone. If F be this force, we have—

$F = h \tan \theta$, where h = horizontal component of earth's field, and θ is the deflection. We have, therefore,

$$\frac{m}{a^2} - \frac{ma}{b^3} = h \tan \theta,$$

from which m can be found for each current in the solenoid.

The magnetizing force applied to the specimen by a current of C amperes in the solenoid is equal to

$$\frac{4\pi NC}{10} = H.$$

The induction density (B) being found by dividing $m \times 4\pi$ by the cross-section of the specimen, the permeability is given by the ratio of B to H .

To obtain a hysteresis loop by this method the magnetizing force is carried through a complete cycle, and θ noted for each value of the current.

The deflections of the needle are usually read by a lamp and scale arrangement, a mirror being fixed upon the magnet, which is enclosed in a wooden or brass case to keep off draughts.

Rowland's Ring Method.—The specimen here is in the form of a ring. It is wound uniformly with a magnetizing coil, while a small search coil encircles it, and is joined in series with a ballistic galvanometer and an earth-inductor. The latter is for calibrating the galvanometer, and consists of a coil of fine wire of known area and number of turns. It is placed on a horizontal axis which can be turned suddenly through 180° . During this motion there will be a total number of lines cut equal to $n_1 VA$, where n_1 = number of turns in coil, A = effective area of coil, and V = vertical component of earth's magnetic field. This will produce a throw on the galvanometer needle. Let this throw be d_1 divisions. Then the throw per magnetic line reversed will be $\frac{d_1}{n_1 VA}$. If now the reversal of C amperes in the magnetizing coil produce a throw of d_2 divisions, we know that the number of lines reversed in the ring is equal to $VA \frac{n_1 d_2}{n_2 d_1}$, where n_2 = number of turns in search coil. The above expression divided by the cross-section of the ring in sq. cm. gives the induction density B . The magnetizing force (H) being equal to $\frac{4\pi nC}{10}$, where C = current in

amperes, and n = turns per cm. length on ring, the permeability is obtained by dividing B by H .

Another, and more usual method for calibrating the ballistic galvanometer consists in employing a long solenoid around the middle of which a small secondary coil is wound. Or, the latter is wound on a wood cylinder of known dimensions and slipped down inside the solenoid. The secondary is connected to the galvanometer, and the throw taken due to the reversal of a known current in the solenoid. If A be the amperes reversed in the solenoid, the field strength at the centre is $\frac{4\pi n A}{10}$ C.G.S. units,

where n = turns per cm. length. If the area enclosed by the secondary coil be $= a$ sq. cm., and the number of secondary turns $= n_1$, the total flux-change due to the reversal of A amps. will be $0.8 \pi n n_1 a A$, from which the ballistic throw per magnetic line may be found. If the secondary coil used for calibrating is connected in series with the search coil on the ring, and kept so during the test, the constant found as above may be used direct. If the resistance of the circuit is different when calibrating, the constant must be multiplied by the ratio of the total resistance of galvanometer circuit when calibrating to the total resistance when testing.

The hysteresis loop is obtained by taking the magnetizing current through a cycle in small steps, the throw on the galvanometer being noted for each sudden increment (or decrement) of the current. The sum of all the individual throws up to any point gives the total change of induction up to that point.

Hopkinson's "Bar and Yoke" Method.—For this method the specimen is in the form of a rod divided into two equal halves T_1 and T_2 (see Fig. 33). They are fitted as shown into a massive wrought-iron yoke A , whose reluctance is negligible compared to that of T_1 and T_2 . The latter are magnetized by



FIG. 33.

two coils C_1 and C_2 , and a search coil is threaded on between them. The butt-joint between T_1 and T_2 is made as perfect as possible by careful facing of the ends of T_1 and T_2 . The measurement of the flux in the specimen is made by suddenly opening the joint between T_1 and T_2 , and thus allowing the

search coil to be pulled out by a spring. A ballistic galvanometer and calibrating coil are included in series with the search coil as with the ring method, and the apparatus is used in the same way.

Ewing's Hysteresis Tester, for rapidly testing the hysteresis loss in a sample of sheet iron, consists of a *C*-shaped permanent magnet which is mounted upon a knife edge, and provided with a pointer and scale, by means of which the deflection of the magnet can be observed. Between the poles of the magnet, the specimen, consisting of a bundle of six or seven strips each $3'' \times \frac{5}{8}''$, is pivoted, and can be rapidly revolved by means of a handle. The handle is turned at such a speed as to make the separate impulses given to the magnet blend into a steady deflection. The hysteresis loss is not strictly proportional to the deflection, as the latter contains a small component due to loss in the magnet itself. The instrument is calibrated by means of two standards having known hysteresis constants, and from the straight line drawn between the two points representing hysteresis plotted against deflection for the standards, the loss corresponding to any other deflection can be read off. This line does not pass through the origin owing to the small constant deflection alluded to above.

It is advisable to frequently check the standard pieces, as their hysteresis loss is liable to variation with lapse of time.

Permeability Bridge.—The principle of this instrument for rapidly measuring the permeability of sample rods is shown by Fig. 34. *A* and *B* are two rods, *A* being a standard of known permeability, and *B* the test rod. They are united at both ends by yokes *C* and *D*. A magnetizing coil is slipped over each rod.

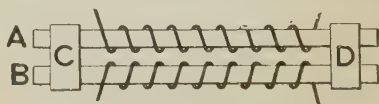


FIG. 34.

The currents in each coil are in such a direction as to drive the flux round the circuit formed by the two rods and yokes. It is evident that if the currents are so adjusted as to produce the same flux in *B* as in *A*, the yokes will be at the same magnetic potential, and will exhibit no polarity. The magnetizing force applied to both rods is known from the number of turns and the current. The induction density corresponding to that force is known for the standard, and hence for the sample, and the permeability of the latter can then be obtained.

To indicate the point at which the yokes are at the same potential, Mr. Holden, in America, has used a magnetic needle placed half-way between the yokes. Prof. Ewing, in his instrument, places a magnetic needle in the gap between two horns attached one to each yoke. When reversal of the currents produces no effect upon the needle, the balance is correct.

Drysdale's "Plug" Method.—This method was devised for testing the quality of iron in bulk. (See *Proc. Inst. Elect. Eng.* for January 1902.) A hole $\frac{3}{8}$ " diam. and $\frac{5}{8}$ " deep is drilled in the metal, leaving a pin standing in the centre $\frac{1}{16}$ " diam. (see Fig. 35). A plug of conical shape, and provided at its lower end with magnetizing and search coils (the latter being placed inside the former), fits the hole, and so produces a miniature permeameter within the bulk of the material. The tests with this apparatus can be made either with a ballistic galvanometer, or by means of a secolimeter and standard of self-induction. With the latter arrangement the permeability can be read direct, though the former method is to be preferred. Though the results obtained by the Drysdale method agree very well with one another, the curves appear to differ somewhat from those obtained by the ordinary methods.

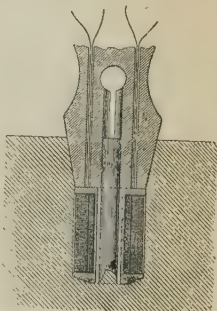


FIG. 35.

Wattmeter Method.—The methods described above may all be used for the measurement of hysteresis loss alone. In the wattmeter method, which has come into more general use during the last few years, the *total* loss in the iron (*i. e.* that due to eddy currents as well as hysteresis) is measured directly at working frequency. (See Fig. 36.) A magnetic circuit *A*, preferably ring shaped, is built up of the iron to be tested, insulated thoroughly between sheets, and wound with a suitable magnetizing coil *B*. The latter is connected to an alternating supply of known frequency, a suitable wattmeter *W*, and voltmeter *V*, being included in the circuit. Readings of watts are taken with known voltages applied to the coil. If the *CR* drop in the magnetizing coil be negligible, the induction density in the core for a given voltage and frequency may be calculated as follows—

Let W = wt. of iron in core,
 l = mean length of magnetic circuit,
 T = no. of turns in coil,
 n = cycles per second,
 B = induction density (lines per sq. inch),
 V = applied voltage.

Then the section of iron in the core = $\frac{W}{.28l}$.

Also—

$$\text{flux} = \frac{V \times 10^8}{4.44 T n} \text{ (for a sine voltage-wave),}$$

$$B = \frac{\text{flux}}{\text{section}} = \frac{0.28 V l \times 10^8}{4.44 T n W} \text{ lines per sq. inch.}$$

In order to analyze the total loss at a given B , and find what proportion is due to hysteresis, measure the total watts at two different frequencies, while keeping B the same in both cases. If W_1 be the watts measured at n_1 cycles, and W_2 at n_2 cycles,

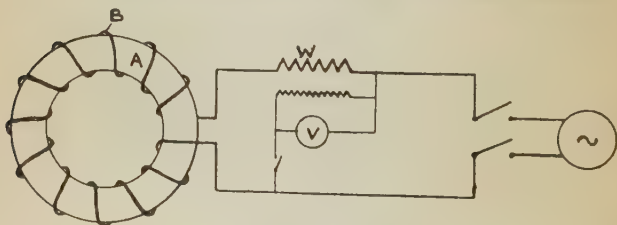


FIG. 36.

while h and e stand respectively for the hysteresis and eddy losses at n_1 cycles, we have—

$$W_1 = h + e,$$

$$W_2 = \frac{n_2}{n_1} \cdot h + \left(\frac{n_2}{n_1}\right)^2 e,$$

from which h and e may readily be found. It is convenient to take $n_2 = 2n_1$, in which case

$$e = \frac{W_2 - 2W_1}{2}.$$

The same result may be obtained by plotting the quantity watts / frequency as ordinates against frequency as abscissae, and producing the line so obtained back till it cuts the line of zero frequency at a point which gives a hysteresis constant. This constant, when multiplied by any frequency, gives the hysteresis loss at that frequency.

In the analyzing tests, where B is to be kept constant, the voltage must, of course, be varied exactly proportionally to the frequency.

In building up a core suitable for the above tests, it is best to use ring-shaped punchings to avoid joints. It is easier, however, to use a rectangular core, as the strips may be passed through coils already fixed in position. The joints in such cases must be as good as possible, as otherwise the loss will be increased by eddy currents occurring where the flux passes at the lap from the face of one sheet to the next.

A drawback to the wattmeter method of iron testing is that unless a low reading wattmeter is used, a large sample is required for the test. This is obviated by a method of Wild's* in which two coils are wound on the core (see Fig. 37). One (A) has few turns and is placed in series with the wattmeter current coil, while the other (B) has many turns and is connected across the potential coil of the wattmeter. If N_1 and

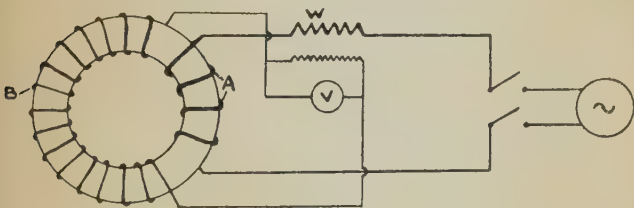


FIG. 37.

N_2 are the respective numbers of turns in these coils, then the reading of the wattmeter must be divided by N_2/N_1 , after correcting for any resistance drop due to potential current in coil B .

The foregoing method applies only where there is no magnetic leakage between coils A and B . This condition can always be realized in practice, as the shunt current is very small, and the coils can be wound concentrically on the core. Beattie† has applied a somewhat similar method to the measurement of the loss in a bundle of straight strips.

* See *The Electrician*, vol. liv, Nov. 11, 1904, p. 128.

† See *The Electrician*.

Measurement of Resistance.

Substitution Methods.—I. Place the resistance, X , to be measured in circuit with a source of E.M.F. and a current indicator, and note the deflection. Now replace the unknown resistance by a known variable resistance R , and adjust the same until the same deflection is obtained. The resistance of the variable resistance is then equal to that of the unknown resistance. The resistance of the rest of the circuit should be small compared with that to be measured, and bad contacts must be avoided. This method is employed in practice for insulation resistance measurement.

II. If the resistance (g) of the galvanometer is known, and that (r) of the rest of the circuit, then the unknown resistance X may be compared with any known resistance R , preferably of about the same value, by noting the relative current values, C_1 and C_2 when X and R are in circuit respectively. Then—

$$\frac{C_1}{C_2} = \frac{R + g + r}{X + g + r}$$

and, if g and r are negligible, then—

$$X = R \frac{C_2}{C_1}$$

III. —If, in either of the above methods, the only available R is much smaller than X , then the galvanometer may be shunted by a resistance s when R is in circuit. Then, in the first method—

$$X = \frac{g + s}{s} R,$$

and in the second method—

$$\frac{C_1 s}{C_2 (g + s)} = \frac{R + g + r}{X + g + r}$$

and

$$X = R \frac{C_2}{C_1} \times \frac{g + s}{s}, \text{ if } g \text{ and } r \text{ are negligible compared with } R.$$

Differential Galvanometer Method.—Arrange the unknown resistance X and a known variable standard resistance R in the two circuits of a differential galvanometer, connecting the circuits in parallel. Adjust R until zero deflection is obtained, then $R = X$. If the instrument is sensitive, and of not too high a resistance as compared with X and R , very accurate results may be obtained. If zero deflection cannot be obtained, let a = deflection when R has nearest value below, and B = deflection when R has nearest value above the correct value, and let d = the difference between the two values of R used. Then—

$$X = R + d \frac{a}{a - b},$$

“ b ” being considered negative when in the opposite direction to “ a .” If R is much smaller than X , the coil in series with R may be shunted by a resistance s . Then, calling the resistance of the galvanometer g —

$$X = \frac{g + s}{s} R.$$

Workshop Method.—We may obtain X from Ohm’s law by measuring the difference of potential V between the two ends of X when a current A is passed through. If the ammeter has a known resistance r , and the voltmeter is connected across both ammeter and resistance, then—

$$X = \frac{V}{A} - r.$$

If the voltmeter has a known resistance r , and it be connected to the ends of X —

$$X = \frac{V}{A - \frac{v}{r}}.$$

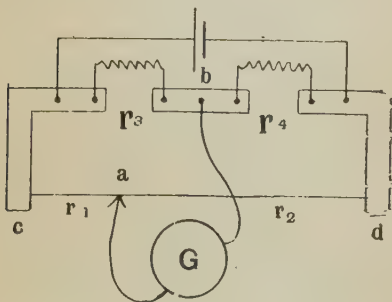


FIG. 38.

If in the first case the ammeter resistance is negligible, or if in the second case the voltmeter current is negligible, then—

$$X = \frac{V}{A}.$$

Wheatstone Bridge.—The essential parts of a Wheatstone Bridge are a wire cd of uniform resistance, a sliding contact a , certain connecting strips of negligible resistance indicated in the figure, and a scale for measuring lengths on the wire (see Fig. 38).* The unknown resistance is r_1 , and r_3 is any known standard resistance. The ratio between the resistances r_1 and r_2 is indicated by the lengths ca and ad of wire on either side of a . The sliding contact is adjusted until no deflection is obtained in the galvanometer, in which case

$$\frac{r_1}{r_2} = \frac{r_3}{r_4}, \text{ or } r_4 = r_3 \frac{r_2}{r_1}.$$

Post Office Bridge —In the Post Office Bridge the resistances r_1 , r_2 , and r_3 consist of a number of wire resistances mounted across gaps in a large brass bar, the gaps being short-circuited by plugs. On the removal of a plug, the corresponding resistance is inserted.

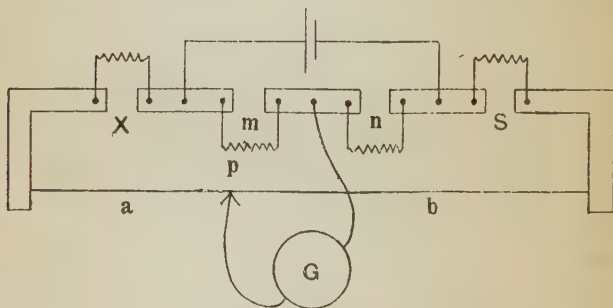


FIG. 39.

The ratio arms contain resistances of 10, 100, and 1000 ohms, or some similar arrangement, and the variable arm contains resistances so graduated that a maximum number of combinations may be obtained. By this arrangement, any resistance from 1 to 10,000 ohms, stepping up by an ohm, may be obtained, and by means of the ratio arms this range may be extended so that unknown resistances from .01 to 1,000,000 ohms may be measured.

* Various modifications of the Wheatstone Bridge have been devised ; see amongst others, p. 165.

The **most sensitive arrangement** is that which will give the greatest galvanometer deflection for a given small variation of r_3 . The following rules may be observed—

Let the galvanometer resistance be g and the battery resistance b . Whichever of these is the lower should be connected across between the junction of the two lowest resistances, out of r_1, r_2, r_3 , and r_4 , and the junction of the two highest.

The best value for r_1 is that most nearly equal to $\sqrt{b \times g}$.

The best value for r_2 is that which $= \sqrt{\frac{b + r_4}{g + r_4} \times g \times r_4}$.

Comparison of Standard Resistances.

Carey-Foster's Method.—This is a very accurate method for comparing two resistances of nearly equal value (see Fig. 39). As usually employed, the resistance X and the standard S are

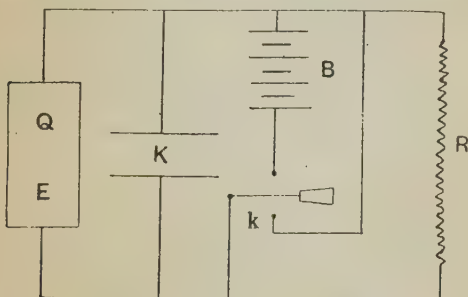


FIG. 40.

connected in the bridge as indicated, also two resistances m and n , preferably of about the same resistance as S and X . Balance is obtained at some point P in the wire. X and S are now interchanged and balance again obtained, this time at some point P' . The resistance of the length of wire between P and P' will be equal to the difference between X and S . When the resistances are interchanged, the point of balance moves towards the larger of the two.

The contacts between X and S and the bridge are best made by mercury cups. The resistances of the contact strips and soldered joints in the bridge cancel out, and are therefore negligible.

The position of the points P and P' may be altered by increasing or decreasing the resistance m or n .

Readings should be taken on several parts of the wire and the mean taken.

In a modification of this method introduced by Dr. C. V. Drysdale and Mr. A. D. Hughson,* the resistances X and S are placed in the positions now occupied by m and n . In this case $S - X = \frac{S + X}{a + b + m + n} \times (\text{resistance of wire between points of balance})$. The advantages of this method are, amongst others, (1) greater simplicity of connection and commutation; (2) by adding two extra resistances the bridge can be used for the Kelvin method, and so is applicable to existing small resistances. (The Carey-Foster method is not suited for the measurement of resistances of less than $\frac{1}{10}$ ohm.)

Measurements of High Resistances by Loss of Charge Method.

QE = Quadrant Electrometer (see Fig. 40),

K = Condenser,

R = High resistance to be measured,

B = Battery,

k = Charging, discharging, and short-circuiting key.

Hold k against the battery contact, and thus charge the condenser K and the electrometer. Note the electrometer reading corresponding to the potential applied.

Now put the key in the middle position, and the condenser will slowly discharge through the resistance R . Note the electrometer deflections frequently.

Let Ω = resistance of R in megohms,

K = capacity of condenser in microfarads,

D = deflection just before discharge commences,

D_1 = deflection t seconds after discharge commences.

Then

$$\Omega = \frac{t}{2.303 K} \log \frac{D}{D_1}.$$

NOTE.—The condenser discharges through its own internal resistance and through the insulation resistance of the apparatus used, in parallel with the resistance R . For accurate results, therefore, a reading should be taken of all those leakage resistances with R disconnected. Let this reading be Ω' , and let the g with R connected as well be Ω'' ; then the true resistance of $R = \frac{\Omega' \Omega''}{\Omega' - \Omega''}$. If the external leakage be negligible, then Ω' will be the internal insulation resistance of the condenser. There is no better way of measuring this.

* See *Electrician*, vol. xlv. p. 883.

Workshop Method.—Place the resistance (R) to be measured, in series with a sensitive voltmeter of known resistance (preferably of the moving coil type). Apply a voltage and take a reading (v). Then short-circuit R and read again (V). Then $R = r \times \frac{V-v}{v}$, where r = voltmeter resistance. This method is often convenient for measuring insulation resistances.*

Resistance of a Battery.—*First Method.*—The resistances used in these methods must not be very large or very small as compared with b , the battery resistance. Let g = galvanometer resistance. Let c_1 and c_2 be the relative current values when the circuit is closed through resistances r_1 and r_2 . Then—

$$\frac{c_2}{c_1} = \frac{b + g + r_1}{b + g + r_2}$$

from which b may be obtained. c_2 should be about twice c_1 . The key contact and other contacts should have a negligible resistance. Take all readings quickly, or polarization will take place.

Second Method.—By means of a resistance r_1 obtain a convenient deflection on the galvanometer of resistance g shunted by a resistance s .

Now remove the galvanometer shunt and increase the resistance until the same deflection is obtained. Let the resistance used be r_2 .

$$b = \frac{s}{g} (r_2 - r_1) - r_1.$$

It is unnecessary to know the E.M.F. or the current obtained.

Third Method.—Place a voltmeter or high-resistance galvanometer, and an ammeter or low-resistance galvanometer (the galvanometers having a known constant), in parallel across the terminals of the cell, with a key in each circuit. Put any resistance in circuit with the ammeter, so that the voltmeter deflection is about halved when the ammeter circuit is closed. Let the voltmeter reading be v_1 when the ammeter circuit is open, and v_2 when there are c amperes flowing through the ammeter. Then $b = \frac{v_1 - v_2}{c}$.

Fourth Method.—With a voltmeter, or high-resistance galvanometer of known law but unknown constant and a known variable resistance connected across the cell terminals, let r_1 be the reading. If the resistance r be so chosen that v_1 is doubled when the circuit is opened, then $b = r$.

* The application of this method is limited by the resistance of the voltmeter; it will usually be found possible to read (say) 1 megohm per 100 volts with a fairly sensitive moving-coil voltmeter.

Fifth Method (Mance's Test).—Connect up as shown in Fig. 41 with three resistances— r_1 , r_2 , r_3 , a galvanometer and a short-circuiting key. Of the three, r_2 should preferably be nearly equal to the resistance of the cell. Adjust until the galvanometer deflection does not change on depressing the key. Then $b = r_3 \frac{r_2}{r_1}$. If the galvanometer is a sensitive one, work to a false zero.

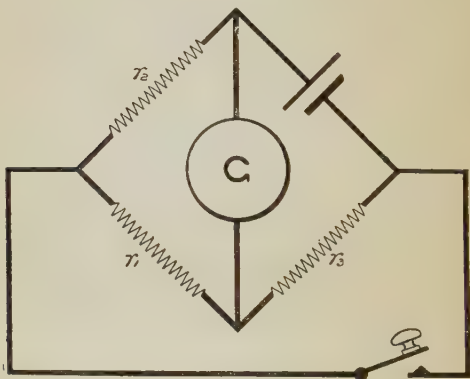


FIG. 41.

Galvanometer Resistance.

First Method.—It is usually most convenient to measure the resistance of a galvanometer by means of another galvanometer and one of the usual methods for measuring resistance.

Second Method.—Connect up as for the first method of measuring battery resistance. The resistance b must be known or negligible. The same formula is used.

Third Method.—Connect up as for the fifth method of measuring battery resistance, only interchange the battery and galvanometer in the connections and formula.

Measurement of Electromotive Force.

First Method of Comparison.—Use any suitable galvanometer (the constant of which need not be known), shunted or unshunted, of known resistance, and a known variable resistance. Find the values of the resistances r_1 and r_2 , which will give the same deflection when in series with the E.M.F.s E_1 and E_2 . Then

$$E_1 = E_2 \frac{r_1 + g}{r_2 + g},$$

where g = galvanometer resistance. Resistances used should be large in comparison with the battery resistance.

Second Method of Comparison.—With a suitable high-resistance galvanometer of known constant, or a voltmeter, compare the deflections obtained with the two E.M.F.s.

Third Method of Comparison.—Specially suitable with cells that polarize readily. With a suitable ballistic galvanometer and condenser, connected up as shown in Fig. 42, compare the deflections obtained on charge and discharge with either E.M.F. in circuit. The deflections, if small, will be proportional to the E.M.F.s.

Fourth Method of Comparison.—Connect up the cells with two resistances r_1 and r_2 and a galvanometer, as shown in Fig. 43. Note that opposite poles of the cells are connected together. Adjust r_1 and r_2 until there is no deflection, then $E_1 = E_2 \frac{r_1}{r_2}$, assuming

the battery resistances are negligible.

Fifth Method—Direct.—Connect in circuit with the E.M.F. to be measured, an ammeter (or galvanometer of known constant) of known resistance r_1 and a resistance R to adjust the current within the range of the instrument. Then, having read the current C , $E = CR$.

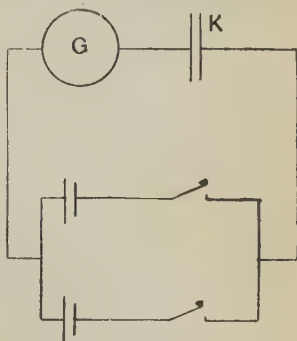


FIG. 42.

Standards of E.M.F.

The original standard, devised by **Latimer Clark**, which has an E.M.F. of 1.433 * B.O.T. volts at 15° C., and has been adopted by the Board of Trade, consisted of a mercury zinc cell, using mercurous and zinc sulphate as the electrolyte. For the setting

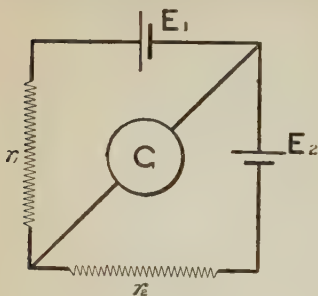


FIG. 43.

up of the cell a specification was drawn up, giving minute details for the purification and treatment of the materials used in its construction (see Fig. 44).

Dr. Muirhead devised a cell containing very much less mercury, which was held by surface tension in a spiral of platinum wire amalgamated by plunging it red-hot under mercury or by boiling it in the latter. The cells

were also rendered more uniform, and in the latest pattern of Muirhead cell the spiral of platinum wire is in the form of a flat helix at the bottom of the containing vessel. Such a cell has an

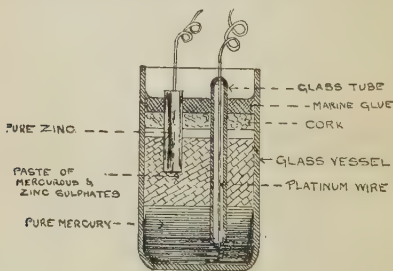


FIG. 44.

"Board of Trade" Clark Cell.

E.M.F. of 1.433 volts at 15° C., and the voltage decreases .078 % or .077 % per degree C. rise of temperature.

* The E.M.F. is usually given as 1.434, but the Reichsanstalt (1903) finds the mean value to be 1.4328 volts at 15° C. See also Kahle Wiedemann, *Annalen*, lix. p. 582, and *Phys. Rev.*, vii. p. 257.

Another form of cell has been devised by Professor **Carhart** of the Clark type which has very small temperature coefficient, amounting to only $\cdot 038\%$ per degree C. It also readily takes up the temperature of the room.

When using Clark cells it is not advisable to work them through less than 10,000 ohms.

Dr. Hibbert, working on the lines of a cell originally designed by Helmholtz, has produced a very satisfactory **1 volt standard**. In this cell the sulphates of mercury and zinc are replaced by chlorides of these metals, while the mercury and zinc electrodes

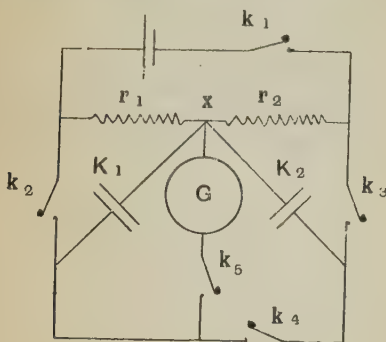


FIG. 45.

are still retained. One valuable feature of these cells is that they can be constructed to give accurately 1 volt at any given temperature, and their temperature coefficient is only $\cdot 01\%$ per degree C.

Yet another form of standard is the **Cadmium cell**, which is constructed similarly to the Clark cell, but using cadmium amalgam instead of zinc, and cadmium sulphate crystals instead of zinc sulphate.

The Cadmium Amalgam has the composition of 1 part of cadmium to 6 parts of Hg. The E.M.F. is approximately 1.0188 volts. The temperature coefficient is from $\cdot 002$ to $\cdot 004\%$ per degree C., when the cell is carefully set up with specially purified chemicals. The time lag for temperature is, however, very great, but the recovery from polarization fairly rapid.*

* Henderson, *Phil. Mag.*, July 1899.

Measurement of Capacity.

First Method of Comparison.—A ballistic galvanometer is required, a standard condenser, and a source of E.M.F. Charge the condensers separately from the same E.M.F. and note the respective deflections on discharge. The latter will, if small, be proportional to the capacities.

Second Method of Comparison.—A sensitive galvanometer (preferably ballistic) is required, also two variable resistances, and a source of E.M.F. Connect up as shown in Fig. 45, with five keys in circuit. Depress k_1 , k_2 , and k_3 , and charge the condensers. Release k_3 , k_2 , and k_1 , and close k_4 . After a few seconds, close k_5 . If there is no deflection, then $K_1 = K_2 \frac{r_2}{r_1}$. If there is any deflection, adjust r_2 and r_1 , and try again. Note that the biggest resistance is associated with the smallest capacity.

If the capacity to be measured is that of a cable, the junction of the condensers at the point x is made through the earth.

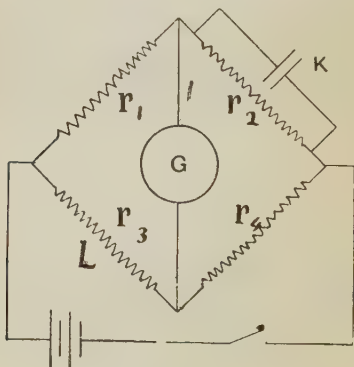


FIG. 46.

Third Method.—Connect up as shown for measurement of "high resistance by loss of charge" (p. 148). Use a known high resistance R_1 and make the test as described. Then

$$K = \frac{t}{2.303 R \log \frac{D}{D_1}}$$

Measurement of Self-Induction.

Let K = capacity of condensers in farads. The resistances r_1 and r_2 should be high-resistances and the galvanometer a high-resistance one (see Fig. 46).

Depress the battery-key and adjust to zero deflection. Release the key, and note the throw d_1 . Remove the condenser, and obtain a second throw d_2 .

$$L = \frac{d_2}{d_2 - d_1} Kr_1 r_2.$$

Workshop Method.—Measure the impedance of the circuit whose inductance is required by applying a known alternating voltage and measuring the current flowing. If r = resistance, V = voltage at frequency n , C = current, and L = inductance in henries,

$$\text{Then } L = \frac{1}{2\pi n} \sqrt{V^2/C^2 - r^2}.$$

If r^2 is small compared with V^2/C^2 , then

$$L = \frac{V}{2\pi n C}. \quad (\text{See p. 122.})$$

ELECTRICAL MEASURING INSTRUMENTS.

The **Kelvin Balance** and the **Potentiometer** are perhaps the most generally used standards for current measurement. The former consists of a beam carrying a coil at either end and hung by flexible ligaments, which also serve to lead the current into and out of the moving system. One coil is attracted and the other repelled by a pair of fixed coils connected in series with the movable coils. Balance is restored by a fixed counter-weight in a pan at the end of the beam, together with a sliding weight which can be pulled along the graduated beam until equilibrium is restored. The reading is, of course, proportional to the square of the current, but in order to save calculation there is a fixed

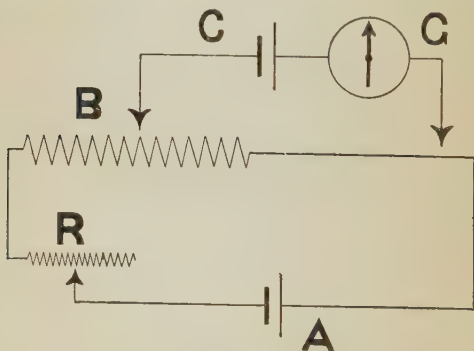


FIG. 47.

scale graduated in amperes (or fractions of amperes, according to the weight used), which can be employed in all cases where extreme accuracy is not required. In instruments which are to be used for alternating currents, a stranded cable is employed in which each strand is insulated so as to reduce skin effect and eddy currents. Voltmeters and wattmeters are also constructed on the same principle.

For direct current standardization the **potentiometer**, in which all measurements are made in terms of standard resistance and standard E.M.F., will be found most useful. The best-known potentiometer is that of Crompton, in which a stretched wire and 14 coils of equal resistance are employed. A constant

current from a small accumulator (*A*, Fig. 47) is sent through the system, and the regulating resistance (*R*) is adjusted until the galvanometer (*G*) shows no deflection when contact is made with the slide wire at a given point. When this is the case it is clear that the fall of potential between this point and the point *B* is equal to the E.M.F. of the cell *C*. A Clark or Cadmium cell is usually employed for this purpose, and when once the wire has been standardized in this way any other E.M.F. less than 1.5 volts can be measured. Currents are measured by noting the drop of potential produced over a standard resistance (usually constructed of manganin). Resistances can be measured in like manner by noting the drop produced by a known current. Voltages higher than 1.5 are measured by "tapping" off a known fraction, say $\frac{1}{100}$ or $\frac{1}{500}$. Special precautions are taken to prevent thermo E.M.F.s, and care must be also taken to insulate the battery *A* from the circuit under test.

The **Siemens Electro-Dynamometer** is sometimes used as a secondary standard for current measurement. In this instrument a fixed coil and a moving coil are normally at right angles to one another. If a current be sent through them in series the movable coil will tend to deflect. This motion is opposed by turning a milled head carrying a spring, one end of which is fixed to the movable winding. The angle turned through by the milled head is proportional to the torque on the spring, which is, in its turn, proportional to the square of the current.

If care be taken to keep all masses of metal away from the instrument, it will read equally accurately with the alternating currents of any frequency and with continuous currents. If eddy currents are produced their presence can be detected by sending an alternating current through the movable coil only, when, if any deflection is produced, it is due to eddy currents. If a continuous current be employed and a deflection is produced, it will be due to an external magnetic field. Even the earth's field may produce an error of as much as 2 %.*

Instead of sending the same current through the two coils in series, one may be wound with fine wire and can be connected across the load while the other coil carries the main current. The instrument will then act as a wattmeter; if the self-induction of the fine coil be negligible it will read correctly with alternating currents of any frequency and any phase difference. (See also p. 129 and p. 162.)

Galvanometers may be divided into two main classes—

- (1) Fixed coil and moving magnet.
- (2) Fixed magnet and moving coil.

* *Pro. Roy. Soc.*, 1899, and *Phil. Mag.*, S. 5, vol. 47, No. 8.

Of class (1) the **Kelvin** is best known, and is probably the most sensitive galvanometer which can be obtained. (Fig. 48 shows the needle of such a galvanometer.)

Instruments belonging to class (2) (due to **D'Arsonval**) have the great advantage of being capable of perfect "dead-beatness," though they cannot be made quite so sensitive, chiefly owing to

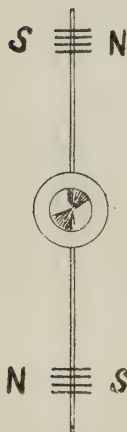


FIG. 48.

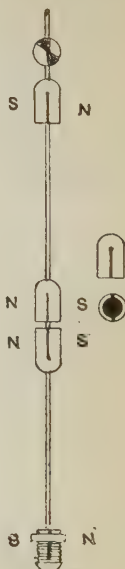


FIG. 49.

the necessity of providing metallic connection with the moving coil. They may have (a) wide coils with soft iron cores (ordinary type); (b) narrow coils (due to Ayrton) in which the coil is used without any core at all. A phosphor-bronze strip (.3 to 1 mil. in thickness, and 10 to 25 mils. in width) is usually employed to support the coil from above, while a spiral of many turns of the same metal leads the current out. Or the coil may be hung between the two strips stretched tight* (c. g. Sullivan and others).

* Crompton uses a bifilar suspension and a round coil.

For condenser work a "**Ballistic Galvanometer**" (*i. e.* one having a long time of swing and little "damping") must be used (see p. 154). This is usually of class (1), though a D'Arsonval galvanometer can be used for rough work, so long as the coil is not wound on a metal frame. The magnets in the former case are usually of the bell shape, so as to possess a considerable moment of inertia, and yet not to produce much air friction. They are usually suspended by unspun silk, or better still by a fine quartz fibre (C.V. Boys). Fig. 49 shows a usual form of needle for a ballistic galvanometer.

The **permanent magnets** of D'Arsonval galvanometers are usually "**aged**" by being immersed in hot or boiling water for an hour or so. Some firms also subject them to mechanical vibration. If a magnet is to keep its strength well, it is important that the air-gap should be short and of large area, and the ratio of length to area of steel as great as possible.

Commercial Measuring Instruments.

These may be divided into—

- (1) Electro-magnetic.
- (2) Electro static.
- (3) Thermal.
- (4) Induction of Ferraris type.

Electro-magnetic Instruments are the most widely used, and may be subdivided into—

- (a) Direct pull (moving iron).
- (b) Fixed coil, and moving permanent magnet.
- (c) " " " " soft iron needle (moving iron type).
- (d) Moving coil and permanent magnet (moving coil type).
- (e) " " " " electro-magnet (dynamometer).

Only a very few types can be mentioned here.

Of class (a), the **Kelvin** gauges, of which a solenoid attracts an iron wire attached by a cord to the moving system, may be alluded to, although little used now.

The majority of instruments at present in use belong to class (c). As a rule, a fixed iron repels a movable iron fixed to the axis carrying the pointer. All instruments of types a, c and e are suited for use with **alternating currents**, though they require special calibration. In the case of ammeters, the hysteresis in the iron causes the instrument to read low with alternating currents, and with voltmeters the error is still further increased by the self-induction of the coil.

In the "Universal" type of instrument (Class a), of Messrs. Everett, Edgecumbe & Co., **errors** from this cause are made quite **negligible**, partly by employing a special form of coil, and partly by reducing the induction to a minimum. These instru-

ments can be relied upon as correct to within 1 % of the maximum reading, whether used with continuous or with alternating currents of any frequency or wave-form.

Electro-magnetic voltmeters require from 30 to 100 m.a., and from 300 to 400 **ampere-turns**, while an ammeter is usually wound to 400 or 500 ampere turns, and absorbs (say) 1 watt.

Of instruments belonging to class (*d*), which can only be used for continuous current, the earliest were the **Weston**. The principle upon which they work was, however, originally due to **D'Arsonval**, and consists of a coil turning about its own axis in the field of a powerful permanent magnet. The motion is almost always controlled by a spring or springs which frequently also lead the current into and out of the moving coil.

The arrangement is somewhat complicated, and requires the utmost care both as regards workmanship and the selection and ageing of the **springs** and magnets. For the former phosphor-bronze is almost exclusively used, while the **magnets** are usually **aged** by heating to (say) 80° C. for a more or less prolonged period. In order that they may keep their magnetism the smallest possible air-gap is essential.

As regards the **power absorbed**, a voltmeter of this type usually takes about 10 m.a., while ammeters, which have, of course, to be shunted, require a drop of .05 to .1 volt.* In the case of ammeters the moving coil is shunted by a resistance (spoken of as a shunt) usually of constantan, manganin, or the like.

If the moving system be light and the coil wound upon a copper or aluminium frame, perfect "dead-beatness" can be obtained. If the air-gap flux is of uniform strength, the scale will be even over the whole range.

It is of the utmost importance in the case of ammeters that some resistance, having a negligible temperature coefficient, should be put in series with the moving coil, or a very large **error** will result **from changes of temperature** (4 % per 10° C.).

Electro-static Instruments.—These instruments are generally of two main types: (1) an aluminium needle fixed at right angles to a suspension or pivot, repelled or attracted by a pair of quadrants between which it is free to move (Kelvin, etc.); (2) curved blades forming part of a cylinder of which the pivot forms the axis (Ayrton-Mather). The main **drawbacks** to this type of instrument are, that the moving mass is necessarily large, while the force is small, and hence the friction of the pivots is excessive, and there is a great want of "dead-beatness." The force for low voltages (down to 100 volts) can be made greater by increasing the number of needles and sectors (multi-cellular type). Swinburne and others have employed friction-wheels to

* A drop of 0.075 volt has now been standardized by the Engineering Standards Committee.

reduce the friction. Magnetic or oil damping is often used. It is, further, difficult to insulate the moving vanes, as should these become even slightly bent, then they have a great tendency to spark across. Each instrument should on this account be provided with a high resistance in series with it.

For extra high pressures, say over 6000 volts, condensers are often connected in series with the instrument to increase the range.

The scale is usually short, and often somewhat uneven. Owing to these various disadvantages electro-static instruments are not now very largely used except as Leakage Insulators, one being for this purpose connected between each main and earth.

Of class (3) the best-known examples are the following—

(1) **Cardew's Voltmeter**, in which a long wire is heated, and in expanding rotates the pointer. The wire is usually carried round pulleys, and passes several times up and down a long tube having the same coefficient of expansion as the wire itself, so as to eliminate temperature errors as far as possible.

(2) Messrs. **Hartmann and Braun**, some fifteen years ago, introduced a type which is still extensively used on the Continent. The current to be measured flows along a stretched wire of platinum-silver some 6" long. Its expansion is magnified by attaching to its centre, at right angles to it, a phosphor-bronze wire, which is, however, not in circuit. The other end of this second wire is rigidly fixed, and to its centre is attached a silk thread which, after passing round a pulley on the spindle of the pointer, is attached to a spring. Thus the whole system is in tension and the magnification is very considerable. The motion is damped by an aluminium disc rotating between the poles of a small permanent magnet. The whole is mounted upon a plate having the same coefficient of expansion as the wire itself. In the case of ammeters a shunt is used, having a drop of .25 volt, while a thicker hot wire is required, and is generally so connected that the current flows through it in four parallels.

Hot wire instruments have the advantage of being equally accurate with direct or with alternating currents of any frequency. They are also absolutely unaffected by external magnetic fields.

The zero position of the pointer is, however, very liable to alter (and with it the accuracy of calibration). The power consumed is also excessive, and they are easily burnt out by an overload.

Induction (or Ferraris) instruments are often used owing to the fact that scales subtending an angle of 300° or more can be obtained. The principle of action is similar to that

of the meters described on p. 189. In the case of wattmeters the construction is precisely the same as that of the watt-hour meter, a pointer and control spring taking the place of the counting train and damping magnet. For ammeters and voltmeters (in place of two windings, one carrying the main current and the other a shunt current), both circuits are connected in parallel, one being as inductive and the other as non-inductive as possible.

The torque is consequently proportional to the square of the current, or nearly so. As a result, while the scale of an induction wattmeter is nearly evenly divided, that of an ammeter or voltmeter is much more open at the upper end. To correct this objection the disc or drum is often cut away, so that, as the deflection increases, the area of the disc threaded by the magnetic field becomes less and less. In "shaded pole" ammeters and voltmeters part (usually about $\frac{2}{3}$) of the polar area is surrounded by a copper ring or winding in which eddy currents are induced. Two magnetic fields displaced by nearly 90° are thus produced, which give the necessary torque, while only one winding is required. Induction instruments are only suitable for switchboard and approximate commercial work owing to their large frequency and temperature errors, which usually entirely swamp any supposed gain in accuracy due to the long scale.

Wattmeters are usually constructed either on the dynamometer or on the Ferraris principle. The dynamometer type is similar to the Siemens dynamometer (see p. 157), except that most modern instruments are provided with an indicating pointer moving over a direct-reading scale against the opposing torque of a spiral spring. In order that the indications may be accurate at all power-factors and frequencies, the following points must be attended to: * (1) Self-induction and capacity of moving coil must be kept very low. † (2) Eddy currents must be avoided. (3) If transformers are used in connection with a wattmeter, the phase relation between primary and secondary volts must be the same as that between primary and secondary amperes. When carefully constructed dynamometer wattmeters are capable of extreme accuracy. Ferraris type wattmeters are chiefly used for switchboards. Although they have been greatly improved of late years, they are still inferior, as regards accuracy, to the dynamometer type.

Power-factor Indicators, ‡ which are now coming much into use, may be divided into two classes: (1) instruments showing

* For a full discussion see Edgcumbe and Punga, *Jl. Inst. Elec. Engs.*, No. 167, vol. xxxiii., 1904.

† See p. 129.

‡ For theory see Sumpter, *Phys. Soc.*, Oct. 1905.

the wattless component of the load ; and (2) those showing the actual power-factor or angle of lag. An ordinary wattmeter with a highly inductive resistance in series with the volt coil, in place of the usual non-inductive resistance, will indicate the so-called "wattless component" of the power, and several instruments of this type are on the market.

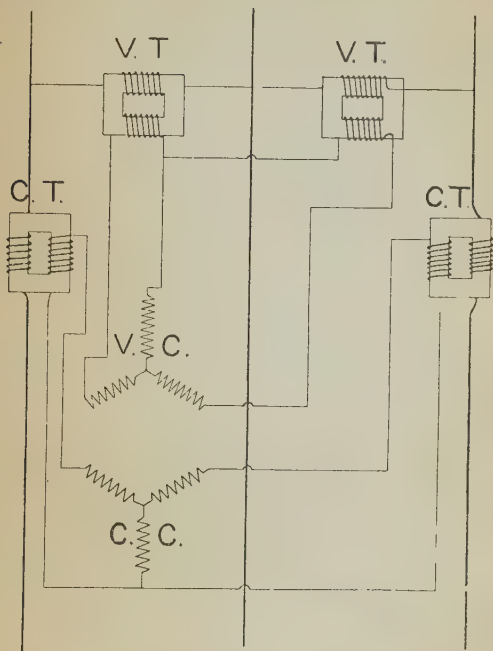


FIG. 50.

Of power-factor indicators proper, that is to say, those of the second class, the simplest arrangement, for a single-phase circuit, consists of a fixed current coil, as in the ordinary dynamometer wattmeter, acting on two pivoted moving volt coils fixed at right angles, or nearly so, to each other, and carrying a pointer. The current is led into and out of the coils by means

of three fine strips exerting as little control as possible on the system. In series with one coil is connected a non-inductive resistance, and in series with the other an inductive resistance. A two-phase rotary field is thus produced, and the moving system will tend to set itself in such a position that at the instant the current in the fixed-current coil is a maximum the axis of the rotary field is parallel to it. Thus the position gives an indication of the angle of lag or lead between amperes and volts. The indications are, however, much affected by changes of frequency, though not of voltage. To overcome this objection it was proposed by F. Punga* to use, in the case of two- or three-phase systems, the actual rotary field of the system, and this method is employed by Messrs. Everett, Edgcumbe and Co. in their power-factor meters, which can be used for either balanced or unbalanced loads. Fig. 50 shows the connections for a H.T. three-phase unbalanced load. The readings are quite independent of frequency, and are as accurate at low power-factors as at high. The deflection is directly proportional to the angle of lag or lead.

It is now customary to employ **transformers** in connection with all high tension instruments, whether ammeters, voltmeters, or wattmeters. The design of satisfactory volt transformers presents no difficulties, apart from the question of insulation, but a current transformer requires considerable care.†

Recording Instruments have come into almost universal use in central stations in this country, although in America and on the Continent they are only used to a limited extent. As usually constructed, a recorder consists of an ordinary instrument carrying at the end of the pointer a pen, which traces a line on a paper chart moved forward by clockwork.

The disadvantages of this construction are: (1) excessive friction between pen and paper; (2) drying up of ink; (3) blotting of record; and (4) inaccuracy of timing of clock. These defects seem to be inherent to recorders of the ordinary type, but are obviated in the "inkless" recorders of Messrs. Everett, Edgcumbe and Co. In these the pen is replaced by a hardened steel point which normally swings free of the paper, thus introducing no friction. Every five seconds (as a rule) a tapper comes down and presses the point on to the chart through an inked tape, such as is used in typewriters. By this means a dot is made on the chart, and the dots are so often repeated that a practically continuous line is traced even with a varying load.

* *Helios*, July and August 1902, Nos. 31 and 32.

† See Campbell, *Phil. Mag.*, xlii. p. 271; F. Punga, *Elect.*, vol. li. p. 1009; Edgcumbe and Punga, *Jl. Inst. E. E.*, part 167, 1904; Drysdale, *Elect.*, Nov. 23, 1906, p. 201, and *Phys. Soc. paper*, March 1908.

The tapping device is worked by a momentary electric current sent every second or five seconds by means of a special master-clock, and these impulses also drive forward the recorder chart in absolute synchronism. Any number of recorders and clock dials can, moreover, be worked by a single master-clock.

For **Watt-hour Meters and Ampere-hour Meters**, see p. 173.

Ohmmeters, or resistance measurers, usually consist of two coils at right angles to one another (or nearly so), one of which is connected across the terminals of a battery, say, while the other is connected in series with the resistance to be measured. A pivoted magnetic needle (or, better, a soft iron needle) carrying a pointer takes up a position along the resultant field, the direction of which, it will be seen, depends upon the resistance to be measured. The indications are independent of the voltage, and the chief disadvantages of the arrangement are that it is much affected by external fields, that it is far from dead-beat, and that the scale is usually somewhat uneven. Evershed, who greatly improved the instrument, suggested the use of a portable magneto machine in place of the battery. In the latest form of Evershed ohmmeter, known as the "Megger," a different principle is adopted. Instead of the two coils being fixed and the magnet pivoted, the coils are movable and the magnet fixed. The advantage of this arrangement is that a very intense magnetic field can be obtained, whereby the sensitiveness is greatly increased, besides which the effects of stray fields are done away with.

In the ohmmeter of Messrs. Everett, Edgcumbe and Co. the Wheatstone bridge principle is employed, the construction being, however, so far simplified that the instrument is absolutely direct reading and is entirely self-contained. With a generator pressure of 200 or 500 volts measurements up to 50 or 200 megohms can be made. For mains testing a battery is employed instead of a generator. The accuracy, it will be seen, is independent of the voltage, and is not affected by external fields, so that it can be used close to dynamos, etc.

The chief **errors in commercial instruments** are those due to—

- (1) Friction.
- (2) Position of moving system.
- (3) Hysteresis.
- (4) Temperature.
- (5) External magnetic fields.

By careful jewellery (sapphires, or sometimes rubies, garnets, or in extreme cases, agates, are used) **friction errors** can be rendered almost negligible.

A **position error** is found when the poles of the moving iron (if present) are not sufficiently definite. It shows itself by the needle not returning to the same point when it is deflected by hand while a certain fixed current is flowing. In modern instruments it can be neglected.

The presence of **Hysteresis** is evidenced by the upward readings being lower than the downward, and the error is often serious with electro-magnetic instruments. It may be reduced either by making the iron so small that it becomes saturated by a small fraction of the maximum current, or else by making the iron so large that **B** (and with it the hysteresis) is small at all currents. Other things being equal, the better the iron and the shorter its length along the magnetic axis the less will be the hysteresis error. The hysteresis error is limited to types *a*, *b*, *c*, and *e* of class 1.

All voltmeters, except electro-statics, are liable to **temperature errors** due to changes in the resistance of the instrument with temperature. The only remedy is to employ a large idle resistance of negligible temperature coefficient (*e. g.* "constantan" or "manganin") in series with the copper working coil. Ammeters are only liable to this form of error if shunted (*e. g.* moving coil instruments). If copper alone is used the error will amount to about 4 % for every 10° C. change of temperature.

Stray magnetic fields may have serious effects in the case of switchboard instruments, as bus-bars carrying heavy currents often run close to them. Moving-iron and dynamometer instruments are very much affected. Contrary to the usual assumption, permanent magnet moving coil instruments suffer somewhat, while even electro-static voltmeters are liable to be affected by strong alternating fields (owing to eddy currents being set up). Hot-wire instruments alone are free.

The only remedy is to shield the movement. In the case of switchboard instruments a cast-iron case is usually sufficient protection, but for these in brass or wooden cases a shield of thin sheet iron is the only effective method.

In the case of electro-static instruments serious errors are often introduced by **static charges** accumulating on the glass fronts, owing to friction when cleaning. To avoid this Ayrton and Mather have suggested the use of a transparent conducting varnish upon the glass.* Breathing on the glass will usually dissipate the charge.

In the specification recently issued by the **Engineering**

*.A coating of gelatine and sulphuric acid protected with a coat of ordinary varnish will be found to answer well.

Standards Committee, ammeters and voltmeters are divided into three classes :—

Sub-Standard (*e.g.* Portable Testing instruments).

First Grade (*e.g.* High-class switch-board instruments).

Second Grade (*e.g.* Ordinary “Commercial” switch-board instruments).

The **errors allowed** are as follows :—

Sub-standard voltmeters 0.2 % of top scale reading.

„ ammeters 0.5 % „ „ „

First-grade voltmeters 1 % of reading over upper half of scale;
 $\frac{1}{2}$ % of top reading over remainder.

„ ammeters double the above.

Second-grade voltmeters 2 % of reading over upper half of scale;
 1 % of top reading over remainder.

„ ammeters double the above.

For many purposes **Electrical Thermometers** or “**Pyrometers**” are extremely useful. They are mostly based upon the change of resistance of a wire (usually platinum or copper) with temperature, or on the thermo E.M.F. of a couple (usually copper / constantan up to 500° or platinum / platinum and rhodium up to 1600° C.). The wire is mounted in a long tube of glass (for use up to 500° C.), or quartz (up to 1000° C.), or of porcelain for use above 1000° C.

For Resistance Pyrometers a Wheatstone bridge or ohmmeter (see p. 146) is usually employed, and for the Thermo-couple pattern a sensitive milliammeter. For accurate work a potentiometer is the most satisfactory for both types.

For special purposes, such as the determination of furnace temperatures, **Radiation Pyrometers** have come into use. These are based either on the law connecting colour with temperature (Wiens Law) or that connecting heat radiated with temperature (Stefan-Boltzman Law). The light, or heat, emanating from a small opening in the furnace is received by the instrument and is then either compared as to colour, or falls on a Thermo-junction connected to a galvanometer. The indications of such instruments can be made independent, within wide limits, of the distance from the furnace.

OSCILLOGRAPHS.

By W. DUDELL, F.R.S.

OSCILLOGRAPHS, or instruments for recording alternating current wave-forms or other rapidly varying currents or potential differences, are essentially galvanometers whose deflections at any instant are proportional to the current flowing through them at the time. In order to attain this end it has been shown by Blondel* that an oscillograph should fulfil the following conditions—

1. Very short free periodic time.
2. Perfectly dead-beat.
3. Negligible self-induction.
4. Sufficient sensibility.
5. No hysteresis or eddy current effect.

Many methods have been proposed for making oscillographs ; but up to the present only two, both of which were originally due to Blondel,* have come into general use.

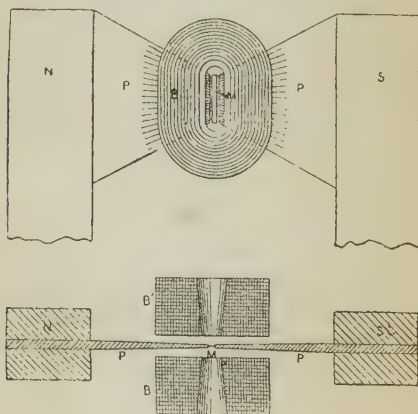


FIG. 51.

A third type, depending on the expansion of wires heated by the current, has since then been devised by Mr. Irwin.†

In the first, or **soft-iron type**, the moving part consists of a

* *Comptes Rendus*, vol. cxvi. p. 502.

† *Journal of the Institution of Electrical Engineers*, vol. xxix. p. 617, 1907.

small piece of iron, to which the mirror M , Fig. 51,* is attached, the whole being pivoted between the poles PP of a powerful field magnet. The varying current to be observed passes through the two coils BB' , one on each side of the pole pieces, and produces a field approximately proportional to the current, and at right angles to the lines of force of the field magnet, thus deflecting the soft iron needle through a small angle sensibly proportional to the current.

In the second, or **bifilar type**, the moving part consists of two metal strips (SS , Fig. 52), stretched side by side in the strong magnetic field given by the magnet NS ; the current to be investigated flows up one and down the other, causing one to move forward and the other back, thus turning a mirror M , fixed to their centres through a small angle which is proportional to the current flowing in the strips at the time.

Both types of oscillograph are in practical use, but the second, which has been perfected by Mr. W. Duddell, is to be preferred on account of its exceedingly small self-induction, the results being equally accurate whether the strips are shunted and used as an ammeter, or used as a voltmeter with resistance in series. This type of oscillograph is generally made with two pairs of strips, so that potential difference and current measurements can be obtained absolutely simultaneously.

These instruments, when constructed with an electro-magnet to produce the field, have a **periodic time** as small as $\frac{1}{10000}$ second and a sensibility of 300 mm. per ampere at a scale distance of 50 cm. With a permanent magnet field the periodic time is $\frac{1}{4500}$ second with the same sensibility. The permanent magnet type is especially suitable for use on E.H.T. circuits, as it can be easily insulated from earth to stand 30,000 volts.

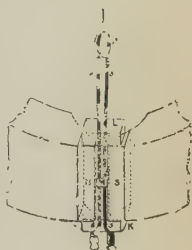


FIG. 52.

In order to observe the deflections of the instrument, which may be varying several hundred times a second, the movements of the spot of light can either be photographed on a moving plate or film, or observed in a rotating mirror. When the variations in the current or potential difference are periodic, as they are in alternate current circuits, a very convenient method of observing and recording the wave-forms is to use a vibrating mirror whose movements are synchronous with the waves of current or potential difference which it is required to record,

* *Electrical Review* (New York).

and whose axis of vibration is at right angles to that of the oscillograph. The beam of light reflected from the oscillograph mirror is again reflected at the synchronous mirror on to a screen where the moving spot of light traces out the wave-form, which, owing to persistence of vision, looks like a stationary bright curve, and can be easily traced or photographed.

Non-periodic phenomena such as the charge and discharge of condensers, and the growth of current in inductive circuits, can often be rendered periodic by means of a contact device attached to the motor, and projected on a screen for demonstration purposes.

By using long lengths of film running continuously the potential rises on switching in cables, and when fuses blow, can be recorded.*

In telegraphy, telephony and the theory of speech, oscillographs are now being used.

Users of the above bifilar oscillograph may find the following hints of assistance. Employ an intense source of light (preferably an arc lamp) to illuminate the mirrors. Always keep the instrument full of damping oil even when not in use. Never allow a high potential difference to exist between one pair of strips and the other pair, or between one pair and the field magnet. Do not change the tension on the strips if it can possibly be avoided. The ratio between the periodic time of the current and that of the oscillograph should exceed 30:1 in ordinary cases, and with very distorted wave-forms should exceed 50:1.

PROTECTIVE RELAYS.

Although not strictly measuring instruments, relays are more conveniently dealt with under the present section than under any other. They fall into 4 groups, corresponding to those into which circuit-breakers (of which they really form a part) are divided.

- (1) Maximum or overload relays.
- (2) Minimum, no voltage or no load relays.
- (3) Reverse or discriminating relays.
- (4) Differential relays.

The use of relays is almost entirely restricted to H.T. alternating circuits. The first group is the most used, and such relays are largely employed to protect machinery of all kinds. They have almost entirely superseded fuses for H.T. circuits, owing to their greater reliability.

* *Journal of the Institution of Electrical Engineers*, vol. xxxvi. p. 139, 1905; vol. xli. p. 40, 1908.

Overload relays are usually of the Solenoid, the Induction, or the Thermal pattern. In the **Solenoid relay** a coil carrying the main current, or one proportional to it (say through a current transformer), attracts an iron core or armature so soon as the current exceeds a predetermined amount (usually an adjustable one). The movable part carries a pair of contacts, often of laminated copper, or, better, of carbon, and thus closes the circuit. The **Induction relay** comprises an induction movement (see p. 161) which winds up a weight arm carrying the contacts. The adjustment of current is made, as a rule, by varying the weight lifted. **Thermal relays** of various forms have been suggested, but the most satisfactory consists of a strip or wire carrying the current, and so arranged as to release the contact arm, so soon as it has expanded to a predetermined extent. The current setting can be varied as required.

Minimum relays are very similar in construction to the overload type, and are usually of the solenoid pattern, in which an attracted armature is released, so soon as the current falls below a certain value.

Reverse relays are now installed in almost all alternating current generating stations to protect the alternators. Supposing one of a number of generators running in parallel to break down, the healthy machines will feed current into the faulty one, and the function of the reverse relays is to cut off the latter from the bus-bars.

Reserve relays are used to a small extent on **direct current** systems, and in that case usually take the form of polarized moving coil ammeters. For **alternating current** work the simplest form is a modified wattmeter, since, in this instance, the direction of deflection depends upon the direction of the current. A serious disadvantage of such relays, however, lies in the fact that the inevitable fall of voltage which accompanies the development of a serious fault, means that the relay will not trip except with a proportionally increased current. To obviate this objection such relays are frequently set to trip with a small fraction of full load, at normal voltage. The result of this, however, is that they are very liable to trip with temporary and quite innocuous reversals, such as those which may occur with a "bad parallel," or even with the blowing of a fuse, or the opening of a breaker. The solution is to employ a reverse relay in which the tripping current is over a very wide range independent of the pressure.

For the protection of inter-connection feeders it is usual to install overload relays at each end, but it has been proposed by Mertz and Price to employ a **differential relay**, which will operate so soon as the current flowing in at one end does not equal that flowing out at the other (this being the condition of affairs on the

occurrence of a fault). Such an arrangement, while ingenious and theoretically good, is complicated and expensive to install, and has consequently not so far been adopted to any great extent.

In most cases, overload relays are called upon to allow a small **time lag** between the occurrence of the overload and the tripping of the breaker, to obviate interrupting the supply for a purely transient disturbance. The heavier the overload the shorter should be this time lag, such an arrangement being known as an "inverse time element." In the solenoid pattern the time lag is usually provided by a dash-pot, a rotating fan, a bellows, or the Statter sucker, the latter consisting of a flat disc resting on an oily surface. In the case of induction relays, a breaking magnet on the disc serves to retard the motion, and in thermal relays, the heat capacity of the strips does the same. Reverse relays are usually instantaneous in their action.

It is often essential that two or more relays connected in series should always trip in the same order, so as to ensure, for example, that a faulty branch circuit should be cut off before the main feeder relay acted. To this end it is important that all the relays in series should give similar time curves, no matter how great the overload, but this is a point which has not been sufficiently attended to by many relay makers.

ELECTRICITY SUPPLY METERS.*

BY H. G. SOLOMON, A.M.I.E.E.

General Remarks.

House Service Meters may be classified in a number of ways : (1) according to their principle of working, into thermal, electrolytic, electro-magnetic and induction meters ; (2) more broadly, with reference to the electrical magnitude they measure, into ampere-hour and watt-hour meters ; (3) depending on the nature of the supply current, into continuous current and alternating current meters. The latter classification is not only the most rational, but the most convenient, as in the selection of a meter the first consideration is the type of circuit to be metered.

In addition to these two main divisions, there is a third class—tariff meters—embracing the special modifications of meters for tariff purposes, and including measuring instruments, such as maximum demand indicators and hour meters, which, however, are not electricity meters proper. Although they neither measure nor register the electrical energy consumed, they form very important adjuncts to meters in conjunction with special systems of charging for electrical supply, and should be considered under this heading.

Of all electrical measuring instruments the house service meter has to work under the most varying and exacting conditions. It is more often than not installed in situations—cellars, underground passages—exposed to damp, dirt, fumes, etc. It receives practically no attention, even when periodically read by the meter attendant, but has, nevertheless, to be capable of functioning correctly with practically the same initial accuracy as when new.

Whatever its nature, an electricity meter consists of two parts—the mechanical and electrical. The two are equally important. Unless the mechanical features are carefully carried out, the meter rapidly deteriorates in service. It is of paramount importance for the various elements of the meter to be so enclosed in a protecting case that they are, as far as is practicable, rendered moisture-, dirt-, dust- and insect-proof. This condition, in the majority of cases, can only be attained by a properly secured and permanently sealed main cover, so that, when once the meter has been delivered by the manufacturer, access can only be had to the vital parts of the meter by breaking the manufacturer's seals, when the latter's liability should at once cease. This sealing, not only of the

* For full particulars and the laws of meters, see the writer's book, *Electricity Meters*, published by Chas. Griffin & Co., Ltd., London.

main cover but also of the adjustments of the meter, is one which deserves most careful consideration and should be insisted upon, as it largely affects the permanence of calibration of the meter, and, unless carried out, precludes the possibility of ascertaining the cause of failure of meters in service. Most meters are provided with two distinct and independent covers, the main cover already referred to and one for closing the terminal compartment. The main cover has, in addition, one or more windows for the reading of the register and the observation of the revolutions of the meter spindle, if of the motor type.

In motor meters the main mechanical retarding influences at work are bearing friction, brush friction (in commutator motor meters), and the friction of the counting train and gear. These should be as small as possible, so as to minimize their adverse effect on the meter curve, especially at low loads. In commutator motor meters brush friction is the main source of trouble. In present-day meters of this type, due to improvements in the construction and reduction in size of the commutator, fewer commutator segments and better brush gear, this source of trouble is very much less than formerly. Mercury motor meters are used mainly with the object of eliminating brush friction, the mercury serving to conduct the current to and from the revolving armature, mostly consisting of a copper disc immersed in the mercury. All jewel bearings should be flexibly mounted, and the revolving element should be as light as is compatible with stability, so as to give a good ratio of driving torque to weight. Some meters of the motor type, notably of foreign manufacture, err on the side of too great lightness of their revolving element. This gives rise to many troubles: sudden increases of current—not necessarily overloads—damage the revolving element, which easily gets bent, and inaccuracies ensue. Ball bearings are generally better than jewel bearings with pivots; the initial friction is slightly higher, but it is less liable to change.

All meters with revolving elements should be provided with a clamping device to prevent damage to the armature, or rotor, during transit, whether this portion be light or heavy. The locking device must, of course, be capable of being sealed after the meter has been installed, to prevent tampering by unauthorized persons.

The integrating mechanism, or register, of a meter consists of dials, cyclometer counters or scales, the former two in motor and clock meters, the latter in electrolytic meters. They should read the units direct (B.O.T. units) without a multiplier or constant. Cyclometer counters offer advantages over the ordinary gas meter dials. The readings are much more readily

taken ; and errors are less likely to be made, as the figures are all in one row. It is, however, very essential that the numbers should not be able to occupy intermediate positions, producing uncertainties, owing to the slow motions in meters. The change from one number to the next should be made by an intermittent motion of the number wheel, so that it does not rotate continuously, as in the dial register and crawling type of cyclometer counter, which should not be tolerated under any circumstances. The first motion wheel of the register revolves continuously, but usually denotes a decimal part of the B.O.T. unit, which is only used for testing purposes. The testing dials and figures (counter type) should be distinct from the remainder, and should be suitably covered by a slide for ordinary working conditions.

The serious disadvantage of the cyclometer counter consists in the difficulty in checking its effect on the working of the meter. This retarding effect, unlike that of the ordinary dials, the component parts of which are always in mesh, is not uniform, but is a maximum when the maximum reading of 9999 has been reached, in the case of a counter indicating up to 9999 units. It often happens that with the counter in this position a small current, such as that taken by a couple of 8 c.p. lamps, will be quite insufficient to actuate the meter, owing to this increased friction of the counter. Units can in this manner be easily consumed without being registered, and the number of such unregistered units may become quite considerable in a supply system. When a few more lamps are switched on, the extra driving torque available overcomes the friction, and the meter is able to rotate and turn the counter as usual, and may then function fairly correctly.

Of the many other requirements of an electricity meter **accuracy** is of primary importance. In motor meters the accuracy depends on the driving torque exerted by the electrical elements on the armature, or rotor, the constancy of the permanent magnets used, if any, on the temperature variations* and friction ; in electrolytic meters on temperature fluctuations, purity of the electrolyte, varying ratio between the shunt and cell circuit resistance (shunted type), formation of crystals, back E.M.F.

The whole revenue of the supply station derived from the sale of electrical energy is determined by the aid of meters, so that as high an accuracy should be demanded as is possible within the limits imposed by considerations of first cost and

* For alterations in the constants of meter due to temperature variations see G. W. D. Ricks, British Association, 1896, and Hooper, *Electrical World* (N.Y.), vol. xxi. p. 384, 1898.

internal losses. The accuracy of a meter is usually expressed in the form of definite limits of error within certain limits of load, and the error limits prescribed by the Board of Trade are those hitherto aimed at in meters. The Board of Trade requirements for "Approval" (see Clause 50, Electric Lighting Act, 1899) are as follows—

"Meters in which the current for the maximum loads exceeds 3 amperes should not have an error exceeding 2% (+ or -) at any point from $\frac{1}{10}$ full load to full load. For meters in which the currents do not exceed 3 amperes, the variation from accuracy at any point from $\frac{1}{10}$ load upwards must not exceed + or - 3%."

For "certified" meters (in other words, for purposes of commercial accuracy) the above limits are modified in the Board of Trade Rules, dated October 24, 1903, of which only Clause II is repeated here—

"II. A meter should be considered by the inspector to be a correct meter if, being a meter intended for more than 3 amperes, the error at $\frac{1}{10}$ of its full load and above this point does not exceed + or - $2\frac{1}{2}$ %. In the case of meters intended for currents not exceeding 3 amperes, the error must not exceed + or - $3\frac{1}{2}$ %."

Quite recently the Engineering Standards Committee have issued the **British Standards Specification** (No. 37) for Consumers' Electric Supply Meters (Motor Type for Continuous and Single-Phase Circuits), October 1907, and the error limits contained in Clause 20 are practically the same as those required by the Board of Trade for "certified" meters, with, however, the difference that the error limits apply to all meters, whatever their capacity, the larger error of $\pm 3\frac{1}{2}$ % for meters taking currents not exceeding 3 amperes being now disallowed. The clause relating to the error limits states that the error at the marked temperature, pressure, frequency and unit power factor shall not exceed $\pm 2\frac{1}{2}$ % from full load down to $\frac{1}{10}$ of full load, and that at $\frac{1}{N}$ of the full load between $\frac{1}{10}$ and $\frac{1}{20}$ full load (inclusive)

the percentage error shall not exceed $\frac{N}{4}$ %. Mention is here made of the power factor in reference to alternating current meters, and further in the specification it is stated that with a variation in the power factor from unity to $\frac{1}{2}$ ($\cos \phi = .5$) the error should not be increased by more than 2%.

Next in importance to accuracy is the question of the **cost** of a meter. By the cost of a meter is meant here not only the initial purchasing price of the meter, but also the cost of the energy absorbed in the meter itself and the charges to cover depreciation, maintenance and interest on capital outlay.

In an ampere-hour meter the loss is confined to the waste of power in the main current circuit (whether shunted or unshunted) of the meter. A drop in voltage is produced in the main circuit between the main current terminals of the meter. This drop depends on the current flowing, reaching a maximum at full load, and being zero at no load. The pressure on the supply side of the meter is, therefore, higher than that on the consumer's side. This drop is the most important loss in the series circuit. With metallic filament lamps the same percentage drop in voltage across the lamp terminals does not produce the same high percentage loss in candle-power as in the carbon filament lamp. Nevertheless, a greater drop than 1 volt at full load should not be allowed, and is easily fulfilled by good present-day meters. In energy meters power is wasted both in the main current and in the pressure circuits. The most important loss is the "shunt" loss occurring in the pressure circuit. The series loss is borne by the consumer, the pressure loss entirely by the supply station. The latter loss represents a considerable annual loss in revenue, as it is continuous, whether the consumer be taking current or not. The pressure loss in the best continuous current meters varies from 4 to 6 watts at 200 volts, and in induction types from 0.5 to 4 watts at 200 volts and a frequency of 50 ~ per sec. The main current loss is on the average 6 watts with the restriction of 1 volt drop at full load.

A meter should be efficiently insulated not only from earth, but its various components amongst themselves; its registrations should not be appreciably altered by a 10 % variation in pressure and frequency; it should be capable of standing continuous overloads of about 25 % for one hour; and its terminals should admit of ready connection to the supply system without errors and without, in the case of energy meters, the possibility of the pressure current being sent through the main circuit of the meter. Moreover, the pressure circuit should be capable of complete isolation at the terminal compartment without the ends of the pressure coils having to be handled, so that for testing purposes the pressure circuit can be energized from a source independent of that to which the main circuit is connected.

Finally, a meter should be capable of starting and running with 1 % of the full load current, and when provided with a pressure circuit it should not run on the "shunt" alone, *i. e.* with no current in the main circuit, even when the supply pressure is increased by 10 %.

CONTINUOUS CURRENT METERS.

I.—Ampere-Hour Meters.

METERS for use on continuous current circuits are divided into ampere-hour (quantity) and watt-hour (energy) meters. The ampere-hour meter depends only on the current flowing in the installation, and is independent of the supply pressure. It measures the quantity of electricity in ampere-hours conveyed in the circuit by the passage of a current during a given interval of time. When the supply pressure is constant the measure of the quantity of electricity is indirectly a measure of the electrical energy, and the meter is then calibrated so that its dials (or scale) give the units consumed at the particular voltage without the use of a multiplier, or constant. A watt-hour meter, on the other hand, depends for its working not only on the main current of the circuit but on the supply pressure, so that it actually measures the electrical energy and registers in units direct.

Ampere-hour meters (sometimes wrongly termed "coulomb" meters) are either of the **electrolytic** or electro-magnetic type. In the first the action of the meter depends on the chemical decomposition produced by a current flowing through certain liquids, such as dilute acids and solutions of metallic salts. The amount of gas set free, or metal liberated, in an interval of time depends on the strength of the current and the electro-chemical equivalent of the substance liberated, and it follows from the nature of the action that meters of this description can only be used for direct current. There are two types of electrolytic meters—the shunted and unshunted varieties. The **Wright** "New Solution" and the **Holden** electrolytic meters are of the shunted type. In the former the electrolyte consists of a double iodide of mercury and potassium, which is a neutral solution and in which the mercury is contained in the mercuric state. This solution practically fills the whole of an hermetically sealed glass tube. The top of the tube terminates in an enlarged bulb—the anode chamber—which communicates at the side with a pear-shaped glass attachment—the anode feeder. In the anode chamber the mercury anode rests on a glass ledge and is prevented from being jerked into the U-shaped syphon tube within the outer sealed tube by a fence of platinum gauze. Below the anode chamber, in the neck of the tube, is situated the iridium cathode. The object of the anode feeder is to keep the level of the mercury in the anode chamber always the same. In series with the cell circuit is a fine-wire resistance to compensate for the temperature changes for a range of about 10° C. below and above the normal. The cell

circuit leads are brazed to the low-resistance shunt of manganin strip, the terminals of which are connected in one of the supply mains. On the passage of a current in the main circuit a fractional part of it flows through the cell circuit. The mercury is deposited in minute globules on the iridium cathode and falls into the U-shaped syphon tube, against which a scale graduated in units gives the amount of energy consumed at the particular supply pressure and is read off at the height of the mercury.

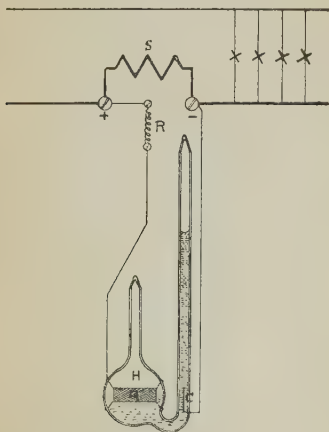


FIG. 53.

It will thus be seen that the fluidity of the mercury is in this case made use of for its measurement by volume instead of by weight. When an amount of mercury equivalent to about 100 units fills the U-tube its level rises slightly above the top bend of the syphon, when the whole passes over and collects in the bottom of the sealed tube provided with a second scale reading in 100 units. For small capacity meters up to about 5 amperes a plain graduated tube reading up to 250 units is used. In the **Holden** electrolytic meter the electrolyte consists of dilute sulphuric acid contained, together with the platinum electrodes, in a sealed glass measuring U-tube with attached bulb, as shown diagrammatically in Fig. 53. The anode *A* consists of a piece of platinum gauze coated with platinum black; and a piece of platinum wire *C*, also coated

with platinum black, serves as the cathode. The anode *A*, situated in the electrolyte in the bulb, partly projects into an atmosphere of hydrogen *H* above the electrolyte. A resistance *R* is used in series with the cell, and the cell circuit is connected in parallel with a low-resistance shunt *S*, which carries practically the whole of the current to be metered. When a current passes, the electrolyte is decomposed into hydrogen at the cathode and oxygen at the anode, the oxygen at the anode combining with the hydrogen absorbed by the anode. The hydrogen liberated at the cathode rises into the vertical tube, which is provided with a scale graduated in B.O.T. units at the particular voltage of supply. Both types of meter are reset to zero by tilting the electrolytic cells after a certain number of units has been registered. They, of course, do not require any refilling.

The **Bastian** ampere-hour meter is an unshunted electrolytic meter. The whole of the current in the installation passes through the cell and effects the decomposition. The cell consists of a large tube of uniform bore containing a solution of caustic soda in which are the two nickel electrodes. These terminate in rods which pass up the tube through a porcelain cap which forms the cover of the cell. The rods are joined by flexible connections to the two meter terminals. The electrolyte is split up on the passage of a current, and the evolved gases escape into the atmosphere. The level of the electrolyte falls, so that the difference in height between two readings of the graduated scale (reading in units at the supply pressure) gives the B.O.T. units consumed in the interval between the readings. To facilitate the reading of the meter a thin layer of oil rests on the liquid column and also serves to prevent evaporation, or loss of water, when the gases are evolved. The meter has to be refilled periodically with water. Only water is required, as the specific gravity of the electrolyte increases towards the bottom of the scale, the water being decomposed when the meter is registering.

Electro-magnetic ampere-hour meters are mostly of the motor type with permanent magnets, and only these will be considered here, as they form the largest class of supply meters in commercial use. There are again two sub-divisions—the mercury and commutator motor ampere-hour meters. In the motor meter there are three essential parts—the motor, the brake system and the integrating counter. The motor comprises an armature actuated by the whole, or a fractional part, of the main current, and it rotates between the poles of a permanent magnet. It drives the counter and also a brake, the object of which is to keep the speed of the armature proportional to the current flowing. The brake in all such

present-day meters is the usual magnetic or Foucault brake. Solid friction (also liquid friction in mercury meters) prevents the proportionality from holding throughout the range of the meter.

In the mercury motor meter the armature is a disc of copper immersed in mercury, the object of which is to convey the armature currents to and from the disc. In the com-

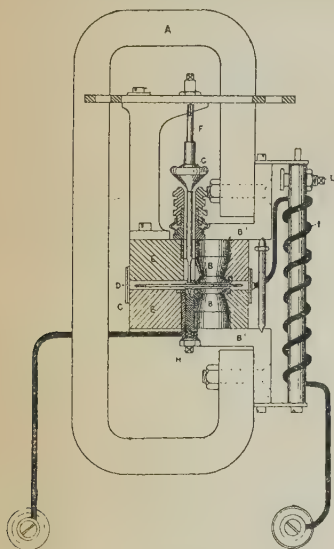


FIG. 54.

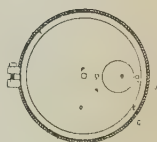


FIG. 54A.

mutator type the armature coils are connected to the segments of a tiny commutator on which bear the brushes conveying the current.

The latest form (1907 Pattern) of the **Chamberlain and Hookham** mercury motor meter is shown diagrammatically in Figs. 54 and 54A. The one permanent steel magnet *A* is used both for the driving and the retarding torques of the meter. To the ends of the magnet are attached the wrought-iron pole-pieces *B'B'* which terminate in the poles *BB*. These are embedded in the upper and lower circular ebonite blocks *EE* which form the

mercury chamber. The side of the latter is closed by the metal ring *C* lined internally with leather. The ring is shown in Fig. 54A, in which *K* and *T* denote the leading-in wires for conveying the current to and from the mercury bath. Inside the mercury chamber and between the narrow air-gap separating the poles is the armature disc *D* mounted on the spindle *F*, which also carries the counterweight *G* to balance the upward thrust of the mercury on the armature. The coil of wire *I* is wound on a stationary iron core, and is in series with the disc. It serves to compensate for the error at high loads due to the fluid friction of the mercury. The speed adjustment is carried out by altering the position of the iron screw at the top of *I*, whereby the reluctance of this magnetic circuit is increased or decreased. The coil is wound so as not to produce any demagnetizing effect on the magnet.

The current flowing in the disc interacts with the magnetic field of the permanent magnet and rotates the disc in which eddy currents are induced by its rotation in the field. When steady motion ensues the driving torque, which is proportional to the current, will balance the retarding torque, which is proportional to the speed, neglecting small frictional disturbances, and the speed will be directly proportional to the current.

In the new **Ferranti** mercury motor meter (Hamilton Patent) the disc revolves between the poles of the permanent magnets *PM*, of which only one is visible in the sectional side elevation (Fig. 55). The one magnet produces the driving field (it also retards the disc) and the other produces the braking field. *MB* is the mercury bath, of which *BP*, *BP* are the top and lower nickel-plated brass plates, and *FR* is a fibre ring. The plates are insulated on their internal surfaces. The disc *CD* is immersed in the mercury within the bath, and is carried on the vertical spindle *S* counterweighted by *W* with the adjusting nuts *N*. The bath is situated between the upper and lower poles of the two magnets, the top poles being united together by the iron bar *I₂B₂* and the lower pair by the iron bar *I₁B₁*. The current to be metered flows radially through the disc on one side and, interacting with the flux of the driving magnet, causes the disc to rotate, the driving torque being proportional to the amperes flowing. The rotation of the disc between the poles of the two magnets produces eddy currents, by which means the retarding torque proportional to the speed is obtained. To compensate for fluid friction at the high loads a compensating coil *CC* in series with the armature is employed. It is so arranged that the magnetic flux it produces increases the driving flux and decreases the field of the retarding poles. Both pairs of poles, however, retard the disc. Since, therefore, the retarding flux of the one is increased, and that of the other

pair is decreased, the total retardation remains unaltered by the coil.

The driving flux is, however, increased, so that at the top loads, where the increased fluid friction uncompensated would slow down the meter, this effect is to a large extent neutralized. Both of these types are unshunted meters, that is, the whole current to be metered passes through the armature in each case. Above 50, or 100, amperes a shunt is employed.

In **commutator** motor meters the same principle is involved, but they are usually of the shunted type, the shunt in the smaller capacity meter being incorporated in the same case. In the "**O'K**" meter the armature consists of fine-wire coils wound on a non-magnetic, non-conducting former connected to a four-part commutator. The armature revolves between the poles of a permanent magnet round a stationary iron core. The brushes are connected to the ends of a slow-resistance shunt. No magnetic or other brake is employed. The motor does no outside work beyond that necessary to overcome friction. On the passage of a current the armature is accelerated until the back E.M.F. generated in it equals the P.D. across its terminals produced by the main current in the shunt. When this is the case the speed remains steady (as long as the current remains so) and is proportional to the main current, only so much current flowing through the armature coils as is required to overcome the frictional retardation to motion.

In the "**EC**" meter, of the Electrical Co., a magnetic brake is used, and the one magnet produces both the driving and the retarding field. The armature coils are placed on the outside of an aluminium bell which rotates round a stationary iron core between the poles of the permanent magnet. The aluminium cylinder acts as the magnetic brake, eddies being induced in it as it revolves in the magnetic field. A feature of this meter consists in the employment of self-adjusting brushes and a specially shaped commutator of three segments. The

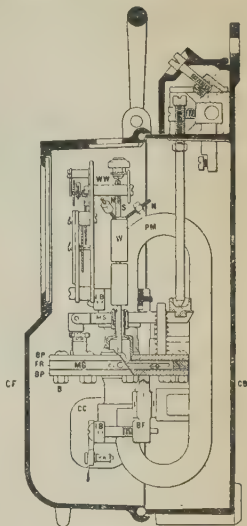


FIG. 55.

electrical elements and connections of this meter are shown diagrammatically in Fig. 56, in which R is the main current shunt, A the armature, C the commutator with its segments twisted at one end, M the permanent magnet, and S the actuating coil carried on the one arm of a bent pivoted lever, which supports the brushes. This coil S is in series with the armature, and is, therefore, traversed by the armature current. In addition, it is located in a stray magnetic field produced by the permanent magnet.

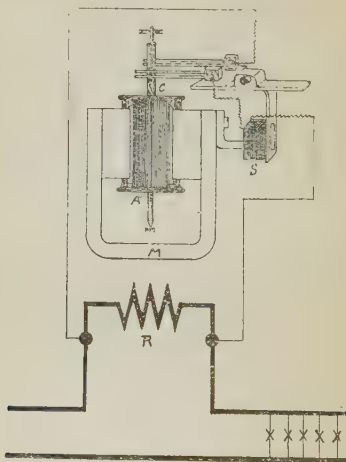


FIG. 56.

Any alteration in the armature current will, due to the electro-magnetic action between the coil and the field, alter the position of the coil and, consequently, that of the brushes on the commutator. The brushes are thus caused to travel up and down the commutator for alterations in the load, contact resistance, or friction. This alone would only produce a mechanical cleaning action. By, however, giving the segments a spiral configuration at one end, the zone of commutation is shifted along this part, and the driving torque at the position corresponding to about one-tenth full load is made relatively a maximum. The action of the brushes along the twisted region is analogous to that produced by giving the brushes of an ordinary motor a more backward or more forward

lead. It is claimed that in this manner the trouble inherent to commutator motor meters, due to contact resistance and friction, is largely compensated for, and just at the low loads where this fault mostly manifests itself. In principle the idea is certainly a very elegant one.

In the **Isaria**, **Adnil**, **Siemens** meters the three coils of the armature are flat, elliptical or oval in shape, and are carried on a light aluminium disc, which rotates between the poles of two oppositely polarized magnets. These, and the shunted, flat-disc type, ampere-hour motor meters made by other firms, differ in matters of detail and arrangement only.

II.—Watt-Hour Meters.

Continuous current watt-hour meters are based on the electro-magnetic principle, and are either commutator motor, clock or oscillating meters. In commutator motor meters of the **Thomson** type the field coils are stationary, and are connected in series in one of the supply mains in the case of a two-wire meter. The armature, either closed-coil or open-coil, consists of a number of fine-wire turns connected to a commutator in series with a compounding coil and high resistance, the armature circuit so formed being placed across the supply mains. No iron is used either in the armature or field system. The field coils are traversed by the main current, and produce a magnetic field which, together with the flux produced by the pressure current in the armature, produces rotation in the latter. The driving torque is proportional to the product of these two fields, *i. e.* the power supplied in watts. The motor spindle carries a brake disc of copper, or aluminium, which revolves between the poles of one or more permanent magnets, and in this manner the retarding torque proportional to the speed is obtained. The motor, in addition, drives the counter for the registration of the units. The straight line law does not hold throughout the range of the meter on account of frictional disturbances, which are mostly felt at the low loads, when the current flowing is small and the magnetic field of the main coils is weak. To compensate for this and to enable the meter to start readily on low loads the compounding, or compensating, coil mentioned above is employed. It consists of a number of turns of fine wire in series with the armature, and so placed relatively to the main current coils that the flux it produces assists the main current field. It thus exerts a constant (so long as the pressure remains constant) supplemental torque on the armature, which, moreover, always exists, whether current be flowing in the main circuit or not. In the *Thomson A* type meter, this compounding coil is made in two sections in

series with one another, each being adjustably mounted within the two main current coils (one in each coil) for effecting the light load adjustment. The two main field coils are rectangular in shape, with their magnetic axes parallel to the base, and the drum-wound armature revolves symmetrically between them. The armature coils are connected to a silver commutator carried on the lower extremity of the spindle, and phosphor-bronze

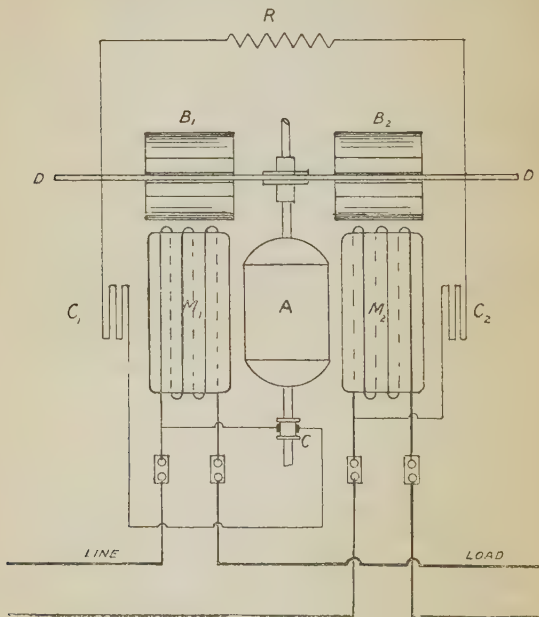


FIG. 57.

wires, with silver bearing surfaces, press on the commutator to convey the pressure current to and from the armature. The brake system is situated above the motor, and the two magnets are adjustably mounted relatively to the brake disc for varying the speed at high loads.

Fig. 57 is a diagrammatic representation of the electrical features and connections of this meter. A is the armature symmetrically placed between the two main current coils M_1 and M_2 . In series with the armature in the pressure circuit are the two halves C_1 and C_2 of the compounding coil, and the high resistance R to cut down the voltage across the commutator C to which the coils of the armature are joined. The spindle of the armature carries the brake disc D , which revolves between the poles of the two brake magnets, B_1 and B_2 , and the worm (not shown) which gears with the dials or cyclometer counter. Meters of this class are extensively used, and differ mainly in details of electrical and mechanical construction, such as the **Isaria**, **Adnil**, **Siemens-Schuckert**, **Duncan**, **Mix** and **Genest**, **Vulcan**, etc. It will be remembered that in these meters a permanent driving torque is being exerted on the armature by the compounding coil, as the pressure circuit is always energized. To prevent "shunt" or "idle" running with no current in the main circuit which might ensue with heavy jars, or increase in the voltage, a non-creeping device is very often employed, and commonly takes the form of a tiny piece of iron on the brake disc. When it comes under the poles of the magnet it is held, and creeping is prevented, the meter starting, however, with 1 % of the maximum load.

Clock Meters. —The only clock meter of practical importance is the **Aron**. It is based on the same principle used in the above motor types, the action between the currents in the stationary and moving coils being utilized to influence an existing pendular motion. It consists of two accurate pendulum clocks, each pendulum of which carries a volt coil and swings above a stationary main current coil. The volt coils of the pendulums are connected in series with a high resistance across the supply mains, and are thus energized by the pressure; they are similarly wound and excited. The main current coils are also in series with one another, but the current to be metered flows through them in opposite directions, so that the one is oppositely polarized to the other. As, however, the two volt coils present the same polarity to the main current coils, the one pendulum is retarded and the other is accelerated by the main current. This causes a difference in speed between the two which is proportional to the watts, and the differences in the rates of oscillation are continuously summed up by a differential gear and counter. The meter readings depend only on the differences in the oscillation rates of the pendulums when a current is in the main circuit. By acting upon both pendulums great sensitiveness is obtained, which is still further increased by the employment of flat springs. The clocks are self-winding electrically, and any want of synchronism between the two

pendulums, when no current is flowing, is neutralized by a reversing mechanism, which reverses the direction of rotation of the counting train. A commutator reverses the direction of the pressure currents in the volt coils simultaneously with the action of the reversing mechanism, as, otherwise, the differences in speed due to a main current would be destroyed. The meter is unaffected by stray fields, these either accelerating or retarding both pendulums equally, and thus producing no alteration in their relative speeds.

Oscillating Meters are based on the same working principle, but by means of special devices an oscillatory motion is produced instead of one of rotation, and the usual brake system is employed.

Intermittent Registering Meters are not used at the present day for ordinary house service, but mainly in connection with certain types of maximum demand indicators, prepayment meters, etc. They all consist of an ammeter, or wattmeter, and a clock mechanism so arranged that the counting train is periodically actuated, the amount of motion imparted to it depending on the amount of current or watts passing through the meter.

For the measurement of the electrical energy consumed in **three-wire continuous** current networks, so-called, three-wire meters are employed. In the Thomson watt-hour motor type the one field coil is placed in series with the positive outer main, and the other field coil in series with the negative outer, the armature circuit being connected direct across the two outers when it is, of course, energized by the total three-wire voltage, or it is placed across one-half of the system. The latter method should be avoided, as the errors due to it may become very considerable for the out-of-balance currents commonly occurring in practice. The first method also gives rise to errors unless the network is perfectly balanced, but they are usually of a small order of magnitude.* Two independent meters connected to the two sides as two-wire meters correctly measure the consumption. Three-wire clock meters of the Aron type are specially well adapted for three-wire circuits, as they measure the total energy correctly, irrespective of the distribution and nature of the loads on the two sides. Meters undergo various modifications to adapt them for special purposes, such as **battery meters** for measuring the charge and discharge of batteries, **switchboard** meters for measuring the outputs of lighting, power and traction systems for mounting on switchboards, and **tramcar** meters.

The **selection** of a meter depends on a number of factors—

* See "Limitations of Three-Wire Energy Motor Meters." H. G. Solomon, *El. Rev.*, vol. lix. p. 327, 1906.

local conditions, type of consumer, price, probable cost of upkeep, losses, etc. For c.c. circuits at constant pressure ampere-hour meters are decidedly more advantageous than watt-hour meters. They are cheaper, the annual charges for maintenance, depreciation and interest on capital outlay are less; they are quite as accurate; they have no shunt loss, and there is no loss in revenue due to this cause; there is also no possibility of shunt running. When the circuit is subject to voltage fluctuation watt-hour meters are to be preferred. The disadvantage urged against ampere-hour meters that they do not directly measure the actual energy is more apparent than real under the conditions of approximately constant pressure. When the voltage drops the ampere-hour meter readings are too high; when it rises they are too low. Electrolytic meters are cheaper than ampere-hour motor meters; they require more attention, however, and have to be periodically refilled or reset. Clock meters have a large range and start registering however small the current may be, but they are complicated and expensive. For general house-service work motor meters are to be preferred. For special purposes, *e.g.* three-wire metering, clock meters are distinctly superior to other types.

ALTERNATING CURRENT METERS.

Induction motor meters are almost exclusively employed for the measurement of the electrical energy expended in alternating current circuits. Formerly continuous current watt-hour meters of the Thomson type (without iron in the armature or field system) and the Aron clock meter were used. Owing, however, to the many advantages of meters on the induction principle they have been quite superseded by these. Induction meters are cheap as regards price, have low energy losses in comparison with the c.c. types, are of extremely simple construction, and have very little friction and no commutator, brushes, or revolving contacts. They are, therefore, free from the troubles inherent to commutator meters, and the revolving element has no current flowing in it from an outside source. An induction motor meter is to-day almost invariably constructed to measure watt-hours, the ampere-hour type (*Shallenberger*, now practically obsolete) no longer being used on account of possible phase displacements between the current and pressure. The motor of an induction meter consists of the stator and rotor. The former comprises one or more magnets wound with coils energized by the main current, or a fractional part of it, and with coils excited by the supply pressure. The latter is usually a simple disc of copper or aluminium inductively acted upon by the shunt and series systems. The motor, which

may be regarded as a two-phase, or split-phase motor, drives the counter and the usual magnetic brake. The rotor constitutes at the same time the magnetic brake disc, as it revolves between the poles of one or more permanent magnets. A choking coil in series with the pressure circuit of the meter is also often employed to make this circuit as inductive as possible, and for pressures exceeding 250 volts up to 500 volts. The two alternating currents of the meter (the main and pressure currents) produce two alternating magnetic fluxes which cut the disc, induce eddies in it, and produce rotation of the disc due to the interactions of the main and shunt eddies with the fluxes. The driving torque exerted on the disc is proportional to the product of the two fields multiplied by the sine of the angle of phase difference between these fields. The fluxes also produce a retarding torque which, however, is usually negligible compared with that of the magnetic brake. When the phase displacement between the two fields for unity power factor is exactly 90° , the driving torque is proportional to the true load in watts, and the speed is directly proportional to the power, neglecting friction. Induction meters have, therefore, to be artificially compensated to produce this phase relationship, and the numerous induction meters on the market differ mainly—apart from arrangements of the elements of the meter, etc.—in the methods employed to achieve this end.* The object is to render the registrations of the meter correct with power factors differing from unity (arc lamps, a.c. motors, etc.), as, otherwise, they would be unreliable on these loads. Without the compensation the angle between the fluxes operating on the disc ranges from about 70° – 85° . For purely non-inductive loads the error with this angular displacement is not very large, as the sine of such an angle is not appreciably different from unity. It is, however, always advisable to use a meter compensated for inductive loads, although the circuit may only consist of incandescent lamps, as the current and pressure are never absolutely in phase. The power in an alternating current two-wire system is $VC \cos \phi$, $\cos \phi$ being the power factor and V and C the pressure and current (R.M.S. values) respectively. Typical induction meters are the **Ferranti, The Electrical Co., A. C. T., Westinghouse, Adnil, Siemens-Schuckert, Isaria, Mix and Genest**, etc.

In Figs. 58 and 59 are given a sectional side elevation and a diagram of connections respectively of the **Adnil** induction meter, which may be taken as representative of this class. E is the shunt magnet and N the shunt coil, and H_1 and H_2 are the

* For the methods of compensation used, effect of over- and under-compensation, law of the induction meter, etc., see the writer's book on *Electricity Meters*: Chas. Griffin & Co., Ltd., London.

two main current coils wound on the two limbs of the series magnet E_2 , one of which only is shown in Fig. 58 at H . A is the light aluminium disc which revolves under the action of the series and shunt fluxes in the air-gap between the poles of the shunt magnet, and between the poles of the brake magnet B which produces the brake action. At W is a small non-inductive resistance by means of which the final adjustment of

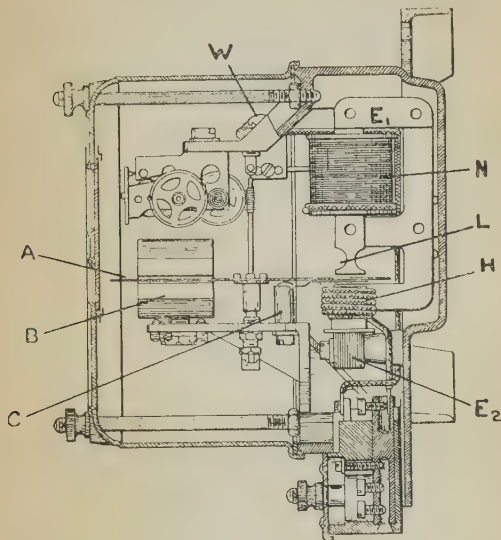


FIG. 58.

the phase displacement to 90° , or unity power factor, is obtained between the shunt and series fluxes. For frequencies from 40 cycles per second and under, this adjustment does not suffice, when a copper shield, so universally used in many induction meters, is employed on the constricted neck at L (the shield is not shown in Fig. 58) to attain the same end. The brake magnet B is adjustably mounted for altering the speed of the meter on the high loads, and by means of the screw C the small iron strip attached to it beneath the aluminium disc A and

above the bottom shunt pole, can be displaced to alter the functioning and starting of the meter at the low loads. This strip forms the no-load regulator, and when unsymmetrically displaced relatively to the shunt poles distorts the shunt field, so producing a torque which either assists or retards the driving torque of the meter. To prevent shunt running a small blob of

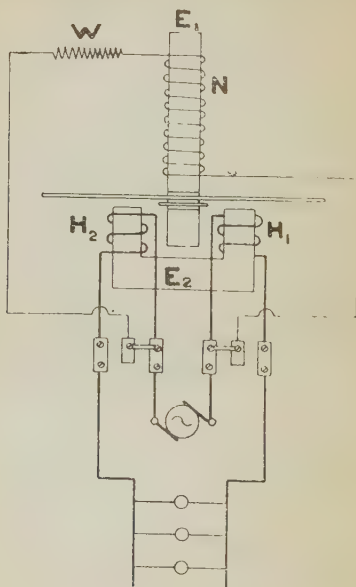


FIG. 59.

iron dust is placed on the aluminium disc, so that under the pressure alone the disc only revolves until this magnetic blob comes under the influence of the brake magnet, when it is held, and further motion is prevented until current is taken in the main circuit.

The method usually employed in measuring the energy consumed in **single-phase three-wire** circuits is analogous to that in continuous current three-wire networks. It should, however, be avoided, as it is very faulty, extra complications being intro-

duced owing to a lag or lead of the current in any one (or both) of the branches. Either two induction meters should be used, each half of the system being independently metered as a two-wire circuit, or an Aron three-wire clock meter should be employed, or a three-phase three-wire induction meter should be used, connected to the system on the two-wattmeter method of measuring power, with the two-pressure circuits of the meter connected between the neutral wire and the respective outer mains.

In polyphase networks (three-phase three-wire star and delta systems, two-phase three- and four-wire circuits, and three-phase four-wire systems), instead of using two or three separate induction meters **polyphase induction meters** are employed. For **three-phase three-wire and two-phase circuits** the polyphase induction meter consists of a combination of two single-phase motor-meter elements operating, as a rule, upon one and the same rotor disc, the usual magnetic brake system being used and a single counter giving the total three-phase energy. The method of connection is that known as the **two-wattmeter method of measuring polyphase power**.

This, however, does not apply to a three-phase system with four conductors, when either three separate induction meters have to be used or a special four-wire polyphase type. The advantage of using a single meter, instead of two or three, for measuring the energy is self-evident.

The two-wattmeter method of measuring power is illustrated diagrammatically in Fig. 27, p. 132.

For heavy currents and high pressures, current and pressure transformers are used in connection with the induction meters. Each motor element of the polyphase type has to comply with the phase conditions as in the single-phase meter in connection with the registration of circuits containing either capacity or self-induction. It should be mentioned here that an induction meter has three adjustments. The one relates to the phase compensation, the other is a light-load adjustment for compensating for friction at the low loads, and the third is the adjustment of the permanent magnet or magnets of the brake for altering the speed of the meter. The light-load adjustment usually consists of a small tongue of iron arranged relatively to the shunt magnet system so as to produce a distortion of the shunt flux and thus create a slight permanent driving torque on the disc, insufficient, however, to start the meter without load. Each half of the polyphase meter is furnished with these adjustments with the exception of that for full load, one brake system being, as a rule, employed, which is, of course, adjustably mounted.

TARIFF METERS.

Different systems of charging for the supply of electrical energy (see p. 523), are in vogue, and in certain cases the meters themselves are variously modified, or special accessory instruments are employed.

In the **Maximum Demand** system of charging, an instrument called a maximum demand indicator is used in addition to the meter proper. It measures the maximum current taken in an installation during a given period, and its scale not only gives these maximum currents but has marked on it the maximum demand units corresponding to them based on the particular supply voltage and duration of the demand. The meter proper registers the units consumed in the usual manner, while the demand indicator shows the units which have to be consumed before the low charge becomes operative. Maximum demand instruments are based on the thermal and electro-magnetic action of a current, and on the principle of intermittent integration. The **Wright** and the **Fricker** maximum demand indicators are examples of the first method. The **Wright** instrument is a differential thermometer consisting of a U-tube filled with a solution of strong sulphuric acid and terminating in two sealed bulbs. The left-hand bulb carries the heating coil through which the installation current flows. The air expands due to the heat generated, and forces the liquid to rise in the right-hand limb and overflow into a reading tube attached to the latter. The height of the liquid column in the reading tube indicates the maximum current, and its scale gives the currents and demand units. The tubes are attached to a hinged support so that they can be tilted for resetting to zero, when the liquid returns to the right-hand bulb from the reading tube. The instrument is naturally sluggish, which is an important feature in this class of instrument, and this sluggishness can be suitably increased by means of an iron cylinder round the heating bulb for use on motor and arc-lamp circuits. The extra capacity of the iron causes the indicator to register more slowly. It can, of course, be used both for c.c. and a.c. circuits.

In the **Fricker** demand instrument the differential thermometer consists of a tube of uniform bore terminating at one end in a sealed, horizontal, cylindrical bulb, and at the other in a vertical, spherical bulb, also sealed. The bore of the tube at the latter end is sealed by a mercury globule which acts as a valve. The thermometric substance is hydrogen. The spherical bulb is surrounded by the heating coil. When a current passes, the enclosed gas expands past the mercury valve into the vertical bulb. On cooling, the gas contracts

and sucks the mercury globule along the bore of the tube, the ultimate position of the valve being an exact measure of the current as indicated on the attached scale. The box containing the thermometer and coil has a water-jacketed cooling chamber attached to it to rapidly cool the thermometer when a reading is to be taken. The resetting is effected by holding the thermometer after its removal from the case in a vertical position and lightly tapping it. The time-lag of the instrument can also be suitably increased by increasing its heat capacity.

The **Atkinson-Schattner** instrument works on the electro-magnetic principle. It is a solenoidal ammeter with a soft-iron core of circular shape attached to a pivoted aluminium frame. The upper sector-shaped arm of the frame carries an hermetically sealed curved glass tube fixed to an aluminium scale, which is attached to the frame. The tube is filled with glycerine, or other viscous liquid, and contains a series of steel balls. On the passage of a current, the iron core is sucked into the solenoid and the tube with its scale is tilted. One or more balls slowly slide down the tube, and those beyond the bend pass into the radial lower limb of the tube. The number of the balls in this limb depends on the angle of tilt, *i. e.* upon the maximum current that has passed through. Temporary shorts do not affect the instrument on account of its sluggishness. Duplicate tubes mounted on scales are provided to prevent loss of time in resetting. To reset the instrument the used scale with its tube is detached from the frame and hung up on pivots in the cover, so that the balls return to the curved part of the tube, and the new scale with its tube is then hung on the frame.

The **Reason** electro-magnetic demand instrument is very similar, the maximum demand registering device taking the shape of a U-tube, having a restricted bend and attached reading tube. A plunger is drawn into a horizontal solenoid and swings over the U-tube, so that a portion of the liquid it contains passes into the reading tube. The sluggishness of the instrument depends on the amount of constriction of the narrowed portion of the U. The **Merz** and the **Aron** instruments are based on the principle of intermittent integration, the latter differing from maximum demand instruments in being a combination of the Aron meter with an intermittently actuated pointer which gives the demand units on a scale. The one instrument, therefore, not only registers the total units consumed, but also the demand units.

All these instruments can be used both for a.c. and c.e. circuits.

In **double-tariff**, or **two-rate systems**, two distinct charges are made according to the time of day when current is taken. The units consumed during the time when the high rate is

operative, during certain hours of the evening, are separately registered from those when the low rate is charged. To effect this the usual method employed at the present time is to install a *double-tariff*, or *two-rate* meter. It consists of an ordinary meter provided, in general, with two integrating mechanisms and a change-over device actuated by a time-switch, or change-over clock, or double-tariff clock, as it is variously termed. The one integrating mechanism (dial) registers the high-rate units, and the other the low-rate units, so that the consumptions are separately given. The change-over device, the object of which is to connect and disconnect each integrating counter with the spindle of the meter at the proper times, is controlled by the time-switch either electrically, when the meter and time-switch form two separate instruments, or mechanically, when the switch is contained in the meter instrument itself. The only modification of the meter is in connection with the registering part. The time-switch is a clock which at certain periods, the setting of which can be varied, alters the position of a mechanical change-gear, or changes from one series of electrical contacts to another. The clock is either hand wound, running for about forty days before it is necessary to rewind it, or it is provided with an electrical self-winding gear. It is unnecessary to give here examples of double-tariff meters, of which there are very many on the market. They differ only as regards the change-over devices and the clock mechanisms, apart from the actual meters to which reference has already been made in the preceding.

In addition to their use with two-rate meters, time switches are employed for a variety of purposes, such as for the control of street lighting, more particularly by incandescent lamps, when the lamps are automatically switched on and off at pre-determined times, for controlling corridor lighting, shop-window lighting after business hours, etc. They are also often fitted with selective devices for special purposes.

It should be borne in mind that a time switch consists of two distinct pieces of mechanism—the clock and the change-gear, and that it is of importance that both these parts should be absolutely reliable.

However ingenious the change-gear mechanism, unless the clock is reliable and accurate, the combination will be worthless. The construction should be simple, strong, accurate in operation, be furnished with a quick and ample make and break and a good, accurate clock, be cheap, well insulated and require no attention beyond the necessary periodical winding-up, unless self-winding.

These conditions seem to be very well fulfilled in the **Venner** time switch, a line drawing of which is given in Fig. 60. It

will be seen that as the hands *H* and *L* rotate with the clock dial the former first engages with the bell-crank lever *A* pivoted at *P*₁. On the smaller arm of this lever is pivoted the detent lever *C*, which is moved downwards by the lever *A* as it gets depressed to the right by *H*. This action continues until the detent comes below the projection *N*₂ on the contact lever *E*, against which the detent lever *C* is held by the spring *F*. When *C* engages with *N*₂ *A* is released by *H*, which is returned to its original position against the stop *G* by its spring *I*. *C* is

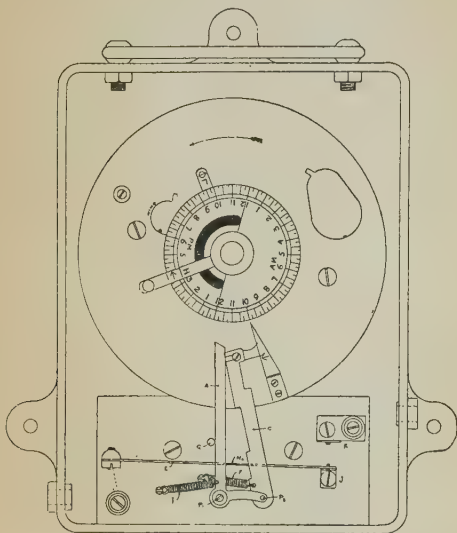


FIG. 60.

also raised and at the same time quickly brings *E* against the contact stop *K*. The circuit through *K* is then completed. It is broken later on by the action of the hand *L*, which pushes back *C*, causing it to release *E*, which rapidly springs back against *J*.

In the **prepayment** system of charging an automatic "slot," or prepayment, meter is used, consisting either of a special prepayment meter, or of an electricity meter of the ordinary motor type in combination, electrical or mechanical, with a prepay-

ment (coin-freed) device. The latter is a coin-actuating mechanism which, on the insertion of a coin in the slot and turning a handle, or knob, first closes a circuit switch in the meter and then winds up a spring-controlled wheel train, or displaces a mechanical gear by an amount corresponding to the value of the coin inserted. On the passage of the current the motor revolves, and either unwinds the spring or returns the mechanism to its original position, this being reached when the amount of energy corresponding to the coin has been consumed. The switch is then automatically opened and current cannot be taken until a further coin is inserted. Several coins can, as a rule, be paid in successively up to a certain number, differing in different meters, when the mechanism becomes locked. A further prepayment is now impossible until at least one coin's worth of energy has been consumed. Generally, the meter does not differ from the ordinary type to which it belongs, the only modification being the connection with the registering part. A prepayment meter should give the coins consumed in the usual manner, the coins unused and the coins used.

The **Watson** slot meter is a prepayment meter of a special type. It consists of a pendulum clock and a registering train which actuates a pointer round a dial. The pendulum clock oscillates a special zigzag lever, the swing of which is electrically controlled by a solenoid. This lever frees the escapement of the registering train of wheels through an escapement lever. When a coin has been inserted and the knob completely rotated, the instrument switch is closed and the registering clockwork wound up, the pointer being moved round the dial face to the number of lamp-hours corresponding to the coin. A current can now be sent through the meter and the zigzag lever is brought into gear with the pendulum clock. The lever oscillates and allows the pointer to be brought back to zero at a rate depending on the current in the circuit. The switch is automatically opened when the prepaid energy has been consumed. Several coins can be prepaid.

In the **Mordey-Fricker** electrolytic prepayment meter, which is, of course, only suitable for direct current, the copper anode of the electrolytic cell is in the form of a thin ribbon wound on a drum and mounted over the cell. When a coin is inserted in the slot and the handle turned, the anode strip is fed off the drum into the electrolyte by a definite amount corresponding to the value of the coin. The cathode is fixed in the electrolyte. The meter is in series with the circuit to be metered, which is completed through the electrolyte when the anode has been fed in. The copper of the anode gets slowly eaten away from the bottom upwards by the current and is deposited on the cathode, so that when it has been consumed the circuit is opened at the

surface of the solution. The whole arrangement is exceedingly simple. The length of the anode within the liquid can be increased by the insertion of coins in succession up to a certain number.

Hour meters are very frequently used in conjunction with special tariff systems, on tramcars and on circuits taking a known constant amount of power. They are simply ordinary clocks the escapements of which are held, either mechanically or electrically, until freed by the passage of a current. The clock then actuates a series of dials on which the hours are recorded during which current is taken. When the current is interrupted the clock stops. It usually consists of a hand-wound clock, the balance-wheel of which is arrested and freed by an electro-magnet, either series or shunt wound, so that the clock is only free to go when the electro-magnet is energized. In some cases a tiny electric motor drives the clockwork mechanism of the dials, when the motor is placed as a shunt across the circuit.

METER TESTING.

The various tests which should be applied to a meter are : time tests (usually at full load only), speed tests (motor meters) for accuracy at different loads, determination of the losses of a meter, effect of variation of pressure (energy meters), effect of change of frequency and power factor (induction meters), temperature effects, effect of stray fields (mostly restricted to c.c. switchboard meters), action of a short circuit current, and test for creeping (energy meters). In the case of electrolytic and Aron clock meters time tests are the only ones which can be carried out for the determination of the accuracy of the meter. The full load current is sent through the meter, and both the current and the pressure are maintained constant during the test, which should be long enough to enable reliable dial readings to be taken, the actual units consumed (measured by an ammeter and voltmeter, or a wattmeter) being compared with those registered by the meter under test. A time test enables a good idea to be formed as to the state of the registering train and gear in the case of motor and clock meters, and can be very conveniently conducted by the aid of a standard testing meter, when it is unnecessary to keep the load constant. In checking an energy meter, whether by a time or speed test, it is very essential that the pressure current of the meter under test and the pressure currents of the voltmeter and wattmeter do not pass through the series coils of any of the instruments. It is best to use two independent testing circuits—the load circuit, in which the main current circuits of the meter (or meters) under test, wattmeter and ammeter

are placed in series with the load, arranged so that it can be suitably varied, and the pressure circuit, to which the voltmeter and pressure circuits of the meter (or meters) and wattmeter are connected in parallel. If two independent testing circuits are not available, the pressure circuits should be connected in parallel across a branch circuit taken off the testing mains on the supply side of the instruments. With either of these arrangements the shunt circuit of the meter under test must be completely isolated from the main current terminals.

A **speed test of a motor meter** consists in checking the revolutions of the meter disc, or spindle, at different loads ($\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full load), for which purpose its **testing constant** has to be used. Very often on account of the speed of the meter being too high at full load to admit of the revolutions being counted with accuracy, at this load the meter testing dials are used, when the pressure has to be taken into account, as the dials read in units.

The **testing constant** gives the relation between the speed of the meter and the current (ampere-hour type), or the speed and the watts (energy type). In many instances the meter cover, or dials, gives the number of revolutions per B.O.T. unit from which the number of watts per revolution per second or amperes per revolution per second is readily deduced.

Let K = declared value of constant of meter in watts per revolution per second.

C = current in amperes.

V = volts.

W = actual load in watts (VC for direct current and $VC \cos \phi$ for alternating current, $\cos \phi$ being the power factor).

N = any convenient number of revolutions.

T = correct time in seconds (*i. e.* time calculated from declared value of constant) of N revolutions.

T' = observed time in seconds of N revolutions.

K' = actual value of constant deduced from observed values of speed and load (current or watts).

$$K' = \frac{W \cdot T'}{N}$$

$$\text{Actual error} = K - K'$$

$$\text{Percentage error} = \frac{K - K'}{K} \cdot 100.$$

The percentage error can be calculated from the watts, or the time taken to execute the given number of revolutions, or the constant as given above. If the watts are taken as the basis of reference, then W = observed watts and the watts given by the meter $= \frac{N}{T'} \times K$, so that the percentage error is

$\frac{\frac{N}{T'} \cdot K - W}{\frac{N \cdot K}{T'}} \times 100$, of course, the same as before, or from the

times = $\frac{\frac{N \cdot K}{\bar{W}} - T'}{\frac{N \cdot K}{W}} \times 100$. When this fraction is positive the

meter is reading high, and when it is negative the meter is under-registering. *E. g.*, suppose a meter should execute 40 revolutions in 1 minute, but takes 63 seconds on test, then the error is 5 % slow.

The percentage error given here refers to the meter readings and can be used directly. Thus, if the meter is reading 5 % high and the units read are 320, then the actual error is 16 units, and the correct value of the reading is (320 - 16) 304. This would be incorrect if the percentage error referred to absolute accuracy.*

When the meter under test is an ampere-hour meter then the current *C* amperes is substituted for the watts in the above. And only an ammeter, or potentiometer, or Kelvin ampere-balance is required in addition to the variable load and stopwatch, which should always be periodically checked, as badly handled and with a zero error it becomes a fruitful source of error. With an energy meter a wattmeter, ammeter and voltmeter should be employed, especially when it is an alternating current one. A three-wire energy meter is usually tested by placing its two field coils in series and checking it as a two-wire meter with its armature circuit energized at the proper voltage. It is best, if possible, to check it as a three-wire meter with two wattmeters arranged in the two halves of the network. With an induction meter it is necessary to know the frequency, and the testing should be carried out on non-inductive loads and repeated, at least at full load, with an inductive load having a power factor of about .5. Three-phase three-wire meters are best tested in a three-phase circuit with two wattmeters connected on the two-wattmeter method of measuring power. The sum of the two-wattmeter readings equals the total power. As a three-phase circuit is not always available for this purpose, each half of the meter is usually checked as a two wire single-phase meter. The two pressure circuits of the meter are kept energized whichever half is being examined. This is very important, as otherwise the tests would be unreliable owing to the braking action of the shunt fluxes. It is almost unnecessary

* See "Calculation of Percentage Error," by H. G. Solomon, *Elec. Rev.*, vol. lix, p. 524, 1906.

to remark here that instruments having different ranges should be used in testing meters so that the same accuracy is obtained in making the measurements at the different loads which vary very greatly ($\frac{1}{10}$ —full load). It is impossible to achieve this with a single instrument.

In **continuous-current meters** the maximum series loss is found by measuring the drop across the main current terminals with a low-reading voltmeter, or by means of a potentiometer, at full load and multiplying the drop by the full load current. The shunt loss is determined by measuring the ohmic resistance of the pressure circuit of the meter (after the voltage has been applied to the terminals for quite an hour) and dividing the square of the voltage by this resistance; the quotient is the wasted power in watts. Or it can be measured by measuring the current in the shunt, with the pressure applied, by means of a low-reading ammeter, or directly by a low-reading wattmeter.

In a.c. meters the drop is measured by a low-reading a.c. voltmeter and the shunt loss by an accurate low-reading a.c. wattmeter. The latter can also be determined by using the three-voltmeter or three-ammeter methods of measuring power (see pp. 127 and 128).

ELECTRIC CLOCKS.

The applications of electricity to horology are so varied that it is necessary to discriminate clearly between self-wound clocks, synchronizing, and electric time circuits.

I. In **self-wound clocks** (independent of one another and self-contained) electricity is simply used to provide the motive power.

II. **Synchronizing systems** require a regulator or master-clock which sends out electrical impulses—at each hour, or once a day—to correct the hands of complete clocks having an independent life of their own, whether key-wound or self-wound; and

III. Circuits of **electrically propelled dials** consisting of a “transmitter,” “controller” or “master clock,” whether self-wound or key-wound, which sends out electrical impulses at short intervals—usually every half-minute or minute—to propel the hands of “receiver,” “indicator” or “secondary” dials.

The first named, self-wound clocks, are numerous and interesting but commercially unimportant, because they merely “go without being wound up” and do nothing to secure uniformity.

The second, synchronization, can only be undertaken by owners of large networks of wiring, such as the Post Office, who have, however, not yet been persuaded to take it up. The cost of such a network, if devoted exclusively to the purpose, is too great compared with its earning power to be commercially attractive; consequently the only service available in England is that of the Standard Time Company, Ltd., established in London over thirty years ago.

We will, therefore, confine our attention to the third division, the only one which seriously concerns the electrical engineer, namely, circuits of electrical impulse dials now coming into general use in colleges, schools, hospitals, institutions, engineering works, and in fact in any large buildings, or groups of buildings, in which uniform and accurate time is required.

The **synchronous propulsion of groups of clocks** dispenses with ordinary clock work and substitutes “dial movements” which propel the hands, usually by means of an armature lever pulled by a magnet and replaced by a spring. Hence the name, “electrical impulse dials,” the impulses being transmitted by a master-clock. All attempts to produce such impulses by the application of contacts to the wheelwork of ordinary clocks have proved more or less unsatisfactory owing to the small power available and to a want of precision in the make and break.

Where such contacts are used it is necessary to reverse the direction of the current at each impulse and use polarized mechanisms in the dials. Broadly speaking, this is the Continental method and is represented by such systems as Grau-Wagner, Peyer-Favarger and Aron. Intermittent or "splashy" impulses can then do no harm because they can only rock the armature in one direction.

The "Magneta" induction system claims to overcome troubles due to failure of battery and contact, more or less inherent in foreign practice. Polarized dial movements are employed, and the master-clock consists of a weight-driven dynamo whose armature is allowed to make a revolution once a minute. This is a great improvement on the induction system of Bain and Wheatstone, but its limitations are obvious. A master-clock capable of operating a large circuit of small dials, or even a small circuit of large dials, must be both costly and cumbersome compared with one which has only to operate a switch, and will require frequent winding.

In his Institution paper of 1899, Mr. F. Hope-Jones, M.I.E.E., introduced a simple method of obtaining, in connection with a pendulum, a non-intermittent impulse perfectly clean in the make and break, and of a duration which automatically adjusts itself to the requirements of the dial movements in series with it. It forms the basis of the "Synchronome" switch, and most installations of electric time service in England are on the synchronome system or modifications of it. It may be described as a switch so combined with a pendulum that it operates every half-minute, the action of the switch being cleverly arranged to impart an impulse to the pendulum to keep it swinging for the next thirty seconds.

The switch consists of two moving parts: (1) the right-angled lever G (see Fig. 61) centred at F and normally supported on the spring catch K . Once every half-minute the lever is let down (in the act of giving an impulse to the pendulum P) upon (2) the armature A . Current from any available source then passes through the series circuit of dials and the magnet M , which attracts the armature A and throws up the lever G on to its catch again.

The pendulum releases the switch by means of the fifteen-toothed wheel C which carries a vane D engaging with the catch K at each revolution. The hook B pivoted upon the pendulum P turns this wheel once every thirty seconds. At the moment of its release the little roller R on the gravity arm G is just above the curved end of the pallet J , down which it runs, giving an impulse to the pendulum *at the moment when it passes through its zero or central position*. Thus the pendulum is free at all times except in the middle of its swing:

not only is the escapement detached, but it operates at zero, a combination which horologists have always been aiming at, but have not hitherto attained.

Indication of impending failure of battery is clearly given by the prolonged duration of contact which results from the switch magnet *M* being unable to replace the weighted lever without the assistance of the pendulum, which is so arranged to render that assistance when necessary. This duration is normally about the thirtieth part of a second, but is rapidly increased to nearly a whole second when the battery is insufficient, yet the circuit will continue to run perfectly for some days or even weeks before finally stopping. If this warning is neglected and the system is allowed to stop, the pendulum will hold the switch open, thereby preventing the battery from completely exhausting itself.

If desired, a single stroke bell may be included in the series circuit, so adjusted that the normal impulses of the short duration will not affect it, but the weaker the current and the longer the contact, the more loudly will it ring.

The dial movement illustrated in Fig. 62 is also that of the Synchronome Company. It is shown with hands attached and dial removed.

These movements are readily applicable to existing clocks of all kinds, from those on the mantelshelf with 3 in. dials to those in a turret with 6 ft. dials. When the latter are illuminated, the movements are fixed to the opal centres of the dials and are relatively so small as to be covered by the bosses of the hands, consequently the clock chamber is

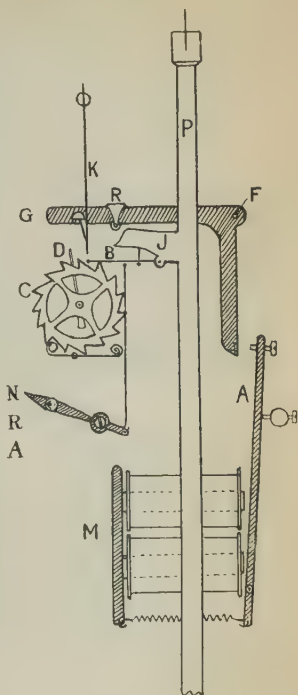


FIG. 61.

entirely empty and available for illumination, no shadows being thrown. Standard sizes of dials for ordinary purposes, such as offices and institutions, are 8 in. and 12 in. They lie flat to the wall and are simply hung like pictures on single nails. All instruments being in series, care should be taken to avoid loose connections, and the terminals should invariably be screwed up with pliers.

Convenient means of setting all the dials to time when first they have been hung and joined up in circuit is provided by the lever on the left-hand side of the controlling pendulum switch. When this lever is at *N* the click *B* releases the switch once every half-minute. This is its *normal* position. When depressed to *R* the switch will be put out of action altogether and the dials *retarded* for as long as may be necessary. When the lever is further depressed to *A*, the dials will be *accelerated*, because the switch will be released at every other swing of the pendulum.

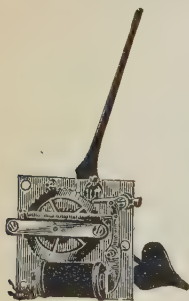


FIG. 62.

Ordinary electric bell wire of 18 gauge is electrically sufficient both for conductivity and insulation, but 3/22 or even 3/20 of electric light grade is recommended in order to minimize the risk

of disconnection and ensure respect for the line. Any convenient source of electrical supply may be used, a primary battery, a set of storage cells or electric light supply, and the current consumed is so small as to be practically negligible. Ordinary wet *Léclanché* cells are the least suitable on account of their high and fluctuating internal resistance, but they may be used if two series sets are provided and joined up in parallel. A reserve battery is never necessary, the consumption of current being so small as to have practically no effect upon the life of the cells, and ample warning of impending failure is given by the assistance rendered by the pendulum in resetting the switch.

This system is the one adopted by Messrs. Everett, Edgumbe & Co. for their inkless synchronised recorders, and in their application of it a pendulum beating half-seconds is used, the switching and self-winding action taking place every second or every five seconds (see p. 164).

ELECTRIC LIGHTING.

The chief **types** of electric lamps are—*

- (1) The Arc lamp.
- (2) The Incandescent or Glow lamp (carbon or metal).
- (3) The Nernst lamp.
- (4) The Mercury Vapour lamp.

The Arc lamp.†—In the case of a direct current arc the + carbon becomes hollowed out to form a crater from which comes the bulk of the light, while the - carbon becomes more or less pointed. The + carbon consumes away roughly twice as fast as the -.

The relative distribution of light in a direct current open arc with + carbon uppermost is shown in Fig. 6 (p. 47).

The **mean hemispherical candle-power** can be obtained from this curve as shown in Fig. 7. The mean hemispherical candle-power is roughly half the maximum in the case of a direct current open arc. As will be seen, the maximum is found at an angle of about 40° with the horizontal. An open arc converts roughly 10 % of the available energy into light. The consumption in a direct current open arc ranges from some 1.2 watts per mean hemispherical candle-power for a 2-amp. lamp down to some .5 watts at 10 amps., and as low as .3 watts at 40 or 50 amps. In an alternating arc the consumption is some 30 % greater, and in the case of lamps for low current still more.

The temperature of the arc may be taken as approximately 4000°C. , according to the Phys. Tech. Reichsanstalt.

The normal voltage required by a direct current open arc is 40 to 45, but three lamps can be run in series on 110 volts and six on 220 volts. An alternating arc requires only 27 to 32 volts.

Arc lamps may be grouped either in series or in parallel. For the former differential lamps are used, while for the latter shunt lamps can be employed, though the differential type is to be preferred.

In **enclosed arcs** the arc burns in an almost air-tight enclosure. The watts consumed per hemispherical candle-power are about 60 % greater than for an open arc, but the light is better distributed owing to the arc being longer (say $\frac{3}{8}$ in.). Such lamps will burn some 150 hours or more and require a voltage of about 80 at the arc.

* For a concise discussion of the merits of various illuminants, see W. Wedding, *Wirkungsgrad und Bedeutung der gebräuchlichsten Lichtquellen*, 1905.

† For a detailed discussion of the merits of various forms of arc lamp, see F. Hope, "Vergleichd. verschiedener Bogenlampe," *Elektrotech. Zeit.*, Sept. 7, 1905, p. 834.

Flame lamps.—In these arcs, which usually give a golden yellow light, the carbons are impregnated (as a rule) with 7 % to 15 % of fluor-spar. Sometimes also with strontium salts, which give a redder or whiter light as required. In some the carbons, instead of being set in line end to end, are placed nearly parallel to one another, whereby the light in a vertical direction is very much increased (*e. g.* Brenner lamp). The efficiency is some 100 % better than that of the open arc.

Globes.—A clear glass globe absorbs some 5 % to 15 % of the light. A “rippled” glass, 20 % to 25 %; ground glass, 30 % to 40 %; heavy do., 40 % to 50 %; heavy opal, 50 % to 60 %. A “holophane” globe absorbs 20 % to 30 %. Large globes absorb rather more than small ones.

The Incandescent or Glow lamp.—In the carbon lamp a carbonized filament is heated to incandescence in an exhausted bulb. Lamps can be obtained for any voltage from 1 to 250, and any candle-power from $\frac{1}{2}$ upwards. As a rule, a low voltage lamp will be slightly more efficient than a high—especially is this the case with low candle-power lamps.

Other things being equal, the greater the efficiency of a lamp (*i. e.* the higher the temperature of the filament) the shorter its life. The following may be taken as a fair average—

3.0 watts per candle-power =	600 hours life.
3.5 “ “ “ “	900 “ “
4.0 “ “ “ “	1200 “ “

It is usual now to rate lamps very high as regards **candle-power**; thus the average candle-power of so-called 16 candle-power lamps will actually be found to range from 12 to 14. A rise of 3 % in the voltage is said to reduce the life of a lamp by 50 %.* An increase of 1 % in the voltage produces an increase of candle-power to the extent of 5 % to 7 % and *vice versa*. There will always be a point in the life of every lamp at which it will be **economical to replace** it by a new lamp, rather than to continue to burn it. This point depends on (1) cost of lamp, (2) cost of electrical energy, and (3) constancy of candle-power. As a general rule it may be said that lamps should be replaced after 300 to 600 hours. It will, moreover, be found economical to run a lamp in such a way that the cost of lamp renewals is about $\frac{1}{3}$ of the total cost (*i. e.* lamp renewals and energy). Probably 3 to $3\frac{1}{2}$ watts per candle-power may be taken as the best all-round figure.

The Tantalum lamp (Siemens) has a filament of tantalum which is incandesced in a vacuum, as with the carbon lamp. The watts per candle-power are 1.3 to 1.7 and the life probably 1000 hours.

* *Electrical World*, vol. xxx. p. 736.

Of **metal filament lamps** the earliest were the osmium lamps of Dr. Auer von Welsbach, but they were only suitable for pressures up to 80 volts and have been superseded by lamps in which the filaments are impregnated with wolfram and other refractory metals. They are made by a number of firms in this country and abroad and sold under various trade names. The consumption is 1·1 to 1·3 watts per candle-power. For some time these lamps were not made with a lower candle-power than 50 for high voltage circuits, but they are now being put on the market for 16 and 32 candle-power at 220 volts. The average life may be taken at 1000 hours.

The Nernst lamp, due to Prof. Nernst, consists of a small rod of the fusible oxide of one of the rare earths which, after being heated by means of a small spiral surrounding it, forms a good conductor and incandesces brightly. The heating spiral is then automatically cut out of circuit. In order to keep the current approximately constant, in spite of small variations of voltage, an iron resistance enclosed in an exhausted glass tube is connected in series with the lamp, and having a high temperature coefficient, automatically regulates the current to a large extent. Even with this precaution, however, these lamps are extremely sensitive to **variations of voltage**, and any increase seriously reduces their life, which may normally be taken as 400 to 500 hours.

The **consumption** ranges from 1·7 to 2·0 watts per candle-power in new lamps. High voltage Nernst lamps are more efficient than low voltage.

The Mercury vapour lamp.—This lamp, perfected by Mr. Cooper-Hewitt of New York, consists of a long tube containing a small quantity of mercury and filled with mercury vapour at a greatly reduced pressure. At either end is an electrode. When once an arc has been started, which can be done either by tilting the tube or by a high voltage discharge, it will be maintained through the mercury vapour.

The **colour** of the light may perhaps be best described as a "bluish-green," and contains no red rays at all. Attempts have been made to improve the colour—with a corresponding loss of efficiency—by combining mercury lamps and glow lamps.

The candle-power gradually falls off owing to a greyish deposit on the glass, giving a life of 2000 hours and an **efficiency** of ·6 watt per candle-power.

Predetermination of Illumination.

Various rules have been given for predetermining the number of lamps required, *c. g.*—

(1) **Incandescent Lighting.**

- (a) 1 16 c.-p. lamp, 8 feet from floor, lights a circle 8 feet in diameter.
- (b) $2\frac{1}{2}$ to $3\frac{1}{2}$ c.-p. per 10 square feet of floor space illuminated with lamps 8 feet or 10 feet above the ground.
- (c) $\frac{1}{2}$ c.-p. per cubic yard.
- (d) Each 16 c.-p. lamp 7 feet above floor will light—
 50 square feet of classroom (well lit).
 80 " " sitting-room.
 100 to 150 square feet of bedroom.
 200 square feet of corridors.
- (e) Watts per square foot, lamps 8 feet from floor, rooms 10 feet high, should vary from .5 (dull) to 2 (brilliant).
- (f) For banqueting-halls allow one 16 c.-p. lamp per 25 m.

(2) **Arc Lighting.**

- (a) $\frac{1}{2}$ watt per square foot, for railway-stations.
 1 " " reading-rooms.
 2 " " museum (intense).
- (b) For street lighting.
 0.4 c.-p. per square yard is fairly lighted.
 0.7 " " well lighted.
 1.5 " " brilliantly lighted.
- (c) For yards, etc., use 10 amp. lamps 25 feet high and 80 feet apart.
 For workshops use 10 amp. lamps 15 feet above floor and 40 to 50 feet apart. Inverted arcs should be placed 25 feet apart, under the same circumstances.
- (d) With 8 amp. continuous current arcs allow—
 2000 square metres per lamp for yards.
 1400 " " stations.
 500 to 600 " " foundries (general).
 200 to 250 " " " (special).
 200 " " machine-shops.
 200 " " textile factories.

From the above estimates it may be gathered that it is almost impossible to give any universally accurate rule. Each case must be taken by itself, experience being the only guide. In two rooms, each 100 feet by 30 feet, one with white walls and the other with dark oak panelling, Kapp* found that the watts per square foot were respectively $\frac{3}{4}$ and 2 for approximately the same illumination.

Dr. Sumpnor† showed that if the walls were capable of

* Discussion on A. P. Trotter's paper, Inst. C. E., vol. cx.

† *Ibid.*

reflecting 50 % (as an average in all directions) of the light received, the illumination would be doubled (for table of % reflection by various surfaces, see p. 53). Also the kind of shade (if any) employed upon the lamps (especially with incandescents and alternating arcs) may make a further difference of some 50 % in the illumination. The introduction of metal filament lamps has considerably raised the standard of illumination demanded, partly owing to the reduced cost but also largely owing to the difficulty of obtaining these lamps of moderate candle power. For this reason the figures given on the previous page might well be increased by as much as 50 %.

TABLE OF ILLUMINATION ON A HORIZONTAL PLANE.*

	Illumination in foot-candles.†
Best for reading by (varies with the individual)	1
Billiard and dining-tables (well lit).....	1
Drawing-offices	2·5 to 4
Lecture-halls, shops, etc.	1·5 to 3
Textile factories.....	1·5 to 3
Picture-galleries (observed).....	1·7 to 3·1
South Kensington Museum—Incandescents (observed)	1·7 to 2·3
South Kensington Museum—Arcs (observed)..	2·2 to 3·2
Stages of theatres.....	3 to 4
Minimum at any point in a well-lit street.....	·08
Metropolitan railway-carriage (breast high) (observed)	·3 to ·9
Diffused daylight (observed).....	10 to 40
Ordinary moonlight—maximum (observed)...	·028
“ “ London—bright (ob- served)	·013 to ·017

For drawing offices, schools, and many factories, **inverted arcs** will be found useful. The ceilings must be whitened, and the lamp is usually suspended inside a funnel-shaped iron lantern, enamelled white inside, and having a concave reflector fixed below the arc. With continuous current inverted arcs

* For further details see Trotter's paper referred to above, also *Electrician*, vol. xxxv. p. 846, 1895, by Blondel; and Russel and Abney's report on action of light on water-colours, Blue-book C. 5453, 1888.

† An illumination of one foot-candle is that produced by one candle-power at a distance of one foot, or of four candle-power at two feet, and so on.

the + is usually the lower carbon, but when extra good diffusion is required, Messrs. Schuckert put the + below, so that nearly all the light has been already once reflected before striking the ceiling. The loss by this reflection is said to amount to 30 % or 40 % (as compared with some 10 % or 20 % with clear glass globes). The following table shows the illumination attained with these continuous current inverted arcs (+ carbon above), in the case of a room 4 metres high, having whitewashed walls and ceiling—

WATTS PER CUBIC YARD FOR CONTINUOUS CURRENT
INVERTED ARCS.

Mean illumination. Foot-candles.	Watts per cubic yard.
1	1·97
2	2·88
3	3·78
4	5·52
5	7·08
6	8·81

For alternating currents these figures must be increased by about 33 %. When there is much dust, especially if it be at all inflammable, it may be necessary to enclose the arc in a glass cover, and in this case another 20 % must be allowed.

Gas-lamps for **street lighting** range from 10 feet to 12 feet in height, and in main streets are placed 25 yards to 35 yards apart (in side streets as much as 50 yards apart). In substituting electricity for gas in the same lanterns, 16 c.-p. lamps would be used for side streets, and 32 c.-p. for main streets, or preferably, metal filament lamps of 50 c.-p. For arc lamps the **height** ranges from 20 feet to 50 feet in Europe, while in America even higher poles are employed. As a rule from 25 feet to 35 feet will be found a suitable height; the **distance apart** would vary from 25 yards to 100 yards, according to height, etc. Trotter is of opinion * that lamps should be placed at a distance apart equal to four times their height, but this figure is usually exceeded.

* For further details see Trotter's paper referred to on p. 210.

SECONDARY BATTERIES.*

By H. L. JOLY.

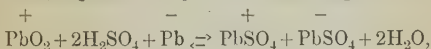
A SECONDARY cell,† storage cell, or electrical accumulator, is a reversible galvanic couple; *i. e.* a cell in which the electrical energy absorbed by chemical changes during the "charge" is partly returned on "discharge" by the reversal of these chemical changes.

Such reversible combinations may consist of a soluble cathode and an insoluble anode in an alkaline, or acid electrolyte such as zinc-lead, zinc-copper and lead peroxide-[Zn-Cd] couples, but this type of cell is not practicable because of the solubility of one of its electrodes, or even of both.

The only **satisfactory cells** consist of electrodes which are insoluble in the electrolyte, and the chemical compounds formed during the working of the cells must also be insoluble or practically insoluble in the electrolyte, whether the electrolyte takes an active part in the reactions or not.

The field is accordingly narrowed down to the "lead" cell, the electrodes of which in the "charged" state are peroxide of lead at the "positive" pole and spongy or reduced lead at the "negative" pole, the electrolyte being dilute sulphuric acid, and to another type the development of which is but recent, namely, the Nickel-Potash-Iron Cell, with which are associated the names of Jungner and Edison.

Theory of the Lead Accumulator. -The theory now generally accepted is that of double sulphation due to Dr. J. H. Gladstone and A. Tribe, represented by the reversible equation—



according to which 3.66 grammes of H_2SO_4 are absorbed for

* For further information the following may be consulted—

E. J. Wade. *Secondary Batteries* (Electrician Publishing Co.).

L. Lindon. *Secondary Battery Engineering* (McGraw, N.Y.).

L. Juman. *Accumulateurs Electriques* (Dunod, Paris).

F. Loppé. *Accumulateurs Electriques* (Gauthier Villars, Paris).

F. Holeczek. *Theory of the lead accumulator* (English translation Spon).

R. Lehfeldt. *Electro-chemistry*.

Papers by Ayrton, Inst. Elec. Engrs.; Hibbert, Inst. Elec. Engrs.;

Blanchon, Soc. Inst. Elec.; Cahen and Donaldson (Electrician); Highfield Inst. Elec. Engr.; Duncan, Amer. I. E. E.; Griseom, Amer. I. E. E.

Periodicals: *Centralblatt für Accumulatoren* (Peters); *Zeitschrift für Electrochemie*; *Electrochemical Industry*, New York; *The Electro-chemist and Metallurgist*, besides the other Electrical Journals.

Sir D. Salomon's *Management of Accumulators*.

† See also p. 516.

every ampere hour liberated by the cell whilst 7.72 grammes of lead are fixed at each plate. The reaction on discharge consists in a formation of sulphate of lead on both electrodes, whilst during the charge, the sulphate is reduced to Pb at the negative pole and transformed to PbO_2 at the positive pole. It must be remarked that in these formulæ no notice has been taken of the possible hydration of the compounds produced, nor of the fact that intermediate reactions may take place; because the final general equation complies with the Kelvin-Helmholtz law, and the theory has been proved correct by the careful work of Dolezalek and Mugdan.

The Electromotive Force of the lead cell varies with the concentration of the electrolyte, the potential difference at the terminals is greater than the E.M.F. during charge and smaller during discharge. When testing cells it is advisable to measure both, and also to make use of Fuch's method with an auxiliary electrode of cadmium or of spongy lead, as this method enables one to discriminate which of the electrodes limits the discharge.

Capacity.—The capacity is measured in ampere-hours, and is, as a rule, calculated for a discharge down to 1.8 volt, or 1.75 volt only, on load. The capacity of the lead cell varies with the rate of discharge, and for each type of cell, according to a law formulated by Peukert expressed by the equation—

$$C = I^n t,$$

in which C is a constant, I the rate of discharge, in amperes, t the duration of discharge, and n an exponential coefficient larger than unity, the value of which must be determined experimentally, and varies within wide limits. The capacity depends on the weight of active material present on both plates, its porosity, its surface, and the quantity of acid in solution.

The product of the capacity in ampere-hours into the average P.D. during discharge represents the energy liberated on discharge. This quantity varies with the concentration of the electrolyte and the rate of discharge; it is also affected by the temperature of the electrolyte.

Efficiency.—The ratio of output to input in ampere-hours is usually spoken of as the efficiency and varies up to 95%; the corresponding ratio in energy rarely exceeds 85%, and obviously both figures are considerably lower at high rates of discharge than at low rates.

Charge.—The charge may be effected at constant current, or at constant P.D., or at graded rates. The first way finds favour in most cases. The charge at constant voltage has been applied to specially designed cells, in traction work, and its only drawback is to be found in the very large intensity of the initial current. The graded rate, derived from it, gives satisfactory

results in electric automobile practice. During the charge the P.D. and the E.M.F. of the cells increase, as also the specific gravity of the electrolyte; when either of these three quantities becomes constant, the charge is ended, but the most reliable indication is given by the hydrometer, care being taken to measure the density of the electrolyte when the cell is "boiling," *i. e.* when the gases are freely but not violently liberated on both sets of plates.

Electrolyte.—The *density* most usually adopted in stationary batteries is 1.210 at the end of charge, in light traction batteries 1.250 is considered quite safe, in fact as much as 1.280 could be used, but for the rapid deterioration of the negatives.

The quantity of electrolyte is calculated so that its density remains about 1.150 at the end of discharge when the P.D. of the cell is 1.8 volt. The electrolyte must be a chemically pure solution of sulphuric acid, *i. e.* pure H_2SO_4 mixed with distilled water; foreign substances, chiefly metallic salts, nitric, hydrochloric acid or their compounds and also arsenic are injurious and must be avoided. Pure acid is easily obtainable, and need not be brimstone acid, as long as it will stand chemical tests for purity. The density of the electrolyte may refuse to rise to its normal during charge, in which case, especially if the gassing does not take place freely, after all possible internal short circuits have been eliminated, the cell must be "milked," *i. e.* separately charged, and not discharged until in a satisfactory condition. No acid of a density above 1.300 should ever be added to the cells, unless all other modes of procedure have failed. The level of the electrolyte must be well above the top of the plates and "topping up" effected when required, with distilled water only.

Acid.—The sulphuric acid used should be so pure that no residue is left after evaporating ten grammes of concentrated acid, any iron present leaving, in that case, a white residue which, after moistening with dilute nitric acid, gives the characteristic blue reaction with potassium ferrocyanide. Lead does no harm. Platinum is rarely found owing to concentration taking place in cheaper porcelain or silica vessels. Copper, tin and antimony are dangerous, and may be found in cells at work. Antimony causes premature discharge of the negatives. Arsenic should not be present; it can be detected by Marsh's test with great accuracy. Nitric and nitrous compounds must not be present; the Brucine test and the diphenylamine are the best to use.

Chlorine is easily detected by means of decinormal silver nitrate, and must be looked for in the distilled water.

Sulphating.—Either through short circuits between the plates or through being left idle between discharge and charge,

the plates may become "sulphated," *i. e.* the active material partly transformed into more or less irreducible sulphate of lead, beyond the ordinary transformation due to the discharge itself; this may also occur spontaneously with plates the grids of which are of hard lead, through local action. The density of the electrolyte will be lower than the normal, the plates somewhat white, and the internal resistance high. A long charge at a low rate will give the best results; this process may, however, be improved by the judicious use of sodium or magnesium sulphate, in small quantities, which, added to the electrolyte, form a double salt with the lead sulphate and facilitate its reduction. If the weight of acid in the electrolyte is known an addition of not more than 4 % of that weight will often be useful. Most authorities advise the removal of the electrolyte after de-sulphating and filling up with fresh acid. Milking boosters are used for the treatment of single cells.

Types and Manufacture of Lead Cells.—The lead cells may be broadly divided into two classes: **Planté** type and "pasted" or **Faure** type.

In the first class the plates are made of pure lead the real surface of which is made considerably greater than the apparent area, by casting, or by mechanical ploughing or by making each plate of superimposed, crimped or corrugated, strips, burnt to one or two conducting bars. The use of pasted negatives in conjunction with **Planté** positives has, however, practically superseded the old method of having **Planté** plates at both poles. Even the Chloride Company, whose negatives were made by the reduction of chloride of lead to sponge lead, in suitable grids, are now discarding this construction in favour of a pasted negative.

Pasted Plates.—The number of types patented or tried is enormous, but the number of satisfactory ones small. The paste of litharge, or minium mixed with dilute sulphuric acid or some binding medium, is placed, as a rule by hand, in the alveoles or pockets of the grid, which may be of hard lead or of pure lead. In the latter case, the grid at the positive pole takes part in the work of the cell after it has undergone partial peroxidization. In other types the grid is formed of two parts, undercut and riveted together, with the paste filling the alveoles.

The manipulation of lead oxides and compounds, being of a dangerous nature, is subject to regulations. Those in sway in the U.K. are the **Home Office Factory Rules**, No. 1004, of November 21, 1903.

The **materials** used in the manufacture must be pure, and it is advisable to submit each consignment to chemical tests. After pasting the plates are left to harden, and when hard they are formed.

Formation.—The work of the formation consists in producing in the case of Planté plates coatings of peroxide of lead and of spongy lead on the positive and negative plates respectively. In the case of pasted plates the action is simply a transformation from PbO to PbO_2 . For its reduction to Pb 240 amp. hours are required per kilog. of litharge; the oxidization of red lead to PbO_2 requires only 160 amp. hours per kilog.

In the original Planté process, numerous reversals of the charging current, with suitable periods of rest, were the only method of formation. The use of nitrates, perchlorates, acetates and other agents helped the manufacturer in obtaining a more rapid result. However, these acids or salts are very difficult to eliminate. The methods most in vogue now consist in the use of dilute solutions of these salts in sulphuric acid, or of alkaline sulphates as electrolyte during the formation.

After formation the plates are discharged, thoroughly washed and stored dry. If shipped as "lead plates" the cost of carriage is less than if the description "Accumulator plates" is applied.

Erection.—The "sections" are preferably burnt in position or bolted with hard lead connectors. The negative sections have one more plate than the positive sections, the end plates may be thinner than the others. The cells up to 800 amp. hours (in ten hours) can as a rule safely be put in **glass containers**, resting on sawdust in trays, the trays themselves resting on some glass insulators (oil insulators being advisable in all cases) on wood or iron stands protected by some antisulphuric paint. Spray arresters made of sheet glass must be placed above the plates.

For large batteries it is necessary to use either lead-lined tanks or solid hard **lead tanks**.

The plates are placed at a distance at least equal to 1.5 times their thickness and separated by means of glass tubes, ebonite forks or wood boards, in sufficient number to minimize the chances of contact in case of buckling.

In some designs the plates rest on stands at the bottom of the containers, but a more general practice with heavy Planté plates consists in casting the plates with an extra lug from which they hang. In any case a few inches are usually left free at the bottom to collect the material shedding from the plates.

After erection the cells are connected to the switch-board and **regulating switches**, preferably by means of copper bus-bars, protected with an acid-proof varnish, the regulating cells being as near the board as possible; when a reversible **regulating booster** is used, besides the working of the cells being improved, these bars are partly dispensed with.

After erection the cells are filled with low density electrolyte (1120 to 1180) acid submitted to a **first charge**, generally lasting

from 30 to 80 hours, until the plates are all gassing and present a normal appearance, the specific gravity of the electrolyte being adjusted to its normal, say 1210 at the end of charge. This charge must be continuous, and it is well to make sure before starting that no hitch may happen in the generating plant. Chalk or slaked lime and sawdust should always be kept handy in battery rooms, in case an acid vat should break.

The **battery room** must be dry, well ventilated, and lighted, no naked flame allowed, except whilst the cells are on discharge and for repairs only; smoking is obviously prohibited.

Maintenance.—Most manufacturers are ready to undertake the maintenance of batteries at rates which vary with the nature of the work performed by the battery. They require a careful record to be kept of the charges and discharges, the extent of which is further checked by the use of charge meters reading in amp. hours and calibrated about 10 % low, and discharge meters reading true. The instruments read alike so long as the amount of charge remains within the pre-arranged ratio. Records must also be kept of the voltages and densities, and such records are advisable even when no maintenance contract has been entered into. Some makers are at present giving a free guarantee for three years for Planté plates in preference to maintenance contracts.

Charge.—The charge, as already mentioned, may be effected either at constant P.D., or at constant current; in most cases, however, a graded rate of intensity will be found to give the best results. In the first method, there is a rush of current at the beginning of charge, harmless to the cells, and which allows of some 50 % of the charge to be put in the battery within an hour, the normal being eight hours for the charge. It is advisable to use a compound generator and to start it on resistances; a change-over switch being used to switch over to the battery.

The charge at constant current has been the most in favour, the E.M.F. of the battery increases with the duration of charge within limits fixed by the concentrations of the electrolyte, and the P.D. at the dynamo must be regulated accordingly. A shunt generator should be used.

In the third method, the initial intensity is greater than the normal, and the rate tapers down with the duration of charge to become quite small at the end so as to reduce waste of energy in liberating gases. A shunt generator is also used. Fig. 63 gives the characteristic curves.

Discharge.—The usual rate of discharge is of about 1.5 to 2 amperes per kilogramme of plates, the discharge is limited in practice to a final P.D. on a load of 1.8 volts per cell.

The E.M.F., P.D., and Sp.g. vary, as shown in Fig. 64.

The cells must be put on charge immediately after they reach

1·8 volts. When cells are pushed much below 1·8 volts or left standing, permanent sulphating of the plates, particularly the negatives, sets in.

The capacity depends upon the rate of discharge, an average figure being 10–12 ampere hours per kilogramme of plates. The formula $I^n t = K$, due to Peukert, represents the dependence of the capacity upon the rate of discharge. It is obvious that the lower the value of n , the nearer the cell comes to complete reversibility; this, however, does not affect the efficiency in the

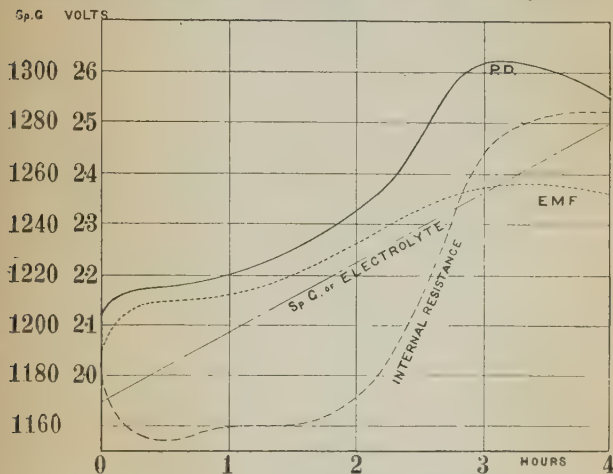


FIG. 63.

same way. The following example illustrates the average variations of capacity in lead cells.

Capacity at 1 hour's rate = ·55	} for stationary cells.
Capacity at 3 hours' rate = ·80	
Capacity at 6 hours' rate = 1	
Capacity at 9 hours' rate = 1·1	

Automobile Cells.—In cells used for automobile propulsion the weight has to be reduced to a minimum, consistent with large capacity and fitness for hard work. The standard automobile cells are therefore made up of a large number of thin plates, with stiff grids, and the specific gravity of the electrolyte used is as high as 1260; with a higher concentration the negatives

give trouble though the capacity is of course greater still. The sections are fitted in ebonite containers and the plates separated by means of ribbed or corrugated pieces of perforated ebonite. Ribs are provided at the bottom of the cells for the plates to rest upon. The cells are connected by autogenous soldering or by special lead-covered connections.

The methods of manufacture vary, but the most in favour consist in using moist powdery oxides of lead, and compressing them into the grids under hydraulic pressure, at the same time perforating the plates with fine holes. Twenty-five watt hours

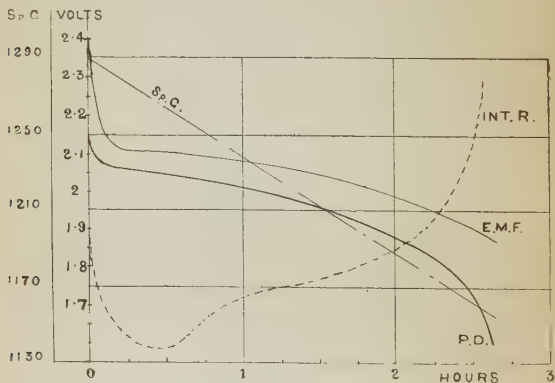


FIG. 64.

per kilogramme of cell have been regularly obtained at rates of discharge averaging 5 watts per kilo. This is essentially a French method of manufacture. Hand-pasted plates are cheaper, and if properly made and treated can give as good results as the hard plates in points of capacity, cost of maintenance and running.

The life of the cells depends of course both upon their construction and maintenance. Cells are on the market which do not require to be washed till after some 2500 3000 ton miles and to be replaced after 5000 6500 ton miles. The negatives last sometimes longer than the positives, but with hard positives, except on very rare occasions or when special treatment is resorted to, they rarely last twice as long as the positive plates.

Thin separators, made of bass wood, with ribs on one side—the wood being treated to remove chlorine and nitrogen compounds left by the sap in the fibre—are used extensively,

especially in combination with thin perforated ebonite laid right against the plate. They have been found very satisfactory in practice, and in some cases have considerably increased the life of the plates. The wood separators, however, are liable to reduce the capacity, and to become metallized.

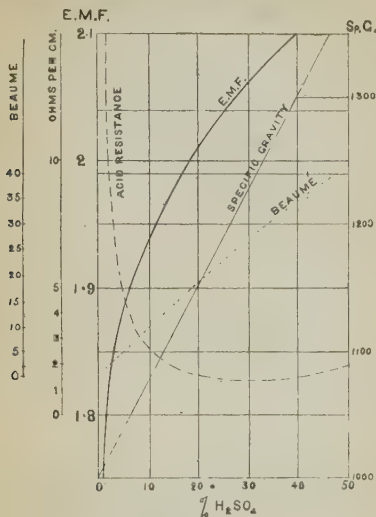


FIG. 65.

Relation between acid concentration and E.M.F.

Nickel Iron Potash Cells.—The Edison cell is up to now the only one of this type commercially manufactured; the H type contains twice as many nickel oxide plates as iron oxide plates.

The capacity depends much upon the gas occluded within the plates during the charge, a rest of twelve hours allowing the gas to escape without doing any work makes a difference of about 10 % in the available capacity. The efficiency in energy is low and the cost rather high. These cells do not appear to be in demand now, outside America.

HEAVY CURRENT RESISTANCES.

By H. W. W. DIX, A.M.I.E.E.

PARTICULARS of the resistances, weights and working currents are given on p. 223 for various metals and alloys, the sizes of wire ranging from 8 to 30 gauge. No. 8 is the **largest size** that can be coiled conveniently; if a larger section than this is necessary, coils of this size or smaller should be coupled two or more in parallel. No. 22 is the **smallest** that can be wound into satisfactory unsupported spirals; if smaller wires are required they should be wound on tubes of asbestos paper, or cylinders of porcelain or slate.

The weights in the Table are given as equal for all materials. This is, of course, not strictly accurate, but is near enough for most purposes, the error being not more than 2 or 3 % for the metals mentioned. The figures in the "ohms per 100 yards" column should, for accurate work, be checked against the resistance of sample coils of the wire to be used, for the specific resistance of alloys varies considerably with slight differences in composition or manufacture. Inaccuracies in wire-drawing have also to be reckoned with. All the currents tabulated give a rise of about 200° F., the resistance being of the open spiral form well ventilated, and the temperature being measured by hanging the thermometer near but not touching the coil.

The amount of **heat dissipated** per yard by a given size of wire, at a given temperature rise, is the same for all wires under the same conditions, and is independent of the nature of the material of which the wire is made. For this reason resistances may be conveniently made of galvanized iron wire, on account of its low cost and good mechanical qualities. The **special resistance alloys** are, as a rule, expensive, but if they combine, as they usually do, a high specific resistance with a low temperature coefficient they can be used with advantage—

- (1) When the high temperature coefficient of iron is inadmissible,
- (2) When wires of iron become too small to be conveniently supported.

Current carried by a Wire.

If the temperature is to remain steady the rates of heat production and radiation must be equal.

If C = current carried by wire; R = resistance in ohms per 100 yards (hot); J = joules equivalent for 1 lb.-deg. F. units.

d = diameter of wire in inches ; T = rise of temperature above the air.

H = heat units (lb.-deg. F.) radiated per square inch of surface per deg. F. temp. rise.

Then the rate of heat production per 100 yards of wire = $\frac{C^2 R}{J}$,

and " " " radiation " " " " = $\frac{TH\pi 3600 d}{J}$;

$$\therefore C^2 R = Kd \text{ where } K = \frac{TH\pi 3600}{J},$$

or $*C = 122.5 \sqrt{\frac{d}{R}}$; K being equal to 122.5 for a 200° F. rise

on well-ventilated open spiral coils. For any other temperature

rise T_1 , K must be replaced by K_1 when $K_1 = 122.5 \sqrt{\frac{T_1}{200}}$.

Making the allowable assumption that the surfaces are in the same condition and radiate heat equally, the coefficient K is the same for all materials.

DIAMETERS AND LENGTHS OF SPIRAL COILS FOR RESISTANCES.

S.W.G.	Diameter.	Area. □ "	B.W.G.	Diameter.	Area. □ "	Maximum Mandril.	Feet per turn.	Turns per inch.	Max. length of Coil.	Yards per Max. Coil.
	"			"		Dia.				
8	.160	.0201	8	.165	.0214	1 1/4"	.38	4	18"	9.1
9	.144	.0163	9	.148	.0172					
10	.128	.0129	10	.134	.0141					
11	.116	.0106	11	.120	.0113	1"	.30	4.5	12"	5.6
12	.104	.0085	12	.109	.0093					
13	.092	.0066	13	.095	.0071					
14	.080	.0050	14	.083	.0054	3/4"	.23	7	12"	6.5
15	.072	.0041	15	.072	.0041					
16	.064	.0032	16	.065	.003318					
17	.056	.0025	17	.058	.002642	1/2"	.16	9	12"	5.6
18	.048	.0018	18	.049	.001886					
19	.040	.0013	19	.042	.001385					
20	.036	.0010	20	.035	.000962	1/4"	.082	14	6"	2.3
21	.032	.000804	21	.032	.000804					
22	.028	.000616	22	.028	.000616					
23	.024	.000452	23	.025	.000491	}	These must be wound on a continuously insulated support.			
24	.022	.000380	24	.022	.000380					
26	.018	.000254	26	.018	.000254					
28	.0148	.000172	28	.014	.000154					
30	.0124	.000121	30	.012	.000113					

* This formula, which agrees with practice, has been used in compiling the Tables on pp. 224 and 225.

WEIGHTS, RESISTANCES, AND WORKING CURRENTS OF SOME METALS.

COPPER. Temp. coeff. = .24% per 1° F.				IRON (Galvanized). Temp. coeff. = .20% per 1° F.				EUREKA. Temp. coeff. = .0001% per 1° F.		BEACON. Temp. coeff. = .031% per 1° F.		Lbs. per 100 yards, at .31 lb. per cubic inch.	Size of Wire.
OHMS per 100 yards at 60° F.		Amps. at 260° F.	OHMS per 100 yards. B W.G. @ 60°		Amps. at 260° F.	OHMS per 100 yards at 60° F. S. W. G. B. W. G.		Amps. at 260° F.	OHMS per 100 yards at 60° F. S. W. G. B. W. G.	Amps. at 260° F.	S. W. G. B. W. G.		
S. W. G.	B. W. G.		S. W. G.	@ 60°		S. W. G.	B. W. G.					S. W. G.	B. W. G.
126	119	120	112	157	147	41	354	384	600	19.5	22.5	24	8
156	148	100	137	192	182	85	435	415	740	16.7	18.2	19.2	9
197	180	88	175	245	224	30	554	510	900	14.0	14.5	15.8	10
240	225	75	213	300	280	25	670	633	113	12	11.8	12.6	11
298	273	65	263	370	336	22	835	770	137	10	9.5	10.4	12
384	358	53	300	420	390	19	107	100	180	8.5	7.4	8.0	13
507	470	43	380	530	490	16	142	132	237	6.8	5.6	6.0	14
618	618	35	470	660	660	13	173	173	310	6.0	4.6	4.6	15
793	765	30	600	840	810	11	222	215	386	5.0	3.6	3.7	16
102	960	25	770	108	730	9.3	284	270	485	4.0	2.8	3.0	17
141	135	19	115	160	155	7.0	394	380	680	3.2	2.0	2.1	18
195	184	15	160	225	210	5.5	545	516	920	2.5	1.46	1.55	19
264	265	12	207	290	300	4.2	700	745	133	2.0	1.12	1.08	20
316	316	10	256	360	360	3.7	886	886	159	1.75	.9	.9	21
413	413	8.5	336	470	470	3.0	115	115	207	1.4	.69	.69	22
562	518	7.0	456	660	590	2.5	157	145	283	1.1	.51	.55	23
670	670	6.0	550	770	770	2.1	187	187	337	.90	.42	.42	24
100	100	4.4	820	115	115	1.5	280	280	504	.70	.28	.28	26
148	165	3.0	121	170	190	1.05	414	464	880	.55	.19	.17	28
210	225	2.4	172	240	260	.84	590	633	1130	.40	.13	.12	30

NOTE.—The Copper Wires are of 98% conductivity. To obtain their resistance at 260° F. multiply the resistance at 60° F. by 1.42.

COMPARATIVE RESISTANCES AND WORKING CURRENTS
FOR VARIOUS METALS.

Name.	Nature.	Comparative Resistance. Copper = 1 @ 60° F.	Comparative carrying cap. Copper = 1 at 260° F.	Increase %	
				Per degree F.	Per degree C.
Aluminum		1·8	·75	·217	·39
Bismuth		19·3	·234	·196	·354
Brussine	{ 61·34 Cu ; 37·4 Mn ; ·8Pb; 11 Fe.	4·16	·542	·092	·165
Carbon	Arc-lamp {	4000 to 8000		— ·0278	— ·05
Constantan } and Eureka }	{ Copper Nickl. Alloy	28	·22	{ practi- cally nil ·23	{ practi- cally nil ·414
Copper		1	1		
German Silver }	{ Copper Nickel Zinc Alloy	12·5	·32	·024	·0431
Graphite		670		— ·05	— ·090
Iron	Pure	6·13	·395	·267	·48
„	{ Commercial Wire	7·8	·35	·267	·48
Kruppin	Nickel Steel	49	·165	·035	·063
Lead	Pressed	12·2	·29	·217	·39
Manganin *	{ Manganese Copp. Alloy	22·7	·25	·0025	·0045
Mercury		55·4		·05	·091
Nickel		7·7	·366	·202	·365
Nickelin	Nickel Alloy	21	·253	·012	·0216
Platinoid	{ Copper, Nickel, Zinc, and Tungsten	21·4	·251	·0122	·022
Platinum		5·5	·456	·135	·243
Platinum Silver }	66% Pt	15	·306	·0172	·031
Platinum Silver }	33·4% Pt	14·5	·311	·018	·032
Rheostene	Steel Alloy	40	·183	·058	·1041
Resista	„ „	40	·183	·058	·1041
Silver		·95	1·04	·209	·377
Steeline	Hard	10·85	·347		
Tin		8·35	·353	·202	·365
Zinc		3·5	·545	·202	·365

* For further particulars of the properties of Manganin see *Electrical Review*, May 6, 1898.

NOTE.—**Platinoid and German Silver**, in common with all alloys which contain zinc, become brittle with prolonged use or overheating. All **steel alloys**, unless very carefully manufactured, are apt to have brittle spots, which prevent their being wound into coils. The **temperature coefficients** are smallest in the case of Eureka, Constantan, and Manganin; the value of the coefficient in the case of these metals being practically nil. Iron, copper, and steel wires can be **hard soldered** owing to their having high melting points. Iron and steel wires should be galvanized or plated to prevent oxidation.

Resistance Frames.

The commonest type of resistance is that formed by suspending wire spirals vertically from insulators secured to a metal frame. This is a satisfactory design for most purposes provided that the coils are stiff and not too long. Suitable proportions for the coils are given in Table on p. 223. The maximum **diameter of mandril** given for each size of wire should not be exceeded, but smaller mandrils can be used if specially stiff coils are desired.

Material of a fireproof nature should be exclusively employed for building resistances, and all insulated conductors liable to be damaged by heat should be arranged at the bottom of the frame. Plenty of heat-conducting material should also be put into the frames, especially in those which are only in circuit for a short time, in order that the **heat capacity** may be large, and the final temperature be only reached after some considerable time. Metal coils can have their heat capacity and effective radiating surface increased without alteration in resistance, by placing them in gilled cast-iron boxes filled with sand, oil, or some other insulating substance which is a fair conductor of heat.

Large resistances may be cheaply made up of galvanized **wire-netting** folded up and down over iron rods supported on insulators.

Galvanized iron wire rabbit-netting * of 1 in. mesh. 12 in. wide, wires 20 B.W.G., has a resistance of about .005 ohm per yard, and carries 100 amperes.

Temporary resistances of large capacity can be made by submerging **coils** of wire **in water**. Iron wire under these conditions will safely carry four times the current given in the table.

Expanded metal and cast grids have both been used in designing resistances.

* Vide E. K. Scott, *Electrical Review*, July 15, 22, and August 5, 1898.

EXPANDED METAL.*

Iron. Thickness B.W.G.	Amps. carried per inch of width.	Resistance per foot of piece 1 inch wide.	Approximate weight per foot of piece 1 inch wide.
16	95	·0025	3·5 oz.
18	75	·0032	2·4 „
22	45	·0041	1·6 „
24	37	·0055	1·2 „

Owing to the relatively small radiating surface possessed by **Enclosed resistances**, they must be made larger than those of the open type. In weatherproof resistance frames the size of the box should be determined by allowing $\frac{1}{2}$ to 1 sq. in. of cooling surface for each watt dissipated, this box being filled as full as possible with wire coils. With a good through draught and the box proportioned on the basis of 1 sq. in. of surface per watt, the maximum temperature rise measured in the air-current will be about 200° F.

Enamelled resistances are neat and compact, and provide a good support for the resistance material. The wires are embedded in a layer of enamel fused on to a ribbed cast-iron plate. Counting one side of the plate only as radiating heat, these resistances will dissipate energy at the rate of 5 watts per square inch. They are a good deal used both on the Continent and in America for shunt regulators, and are quite satisfactory, at all events for frames of moderate size; large resistances of this pattern, however, come out somewhat expensive. Resistances subject to much vibration should not be of the spiral form. In tramway work strip metal and asbestos tape are wound into flat narrow rolls, or zigzags, a suitable number being connected in series. This design gives a good continuous support to the resistance material.

Carbon resistances for heavy currents are used where very fine adjustments are required. They are employed a good deal in laboratories, but are not used to any extent in general engineering work. The resistance consists of a number of plates of carbon held in a frame, the end plates of the resistance being provided with terminals for connecting it in the circuit to be controlled. The resistance is varied by tightening or loosening a screw clamping all the plates together.

A solid arc lamp carbon 18 mm. dia. and 6 ins. long has a resistance of ·032 ohm., and will carry 50 amperes continuously. A cored arc lamp carbon 15 mm. dia. and 7 ins. long has a resistance of ·075 ohm., and will carry 35 amperes continuously.

Resistances for starting continuous current motors should

* Vide E. K. Scott, *Electrical Review*, July 15, 22, and August 5, 1898.

be so graded that equal rushes of current occur at each switch contact. The total amount of resistance with the switch on the first contact should be such as to pass, with the motor standing, an **initial current** that will not be great enough to cause a drop in voltage sufficient to interfere with the proper working of lamps or other apparatus on the same circuit. An initial current of from 5 to 10 amperes is generally specified in the regulations of Public Supply Works, and although these currents can very well be increased for motors working off private plants, a small initial current has the advantage of reducing the spark-

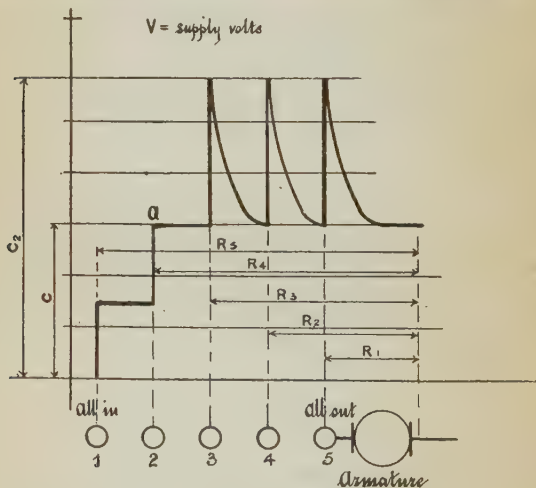


FIG. 66.

ing at the first contact of the starting-switch, when the motor is being stopped.

Public Supply Companies also sometimes require that a motor shall come to full speed with **rushes of current** not exceeding a certain amount, say 10 amperes. Assuming that a motor starts against a constant torque, represented by current C , and that when the motor has begun to rotate the switch is left on each contact sufficiently long to allow the motor to attain a speed corresponding to the amount of resistance in circuit at that contact, then the diagram (Fig. 66) represents approximately the process of switching on under these conditions. The contacts

1 and 2 are used for gradually switching on the starting current C , and the resistances R_5 and R_4 connected to them are readily calculated from Ohm's Law.

The motor starts at contact 3, and, in order that each rush of current may be equal to C_2 ,* the resistance R_4 must be divided in a geometrical progression, the extreme terms of which are equal to the armature resistance and the resistance of the first starting coil respectively.

If the switch has n starting coils (that shown in the diagram, Fig. 66, has three) and the common ratio of the progression is called f , then

$$f = \sqrt[n]{\frac{V}{CR_1}},$$

and the mean terms of the progression are $R_2 = R_1 \times f$; and $R_3 = R_1 \times f^2$. The equation for f can be easily solved with the aid of the approximate logs generally found at the back of the sliding scale of a slide-rule.

Mr. E. A. N. Pochin† has suggested the following useful mechanical method of finding the means, which does not involve the direct use of logs. Find the two extremes on the scale of a slide-rule and mark the length between them on a strip of paper, divide the length so found into a number of equal parts, one more than the number of means required, replace the paper strip on the slide-rule with the ends coinciding with the values of the extremes, and read off the means opposite the equal divisions marked on the paper. The length of one division on the strip of paper is the log of the common ratio as represented on the slide-rule used.

The momentary **rushes of current** are—

$$C_2 = \frac{R_2 C}{R_1} = \frac{R_1 f C}{R_1} = fC,$$

but the real value will be somewhat less than this, owing to the inductance of the armature circuit.

The number of starting coils to give a current rush of C_2 is

$$n = \frac{\log \left(\frac{V}{CR_1} \right)}{\log \left(\frac{C_2}{C} \right)}.$$

The resistance coils at the "all in" end should be designed to carry small currents only, the carrying capacity of the coils being increased as the "all out" end is reached. In no case need the coils be large enough to carry the starting current of the motor for any length of time.

* See *Electrical Review*, August 19, 1904, p. 287.

† *Electrician*, May 7, 1897.

Motor regulating resistances.—The speed of motors is generally *reduced* by inserting resistance in the armature circuit, the field being kept fully excited; or *increased* by weakening the field.

Approximate Rule.

C_x = amperes corresponding to load on motor at which it is desired to reduce the speed.

V = volts at motor terminals.

n = normal speed of motor.

n_s = reduced speed required.

w = ohms to be inserted in armature circuit, the resistance coils being designed to carry C_x amperes continuously.

$$w = \frac{.95 V \left(1 - \frac{n_s}{n} \right)}{C_x}.$$

This rule applies to shunt or compound motors or to series motors with the normal current passing. If series motors have to have their speed reduced when currents less than the normal are passing, then the corresponding weakening of the field must be taken into account.

Motors can also be regulated with boosters to reduce the supply voltage, or to a limited extent a reduction of speed can be obtained by suitably arranging the field winding. In the latter case the motor is designed to work with a weakened field at its normal speed, the speed being reduced by increasing the field strength.

This is an economical method of regulating motors as the waste of energy in large main circuit resistances is avoided; it has, however, the disadvantages that motors must be specially wound to suit it, and that reductions in speed exceeding 40% can only be obtained by increasing the size of motor, otherwise sparking will occur at the commutator.

LIQUID RESISTANCES.*

For absorbing a large amount of power, or for starting motors under load, liquid resistances will be found to have the following **advantages**: (1) They are compact, safe and cheap. (2) There are no metallic contacts to be destroyed. (3) The variation of resistance can be as gradual as required. Against this must, however, be set the fact that it is impossible, as a rule, to entirely cut out the resistance, unless the bath be short circuited.

* For a description of several modern heavy-current liquid resistances see F. Loppé, *Ind. Elect.*, April 25, 1909.

In Lyons' resistances the + and - plates each consist of a cone, fitting one into the other, so as to ensure as good a contact as possible; but even with this arrangement there is a drop of nearly a volt with the normal current.

The **electrodes** usually consist of (1) two plates, or (2) a plate and the containing vessel, or (3) a rod and the containing vessel. The metals most generally used are wrought or cast iron (for alkaline solutions) and lead (for acid solutions), carbon being also occasionally used.

The resistance can be varied either by moving the plates up to, or away from, one another, or by withdrawing one or both from the solution. When the latter method is employed the

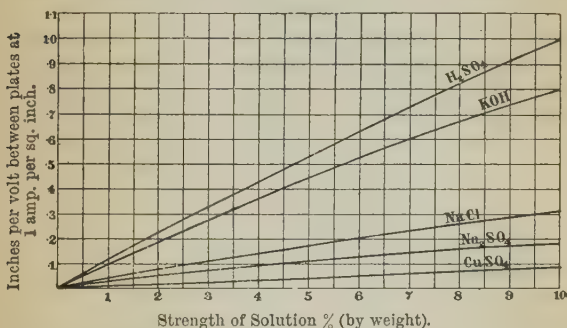


FIG. 67.

plates are often made triangular in shape, so as to increase the possible variation.

A large water resistance installed * at the City Road Station of the County of London Co., and intended to absorb 1000 kilowatts for long periods, consists of an iron tank through which water can be circulated. This water, which serves as the electrolyte, enters through a 4-inch pipe and flows away into an adjoining canal. For continuous currents the tank, which is earthed, serves as the negative electrode, while the current enters by a bottomless inner tank, also of iron, which can be raised or lowered by means of a winch and forms the positive electrode. For alternating currents, with which a much higher resistance is, as a rule, required, two iron rods, which can be

* For description see *Electrical Review*, vol. xlvi. p. 404, 1900. For description of some other Water Rheostats see *Electrician*, vol. xl. p. 696, 1898, and Morcom & Morris, *Jl. I. E. E.*, vol. xli. p. 137, July 1908.

raised or lowered by winches, from one pole (insulated), while the inner and outer tanks are coupled together and form the other electrode (earthed). To ensure effective circulation the two rods are surrounded by porcelain tubes. The over-all dimensions do not exceed 6 feet in any direction.

For voltages of less than 1000 it will usually be found necessary to increase, by some means, the **conductivity** of the liquid. The more usual plan is to add either sulphuric acid or soda to the water. Fig. 67 shows the effect on the conductivity of adding sulphuric acid, caustic potash, common salt, sodium sulphate or copper sulphate to the water (for further details see p. 94). For convenience the conductivity is not given in ohms but by the number of inches between the plates for a drop of 1 volt at a current density of one ampere per square inch. Ordinary tap-water gives a drop of from 2500 to 3000 volts per inch at this density. With Lyons' special solution the drop is about 4 volts per inch under the same conditions, and for incandescent lamp regulation this will be found a convenient value to work to. With sulphuric acid solutions, lead or carbon plates must be used. Common salt is often used, but in some respects sodium sulphate is to be preferred. Copper sulphate with copper electrodes will sometimes be found useful for regulating small currents at low voltages, owing to the absence of polarization. With other solutions iron is usually employed for the plates, and wood or slate for the tanks. The **current density** should not exceed one ampere per square inch, and half an ampere will be found convenient as a maximum value.

There still remains the question of how to **dissipate the heat generated**. In those cases where ordinary tap-water is employed a continuous stream can usually be kept up from the mains. Care must, however, be taken when using high voltages that the tanks are not thereby "earthed," and to this end rubber hose should be employed to lead the water into, and out of, the vats. For potentials up to 2500 volts a length of from 15 feet to 20 feet of hose is found sufficient to prevent "earths," so long as its diameter is less than one inch. For higher potentials longer or smaller hose must be employed. If a rise in temperature of t° F. be allowed, the quantity of water required to carry off one kilowatt = $\frac{3430}{t}$ lb. or $\frac{343}{t}$ gals. per hour. If the allowable rise be t° C., then $\frac{865}{t}$ kilogrammes per hour will be required. This applies also to metallic **resistances cooled by running water**. In this case, however, a good margin should be allowed, as the whole volume of water will

not come into intimate contact with the heated surface. When electrolytes, other than water, are employed, continuous circulation is seldom possible, and in such cases Fig. 68 will be found useful. This curve shows the number of **watts** which can be continuously dissipated, **per cubic inch** of solution, for a given rise of temperature. Theoretically the watts dissipated at a given temperature should depend on the surface area of the liquid and not upon its cubic contents. Experience shows, however, that calculations based upon the former consideration give results which are in no way borne out by experiment, while, on the other hand, the latter method closely agrees with practice.

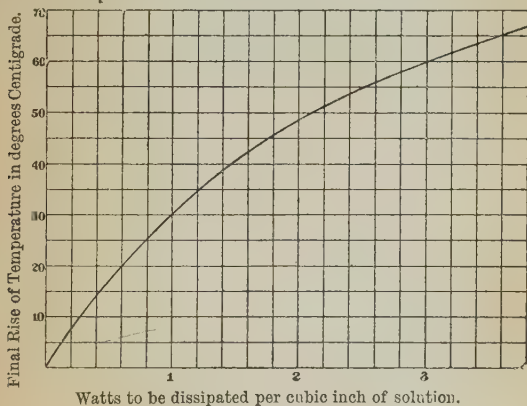


FIG. 68.

In using Fig. 68 considerable allowance should be made for disturbing influences, such as (1) nature of tank (whether of conducting or non-conducting material); (2) size of tank (the larger the tank the more watts per cubic inch it will dissipate); (3) temperature and dampness of surrounding air; (4) back E.M.F. of cell (with low voltages a large proportion of the energy may be absorbed chemically instead of being converted into heat). As a general rule from one to two watts per cubic inch, or (say) from 400 to 800 cubic inches per H.P. dissipated will be found to give good results for continuous running. For motors allow about 20 cubic inches per H.P. for starting resistances, and 60 cubic inches per H.P. for regulating purposes.

With alternating currents, liquid resistances have, of course, a certain **capacity**, but for all practical purposes it will be found negligible. The plates should, however, be chosen of a fair size, so as to avoid "Wehnelt" effects.

In **designing** a liquid resistance for a given purpose, after deciding upon the materials to be employed, calculate the size of plate to give the required current density. Next settle (by means of Fig. 68) the cubic contents necessary to dissipate the required number of watts. For high voltage resistances this latter consideration, rather than the former, will, as a rule, determine the size of plate, as at a high current density the length may, especially when using water alone, come out too small to allow of sufficient regulation. The length of the bath being thus fixed, a convenient strength of solution can be found from Fig. 67 giving the required drop of volts.

DIRECT CURRENT DYNAMOS AND MOTORS.

BY P. R. FRIEDLAENDER.

IN designing a machine for any particular purpose various preliminary points have first to be decided.

(a) Except for small outputs not exceeding about 2 kw., bipolar magnets are now seldom used. The greater the **number of poles**, within certain limits, the lighter will the machine be, but the labour expenses will increase somewhat. The number of commutator segments necessary for satisfactory running also increases with the number of poles. In practice, since a given standard machine must usually be capable of being wound for any voltage between 100 and 500, a compromise has to be made between the lightness and sparkless



FIG. 69.

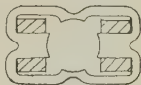


FIG. 70.



FIG. 71.



FIG. 72.

running without an excessive number of commutator segments, and it is very common to find machines up to 75 kw. or 100 kw. having 4 poles, from 100 kw. to about 300 kw., 6 poles, and the number of poles increasing gradually with the output until, for 1000 or 1500 kw. engine-driven sets, 14, 16 or more poles are employed. These figures can, however, only be considered as rough guides, and in special cases comparative designs with different numbers of poles should be got out. In the case of heavy current machines the number of poles often has to be increased above the normal in order to keep the current carried by each armature conductor within safe limits as regards commutation, whilst for H.T. machines, where the permissible commutator segments are often limited by mechanical considerations, fewer poles are preferable in order to ensure satisfactory commutation.

(b) The more usual **magnet types** are shown in Figs. 69 to 72.

Fig. 69 is the ordinary 2-pole horseshoe type.

Fig. 70 is a 4-pole **consequent pole** type sometimes used for car motors and other iron-clad machines.

Fig. 71 is another **iron-clad** type with two poles and one exciting coil. It is a convenient shape for conditions requiring a specially small height of armature centre.

Fig. 72 is the usual form of multipolar magnet, and consists of a cast-iron or cast-steel yoke with cores either of cast steel bolted on to the yoke, or of laminated wrought iron cast in or bolted on.

(c) **Slotted armatures** are nowadays practically the rule. They are mechanically stronger and less liable to damage than smooth ones, and their use also leads to a decided saving in weight of magnet wire. On the other hand, slotted armatures are rather more trouble and expense to build, and there is an increased tendency to spark at the brushes which has to be overcome either by increasing the number of commutator

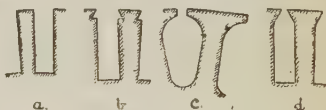


FIG. 73.

segments or by other means (see p. 258). Consequently smooth-cored armatures are preferred for steam turbine generators where, owing to the high speeds, every possible precaution against sparking has to be taken. The four most commonly used shapes of slot are shown in Fig. 73.

Shapes *a* and *b* are suitable for form-wound drum-windings, in which the completed sections have to be passed into the slots through the top. If *a* is used the winding is held in position by binding bands wound over the core as for smooth-cored armatures, whilst with *b* the bands are replaced by long strips of wood forced into the dovetailed nick at the top of each slot. The partially closed slots *c* and *d* are used for hand-windings (the wires being either passed singly through the small opening at the top or else threaded through the slots from end to end), or for built-up bar-windings.

(d) Machines may be **separately excited** from some external source, or they may be self-excited in one of three ways, known as—

Series, Shunt, and Compound excitation.—Figs. 74, 75 and 76 show diagrammatically the connections of the magnet-winding

to the brushes and to the main terminals of the machine in the three cases, together with the methods of regulation. The diagrams apply both to dynamos and to motors.

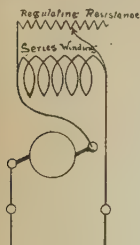


FIG. 74.

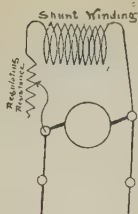


FIG. 75.

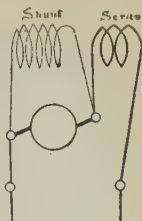
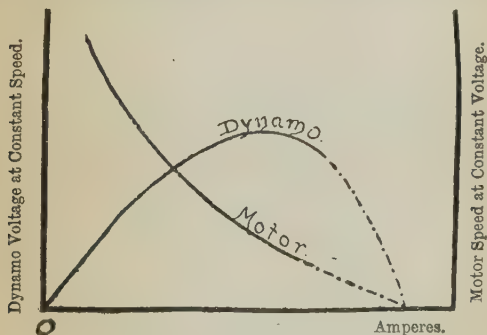


FIG. 76.

The typical external characteristic curves shown in Figs. 77, 78 and 79 represent graphically the behaviour of machines excited in these different ways. The full lines cover the working



Voltage and Speed of Series Machine.

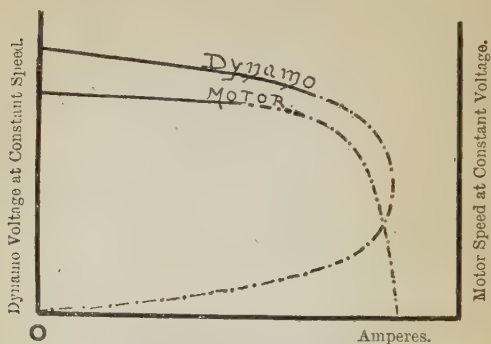
FIG. 77.

range of the machines and the dotted portions show approximately what occurs with overloads.

A modified method of shunt-excitation was patented some years ago by Mr. W. B. Sayers,* and is shown in Fig. 80. A small extra

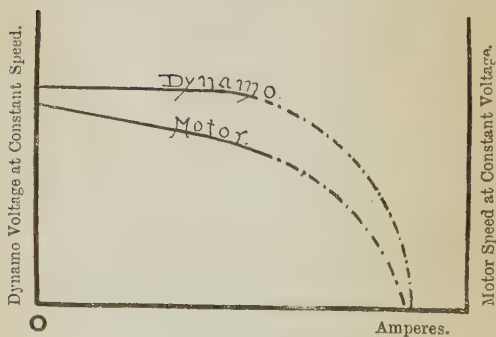
* *Electrician*, No. 1051, 8/7/98, p. 358.

brush is arranged to collect the exciting current, at a point on the commutator about midway between the main positive and negative brushes. The shunt (which is wound for full excitation



Speed and Voltage of Shunt Machine.

FIG. 78.



Speed and Voltage of Compound-Wound Machine.

FIG. 79.

at half the voltage of the machine) is connected between this extra brush and the main brush ahead of it in the direction of rotation.

Both the strengthening of the trailing-horn by the armature cross-magnetization and also the shifting of the main brushes forward as the load increases (the extra brush remaining in one fixed position) tend to increase the exciting voltage at full load and so give a more or less level characteristic. The small extra brush is found to give no trouble from sparking in practice—especially if it be of carbon.

The actual polarity of a dynamo (*i. e.* which brush is positive and which negative) depends on the residual magnetism that happens to be in the iron, and is therefore indeterminate. If at any time a current is sent round the exciting coil, in the reverse direction to the normal, the polarity of the machine will have been reversed, and next time it is started up the pole that used to be the north will be the south, and the positive brush will have become the negative brush. The same result is sometimes produced if an excessive current is taken from the armatures. This point is important in connection with battery-charging and dynamos which have to be run in parallel with one another.

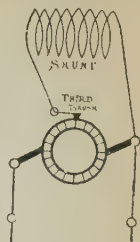


FIG. 80.

Having decided these preliminary points, the design of the dynamo or motor can be proceeded with as follows—

(1) Determine the approximate **Diameter and Length of the Armature Core** by the rule—

(diameter in inches)² × length of core in inches

$$= \frac{C \times \text{watts output}}{\text{revolutions per minute}}$$

where *C* (the “output coefficient”) has the following values—

Output in Kilowatts	<i>C</i>
Up to 5	From 80 to 200,
From 5 to 30	about 60,
„ 30 to 100	„ 40,
Over 100	„ 30.

These figures apply equally to smooth or slotted armatures and to bipolar or multipolar machines. The exact dimensions for a given output are not of very great importance, for though the size may vary considerably, the weight of material and the price of the machine remain practically the same over a large range. Having found the value of (diameter)² × length, it is usual to select such a **diameter** that $D = \text{number of pairs of poles} \times L$ approximately, this proportion enabling fairly square-

shaped pole pieces and circular magnet cores to be employed. At the same time the **peripheral speed** of the armature core must be taken into account and should not exceed—

Up to 25" diameter	3000 to 4000 feet per minute,
30" to 40" ,,	3500 to 4500 ,, ,,
Over 50" ,,	5000 to 6000 ,, ,,

although considerably higher values are employed in the case of the special constructions used for turbo-generators.*

In practice, of course, the core dimensions selected are largely influenced by the fact that stock sizes of discs must be brought in. Each manufacturer has his own line of standard disc diameters such as 5", 10", 15", 20", 30", etc., and each diameter of disc is often employed to make up cores of two or three different lengths. The outputs obtainable at a given speed from these different cores are, with fair accuracy, proportional to the length of core and to the square of its diameter. With the larger diameters, however, the allowable outputs go up more rapidly than these proportions would indicate, owing to the improved ventilation.

(2) Determine, tentatively, the **size of wire** to be used on the armature by means of the permissible current density. The usual **current densities** are—

For small armatures up to 2 kilowatts, 3000 to 5000 amperes
per square inch.

For armatures up to 100 kilowatts, 2500 to 3500 amperes
per square inch.

For large armatures, 2000 to 3000 amperes
per square inch.

The current carried by each conductor is—

$$= \frac{\text{total current of the machine}}{\text{total number of parallel paths connecting positive brushes to negative brushes}}$$

Thus, in an ordinary 2-pole machine, with a single winding, the current in each conductor

$$= \frac{\text{total current}}{2} = \frac{1}{2} \text{ total current.}$$

In a series coupled 4-polar armature with an ordinary single winding each conductor carries $\frac{1}{2}$ total current. In the same way, if a 4-pole armature is parallel coupled the current in each conductor is $\frac{1}{4}$ of the total current for an ordinary single winding, or $\frac{1}{2}$ of the total current for a double winding.

Having provisionally selected the size of wire, the total

* See R. Pohl, *Journal I. E. E.*, Nov. 1907.

number of conductors on the armature can be approximately settled by means of the following table (based on experience), and the wire table given on p. 254.

TABLE OF APPROXIMATE WINDING SPACE ALLOWABLE ON ARMATURES OF VARIOUS SIZES.

Diameter of armature in inches.	SMOOTH CORES.	SLOTTED CORES.	
	Total radial depth of winding space and clearance (= radius of magnet bore - radius of armature core). Inches.	Depth of each slot. Inches.	Width of each slot. Inches.
3	·25	·5	·25*
5	·3	·6	·25*
10	·61	1·0	·35
20	—	1·25	·45
40	—	1·5	·5
100	—	1·75	·6

The insulation on the cores of smooth-cored armatures is from $\frac{1}{16}$ " to $\frac{1}{8}$ " in thickness (radially) according to the voltage. This insulation would consist, in 100 to 230 volt machines, of about four or five successive layers of cotton-tape wound over the core spirally, and each layer varnished as it is put on, or of a similar number of layers of varnished canvas. For 500 volts and over there is usually a complete layer of thin mica (or mica cloth) in addition. The allowance for binding-bands is from $\frac{1}{16}$ " to $\frac{1}{8}$ " in radial thickness (*i. e.* an allowance of $\frac{1}{8}$ " to $\frac{1}{4}$ " in the diameter of the armature). The clearance between the outside of the binding-bands and the magnet bore varies from $\frac{1}{32}$ " or less for 3" diameter armatures to $\frac{1}{4}$ " or $\frac{3}{8}$ " for 100" diameters. In the case of slotted armatures the air gap from the top of the tooth to the magnet face should not be less than half the width of slot opening (see p. 265). The number of driving teeth placed on smooth-cored drum armatures is about one wooden tooth $\frac{1}{8}$ " thick $\times$ 1" wide to every 5" or 6" of core length, the rows being spaced 3" to 4" apart round the circumference. In slotted armatures it is usual to make the width of the slot approximately equal to the width of the tooth at the outer periphery. The **space factor**, *i. e.* the ratio of nett copper

* The slots of such small armatures as these are often partially closed at the top as in C, Fig. 73.

section in a slot to gross slot area should not fall below .3 except in the smallest armatures, and may reach .45 in large strip-wound armatures.

The insulating lining of the slots of slotted armatures consists, up to voltages of about 500, of a trough of varnished canvas or of tough "manilla" paper or presspahn about .02" thick all round. For higher voltages shaped tubes or troughs of thin micanite or of compressed paper are used.

A useful check on the number of conductors found as above is given by the permissible **ampere-wires** per inch. The ampere-wires per inch of armature periphery (*i. e.*

$$\frac{\left\{ \begin{array}{c} \text{total number of conductors} \\ \text{on armature} \end{array} \right\} \times \left\{ \begin{array}{c} \text{current in each} \\ \text{conductor} \end{array} \right\}}{\text{diameter of armature in inches} \times \pi}$$

should lie within the following limits (determined partly by sparking and partly by heating considerations)—

For armatures up to 5" diameter 150 to 200 ampere-wires per inch of periphery

"	"	10"	"	250 to 400	"	"
"	"	30"	"	500 to 600	"	"
		over 30"	"	600 to 700	"	"

The exact number of armature conductors * may be any even number for the single parallel (or lap or multiple circuit) type of armature winding, whilst for the single series (or wave or two circuit) type of winding the number must satisfy the equation—

$$\text{number of coil sides} = (\text{number of poles} \times y) \pm 2,$$

where number of coil sides = number of commutator segments $\times 2$ and y = the span (average of back and front spans) of each coil (stated in coil sides) and would usually be made a number rather smaller than $\frac{\text{total number of coil sides}}{\text{number of poles}}$.

The maximum number of turns per section, and consequently the minimum permissible number of commutator plates, is determined by sparking considerations, and can be arrived at by the rules given on p. 258.

(3) The next step is to determine the **heating**. This depends on the total watts lost in the armature (*i. e.* ohmic loss in the winding + hysteresis loss in the iron core + eddy current losses as calculated on pp. 243-6) and on the available cooling surface.

The permissible **watts per square inch** of cooling surface for the usual Admiralty temperature specification of 70° F. rise above the atmosphere after a six hours' run, and for ordinary

* For a detailed description of the various possible armature windings, see Parshall and Hobart's *Armature Windings* and Hobart's *Electrical Motors*, pp. 50, 86, and 160.

peripheral speeds of 2000 to 5000 f.p.m. are approximately as follows—

For long unventilated armatures. . . 1·5 to 2 watts per □"

For large well-ventilated slotted armatures with one annular ventilation

gap to about every 4" or 5" of core-length 3 to 4½ „ „

For protected motors about ¾ of the above values.

For completely enclosed or iron-

clad machines half the above figures.

NOTE.—For *enclosed or iron-clad machines* it becomes necessary to consider the watts per □" of the outside of the case as well as the separate watts per □" of armature and field windings.

The total watts lost in the machine (ohmic loss in armature winding + armature core losses + ohmic loss in the field coils + ohmic loss at brush contacts + brush friction loss) divided by the total outer surface of the case should lie between ·25 and ·35 as a maximum value.*

For enclosed machines intended for intermittent work, such as traction motors and crane motors, the watts per □" of armature cooling surface may be made as high as 6 or 8 at the maximum rated load of the machine.

These figures refer to block or barrel wound armatures, and the surface to be used is simply the peripheral cylindrical surface from end to end of the winding.†

Armature Losses.

The ohmic **resistance** of the armature winding *hot*, i.e. at 125° F. (52° C.) =

$$\frac{\cdot 000000777 \times \text{mean length of one } \frac{1}{2} \times \text{total conductors}}{\text{conductor in inches} \times \text{on armature}} \\ \frac{\text{sectional area of}}{\text{each conductor } \square''} \times \left(\frac{\text{number of paths from positive}}{\text{brushes to negative brushes}} \right)^2$$

(and it increases or diminishes about ·24 % per 1 degree Fahr rise).

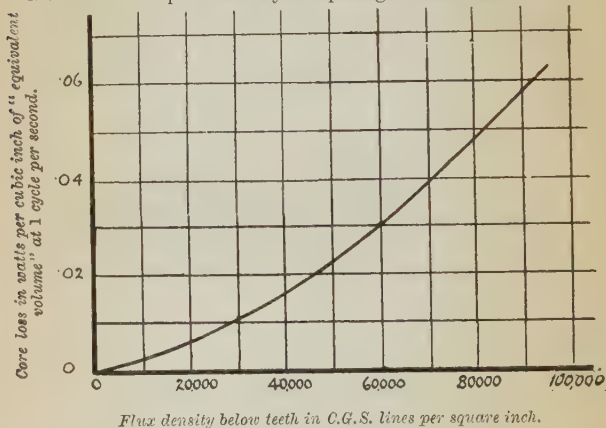
* As this usually entails a very small loss in armature and fields it will often be found advisable to provide the external surface of the case with a series of ribs. Messrs. Brioschii Finzi and Co. of Milan have adopted the plan and obtain from 40 % to 80 % more cooling surface thereby. It is also valuable in the case of induction motors.

† This surface is approximately equal to

$$\pi \times \frac{\text{diameter of armature}}{\text{armature}} \times \left(\frac{\text{length of armature + iron}}{\text{length of armature}} + \frac{\cdot 7 \times \pi \times \text{diameter of armature}}{\text{number of poles}} \right).$$

$$\dagger \text{ This } = \frac{\text{length of armature}}{\text{iron}} + \left(1\frac{1}{2} \times \frac{\pi \times \text{diameter of armature}}{\text{number of poles}} \right).$$

The **core losses**, *i. e.* the difference in the watts absorbed by the armature when run round unexcited and when run fully excited at the same speed are in practice usually found to exceed the combined hysteresis and eddy current losses, calculated by the ordinary theory, by a considerable amount. This is partly due to the fact that the distribution of the flux over the core section is by no means uniform as is assumed by the theory, partly due to additional eddy currents resulting from the burring over, to a greater or less extent, of the armature discs at the periphery through filing or machining, and partly, in the case of slotted armatures, because of the eddy currents induced in the pole faces by the passage of the teeth.



The watts are correct for direct current machines.

The watts are to be multiplied by .75 for alternators with stationary armatures.

The watts are to be multiplied by .6 for stators of induction motors.

CURVE OF CORE LOSSES.

FIG. 81.

Under these circumstances the only useful method of arriving at the core losses is one based on experimental results.* The curve shown in Fig. 81 gives the average results of a large number of no load loss tests carried out on machines of very

* For various other empirical methods of calculating the core losses, see Hobart, *Electric Motors*, p. 29; Hobart, *Electrician*, November 23, 1900, p. 171; *ibid.* June 3, 1904, p. 275; also Esterlin and Wood, *Transactions of American Inst. of Elec. Eng.*, vol. xx, 1903, pp. 1323-38.

varied sizes and speeds, and has been found to give a sufficiently close approximation for all practical purposes.

In order to use it the following values must be obtained—

V_t = the volume, in cubic inches, of the teeth (if the armature has a slotted core) calculated from the teeth dimensions, taking about 90 % of the length of the core as solid iron, the remaining 10 % being occupied by paper or varnish **insulation between the laminations**,

V = volume in cubic inches of the part of the core below the teeth (= the whole core volume in smooth-cored machines),

B_t = average flux density in the teeth, assuming that the number of teeth carrying the flux per pole is equal to the teeth actually under a pole piece together with 5 % to 10 % more to allow for fringing at the pole edges,

B = flux density below the teeth,
 total flux per pole calculated as in (4), p. 245,

$$= \frac{\left\{ \begin{array}{c} \text{length of} \\ \text{core parallel} \\ \text{to shaft} \\ \text{in inches} \end{array} \right\} \times .9 \times \left\{ \begin{array}{c} \text{radial depth} \\ \text{of core discs} \\ \text{below teeth} \\ \text{inches.} \end{array} \right\} \times 2}{120}$$

and p = periodicity of the magnetic reversals
 revolutions per minute $\times$ number of poles.

$$= \frac{\text{revolutions per minute} \times \text{number of poles}}{120}$$

If the **core is slotted** the actual volume of the teeth and of the part below the teeth must be combined into a single "equivalent volume" V_E , the whole of which can be considered as working at the density B . This may be done by making the approximately correct assumption that the loss per cubic inch varies as the 1.6th power of the flux density, so that—

$$V_E = V + \left\{ V_t \times \left(\frac{B_t}{B} \right)^{1.6} \right\}$$

(the value of $\left(\frac{B_t}{B} \right)^{1.6}$ can be read off Fig. 82 for all cases

occurring in practice). The total core loss is then found by reading off Fig. 81 the value of the loss per cubic inch of equivalent volume at a periodicity of 1, corresponding to the given value of B and multiplying this value by V_E and by p .

For smooth core armatures, of course, $V_E = V$.

(4) Having designed the armature to give a satisfactory number of watts per $\square''$, the **magnet** must next be considered.

The required magnetic **flux per pole** in C.G.S. lines =
armature voltage (internal)

$$\frac{\text{Revolutions per minute} \times \frac{\text{total conductors on the face of the armature}}{\text{armature voltage (internal)}} \times \left\{ \frac{\text{number of poles} \cdot \text{number of parallel paths connecting the positive and the negative brushes}}{2} \right\} \times .6 \times 10^{10}}$$

$$= \frac{\text{Revolutions per minute} \times \text{Total conductors on the face of the armature}}{\text{armature voltage (internal)}} \times .6 \times 10^{10}$$

for parallel coupled armatures (single winding), and

$$= \frac{\text{Revolutions per minute} \times \text{conductors on the face of the armature} \times \frac{\text{number of poles}}{2}}{\text{armature voltage (internal)}} \times .6 \times 10^{10}$$

$$\text{for series coupled armatures (single winding).}$$

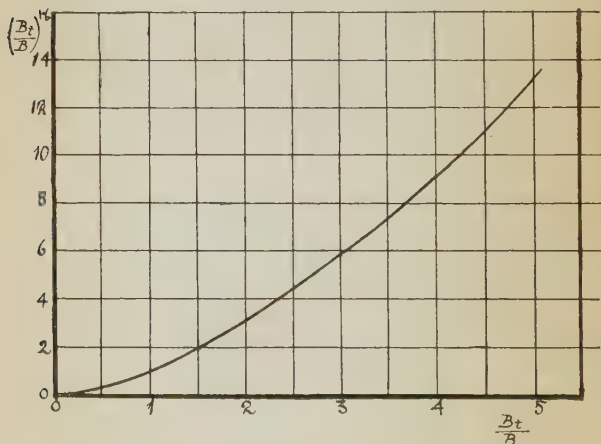


FIG. 82.

The **internal voltage** is the terminal voltage + the voltage lost in the resistance of the armature winding (+ that lost in series field winding, if any) for dynamos, and the terminal voltage - the same amount for motors. The internal voltage

may be assumed to be about 5 % above (in dynamos) or below (in motors) the terminal voltage.

The flux calculated in this way is that which has actually to pass into the armature from each pole and to be cut by all the conductors in turn.

To find the flux that must pass through the magnet-iron, the armature flux must be multiplied by the **leakage coefficient**.

The value of this leakage coefficient can be tentatively taken from the table on p. 266, but unless it has been experimentally determined for a similar machine (p. 362), it should be worked out, as soon as the magnet dimensions and exciting ampere turns have been approximately determined, by the method given on p. 266.

From the flux per pole and the leakage coefficient the sections of the various parts of the magnets can be settled upon by working approximately to the flux densities given in the following table—

TABLE OF AVERAGE WORKING FLUX DENSITIES.

	C.G.S. lines per □ inch.	C.G.S. lines per □ cm.
Magnet-cores or { wrought iron yokes { or cast steel { cast iron	100,000	15,500
Air-gap	40,000	6,200
Cores of smooth armatures, 2-polar	30,000 to 50,000	4,700 to 7,700
Cores of smooth armatures, high speed multipolar	75,000	11,500
Teeth of slotted armatures	40,000	6,200
Core below the teeth of slotted armatures, 2-polar	100,000 to 140,000	15,500 to 21,500
Core below the teeth of slotted armatures, high speed multipolar	75,000	11,500
	65,000	10,000

In special cases (*c.g.* series arc lighters intended to give a drooping characteristic curve) densities as high as 17,000 C.G.S. lines per sq. cm. (110,000 per sq. inch) are usual in the magnets.

Kapp* gives the following figures for the **ampere turns required at high saturations in the teeth** of slotted armatures—

* *Elektromechanische Konstruktionen*, G. Kapp, p. 131.

Actual B. in C.G.S. lines per □ cm.	Actual B. in C.G.S. lines per □ inch.	Ampere turns per 1 cm.	Ampere turns per 1 inch.
19,000	123,000	210	530
20,000	129,000	280	710
21,000	136,000	380	970
22,000	142,000	500	1,270
23,000	149,000	670	1,700
24,000	155,000	900	2,300
25,000	161,000	1,200	3,050
26,000	168,000	1,600	4,050
27,000	174,000	2,100	5,350

It must be remembered that, with high densities of this order, an appreciable proportion of the total armature flux passes down the slots, and not through the teeth at all, so that the **apparent flux density** in the teeth obtained by dividing the total flux by the area of the teeth under the pole-face, is rather higher than the *actual* value. Assuming that the average width of a slot is equal to the average width of a tooth, the ratio apparent flux density in teeth

actual flux density in teeth works out at 1·015, 1·03, 1·06, and 1·1, for values of the actual flux density of 19,000, 22,000, 25,000, and 27,000 C.G.S. lines per sq. inch respectively. If the slots are wider than the teeth the ratio becomes rather larger in each case.

The **length of the polar arc** should be made approximately equal to—

$$\frac{\text{diameter of bore of magnets} \times \pi}{\text{number of poles}} \times .7 \text{ inch,}$$

i. e. 70 % of the pitch of the poles.*

It is now possible to calculate the number of exciting **ampere turns required** on the magnets. For shunt and series wound machines, it is only necessary to calculate the ampere

* According to Fischer-Hinnen, in an article in the *Electrical Engineer* for September 9, 1904, the exact span of the pole-piece relatively to the distance between armature slot centres has an important effect on the humming of electrical machines. This humming is chiefly due to rapid variations of flux, and he states that the humming may be reduced to a minimum by making

$$\frac{\text{pole pitch}}{\text{slot pitch}} = x + \frac{1}{2}, \text{ where } x \text{ is any whole number,}$$

i. e. the pole pitch must be kept as far as possible from covering an exact number of slot pitches.

turns required at full load, but for compound dynamos the calculation must be carried out both for full and no loads. The shunt winding should be arranged to give the no load ampere turns and the series winding the difference between the full load and no load excitation.

The ampere turns required **at full load** are the sum of three separate amounts: (1) for the air-gap; (2) for the iron circuit (the magnet and armature iron can safely be included in one average calculation, except when highly saturated armature teeth are employed; (3) for the armature reaction. However many poles the machine may have, the calculation need only be carried out for a single complete magnetic circuit, consisting of two poles.

$$\left. \begin{array}{l} \text{Excitation for air-} \\ \text{gap in ampere} \\ \text{turns per pair} \\ \text{of poles} \end{array} \right\} = .8 \times \left\{ \begin{array}{l} \text{Length of} \\ \text{double air-} \\ \text{gap in cm.} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Flux density in air-} \\ \text{gap in C.G.S. lines} \\ \text{per sq. cm.} \end{array} \right\}$$

$$\text{or} = .313 \times \left\{ \begin{array}{l} \text{Length of} \\ \text{double air-} \\ \text{gap in ins.} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Flux density in air-} \\ \text{gap in C.G.S. lines} \\ \text{per sq. in.} \end{array} \right\}$$

The **flux density in the air-gap** (allowing for the fringing of the lines at the edges) =
Total flux

$$\left\{ \begin{array}{l} \text{Length of polar} \\ \text{arc + double} \\ \text{air-gap} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Length of pole-piece} \\ \text{parallel to shaft} \\ \text{+ double air-gap} \end{array} \right\}$$

These formulæ apply to smooth cores and to slotted cores with narrow slot openings (slot opening less than $\frac{1}{3}$ of the width of the top of a tooth).

For slotted armatures with wide slots use (instead of "Length of double air gap")—

$$2 \left\{ \delta + \frac{.3S^2}{S+T} \right\}, *$$

where T = width of the top of one tooth,

S = width of one slot opening,

δ = single air-gap from top of tooth to pole face,

all measured in inches.

$$\left. \begin{array}{l} \text{Excitation for} \\ \text{iron in Ampere} \\ \text{turns per pair} \\ \text{of poles} \end{array} \right\} = \left\{ \begin{array}{l} \text{Ampere turns per} \\ \text{cm. corresponding} \\ \text{to the average flux} \\ \text{density in the iron} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Mean length of} \\ \text{iron path in cm.} \\ \text{per pair of poles.} \end{array} \right\}$$

$$\text{or} = \text{Ampere turns per inch} \times \left\{ \begin{array}{l} \text{Mean length of} \\ \text{path in inches.} \end{array} \right\}$$

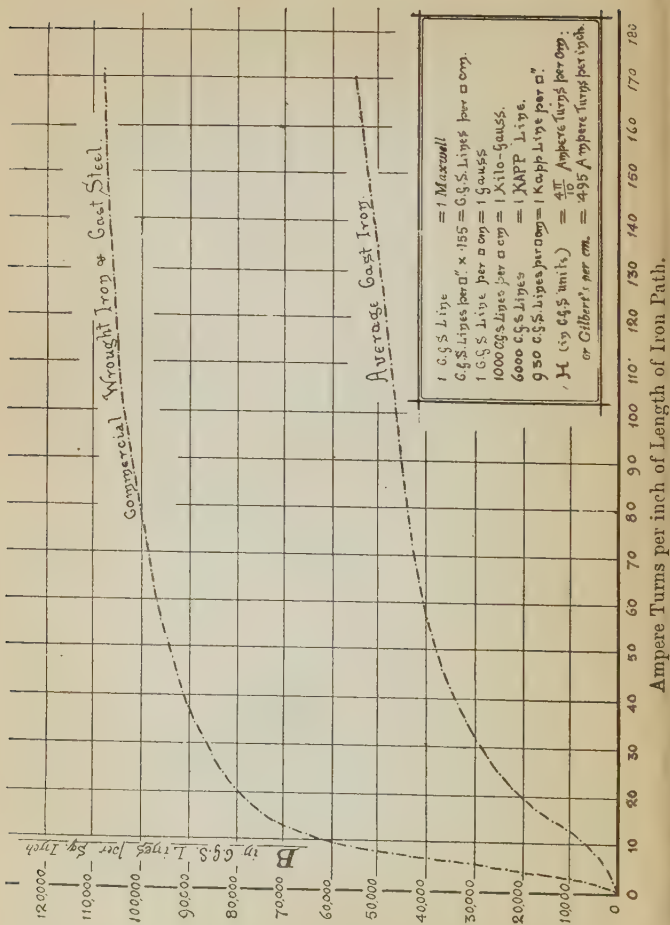
Using the curve on p. 250, Fig. 83, or the table on p. 251.

* This method of allowing for the effect of the slot opening on the reluctance of the air-gap has been found to give reliable results for the ordinary ratio of slot opening to length of air-gap occurring in practice. For a discussion of various other methods see Prof. Hele-Shaw's paper before the I. E. E., Nov. 24, 1904, and *Electrician*, Nov. 25, 1904.

Commercial Wrought Iron & Cast Steel.

Average Cast Iron.

1 C.G.S. Line = 1 Maxwell
 C.G.S. Lines per sq. in. $\times .155$ = C.G.S. Lines per sq. cm.
 1 C.G.S. Line per sq. cm. = 1 Gauss.
 1000 C.G.S. Lines per sq. cm. = 1 Kilo-Gauss.
 6000 C.G.S. Lines = 1 KAPP Line.
 950 C.G.S. Lines per sq. cm. = 1 Kapp Line per sq. in.
 H (in C.G.S. units) = $\frac{4\pi}{10}$ Amperes Turns per cm.
 or Gilbert's per cm. = 495 Amperes Turns per inch.



Ampere Turns per inch of Length of Iron Path.

MAGNETIZATION TABLE FOR IRON.

FLUX DENSITY.		AVERAGE AMPERE TURNS.			
C.G.S. lines per □ cm.	C.G.S. lines per □ inch.	Commercial wrought iron or cast steel.		Commercial cast iron.	
		Per 1 cm.	Per 1 inch.	Per 1 cm.	Per 1 inch.
1,000	6,450	1	2.5	4	10
2,000	12,900	1.25	3.2	6	15
3,000	19,400	1.6	4	7.5	19
4,000	25,800	2	5	10.2	26
5,000	32,200	2.4	6	14	36
5,500	35,400	2.5	6.3	17	43
6,000	38,800	2.6	6.6	21	54
6,500	42,000	2.75	7	26	66
7,000	45,200	3	7.5	35	88
7,500	48,400	3.2	8	49	124
8,000	51,500	3.3	8.3	61	154
8,500	54,800	3.5	8.8	67	170
9,000	58,000	3.6	9.2	—	—
9,500	61,200	4	10	—	—
10,000	64,500	4.3	11	—	—
10,500	67,600	4.7	12	—	—
11,000	70,800	5.4	13.7	—	—
11,500	74,000	5.9	15	—	—
12,000	77,200	7.1	18	—	—
12,500	80,500	8.3	21	—	—
13,000	83,700	9.8	25	—	—
13,500	87,000	12.2	31	—	—
14,000	90,000	14.5	37	—	—
14,500	93,500	19	48	—	—
15,000	96,500	23	59	—	—
15,500	100,000	31	78	—	—
16,000	103,000	38	97	—	—
16,500	106,200	54	138	—	—
17,000	109,500	79	200	—	—

As the length of the magnet-limbs has not yet been determined, the mean length of iron path can only be approximately estimated, but great accuracy in this point is not necessary, as a considerable error in the ampere turns for the iron will not affect the total excitation appreciably.

$$\left. \begin{array}{l} \text{Excitation to compensate for} \\ \text{armature demagnetization} \\ \text{and cross magnetization} \\ \text{per pair of poles} \end{array} \right\} = \left\{ \begin{array}{l} \text{Total armature} \\ \text{ampere turns} \\ \text{per pair of} \\ \text{poles} \end{array} \right\} \times .33.$$

The total armature ampere turns per pair of poles

$$= \frac{\text{Total number of face conductors} \times \text{current in each conductor}}{\text{Number of poles}}$$

The figure .33 is only a practical approximation and represents the average value of the ratio—

$$\frac{\text{Angle of brush lead} \times 2}{\text{Angular distance from} \\ \text{the centre of one pole to} \\ \text{centre of the next}}$$

together with a small allowance for the saturating effect of the cross magnetizing ampere turns on the armature.

For the limit to the value of the armature ampere turns introduced by sparking considerations, see the note on sparking on p. 258.

The sum of the above three excitations gives the **total ampere turns required** on each complete magnetic circuit at full load. If, as is usually the case, each magnetic circuit consists of two limbs with a bobbin on each, half the ampere turns obtained as above are placed on each bobbin.

At **no load** the **excitation** required is simply—

Ampere turns for air-gap + Ampere turns for iron, using the no load flux densities (which in a dynamo are slightly less, and in a motor slightly more, than at full load for the same speed and terminal voltage).

The Field Magnet Winding.

(5) Having decided on the total number of ampere turns to be placed on each bobbin or former, sufficient **winding space** must be allowed to get this number in without overheating. It is most economical, as regards weight of wire for a given temperature rise, if the winding space be made long and shallow, on account of the great cooling surface so obtained. But against this must be set the fact that the weight of magnet iron and the over-all dimensions of the machine are increased.

In practice an allowance, in machines of 50 kilowatts and over, of about 1 inch **length** of winding space for every 1000 ampere turns is found to give good results. For reasonable heating this leads to a **depth** of winding space of from 2 to 3 inches. Small machines are often allowed as much as 2 inches per 1000 A.T.

For **series windings** the cross section of the wire should be selected, preliminarily, to give a current density at full load of 1000 to 1500 amperes per sq. inch.

The total number of turns required = $\frac{\text{Ampere turns.}}{\text{Full load current.}}$

The **mean length** of each turn is—

Periphery of magnet core + $\left\{ \begin{array}{l} \text{about 2 ins.} \\ \text{for insulation, etc.} \end{array} \right\} + (\pi \times \text{depth of winding})$

The **resistance** of the whole winding is (at 125° F. = 52° C.)

$\cdot 000000777 \times \text{mean length in inches} \times \text{Total no. of turns}$

Sectional area of the wire in square inches

(and it increases or decreases about 24 % per degree Fahr. rise).

Hence the ohmic (C^2R) loss in the winding can be calculated.

The **cooling surface** is—

$\left\{ \begin{array}{l} \text{Mean length} \\ \text{length} + (\pi \times \text{depth of winding}) \end{array} \right\} \times \left(\frac{\text{Length of winding}}{\text{space per bobbin}} \right) \times \text{No. of bobbins.}$

NOTE.—In cases where the ends of the winding are not covered by flanges the end surface will also rank as cooling surface.

The C^2R **watts per sq. in.** of cooling surface calculated in this way should be—

For ordinary machines 4 to 6 according to size.

For machines in which the formers are specially well ventilated by the draught from the armature (such as high-speed multipolar machines) as high as 7 to 9

(the smaller machines having the higher figure in each case).

These are calculated for a temperature rise, above the atmosphere, of about 70° F. (39° C.).

For **shunt windings** the **sectional area** of the wire is determined by the formula—

Area of Shunt wire in sq. ins. $\left\{ \begin{array}{l} \cdot 000000777 \times \text{Mean length in inches} \times \text{Ampere turns per former} \\ \text{Volts across one former} \end{array} \right\} =$

The mean length being approximately determined by assuming a depth of winding of 2 or 3 inches.

The volts across one former are the terminal volts of the machine divided by the number of formers in series across the terminals.

The necessary wire (or pair of wires—one below and one above the required area—if great exactitude is necessary, as in compound dynamos, and no single wire of the right area is obtainable) is next selected from a wire list such as that on p. 254, and the number of total turns that will go into the selected winding space is found.

The resistance of the shunt can then be worked out by the same formula as that for series windings given above.

The ohmic (C^2R) loss in the shunt in watts is—

$$\frac{(\text{Terminal voltage})^2}{\text{Resistance of shunt winding}}$$

DOUBLE COTTON-COVERED WIRE.

Gauge Numbers.		Diameter of Bare Wire. Inches.	Sectional Area in sq. ins.	Covered Diameter of Wire.	% additional Weight to be allowed for the Insulation.
Standard Gauge. S.W.G.	Birming- ham Gauge. B.W.G.				
7/0	...	·5	·196	·525	2%
6/0	...	·464	·169	·489	"
5/0	...	·432	·146	·457	"
...	4/0	·454	·1618	·479	"
4/0	...	·4	·125	·425	"
...	3/0	·425	·1418	·45	"
...	2/0	·38	·1134	·405	"
3/0	...	·372	·108	·397	"
2/0	...	·348	·095	·373	"
...	0	·34	·0907	·365	"
0	...	·324	·082	·349	"
1	1	·3	·0707	·325	2½%
...	2	·284	·0633	·309	"
2	...	·276	·0598	·301	"
...	3	·259	·0527	·284	"
3	...	·252	·0498	·277	"
...	4	·238	·0445	·263	2½%
4	...	·232	·0422	·257	"
...	5	·22	·038	·245	"
5	...	·212	·0353	·237	"
...	6	·203	·0323	·228	"
6	...	·192	·0289	·215	"
...	7	·18	·0254	·203	"
7	...	·176	·0243	·199	3%
...	8	·165	·0214	·188	"
8	...	·16	·0201	·183	"
...	9	·148	·0173	·171	"
9	...	·144	·0163	·167	"
...	10	·134	·0141	·157	3½%
10	...	·128	·0129	·150	"
...	11	·12	·0113	·142	4%
11	...	·116	·0106	·137	"
...	12	·109	·00933	·129	"
12	...	·104	·00849	·124	5%
...	13	·095	·00708	·115	"
13	...	·092	·00664	·112	5½%
...	14	·083	·00541	·10	"
14	...	·0 0	·00502	·097	6·0%
15	15	·072	·00407	·089	7·0%
...	16	·065	·00332	·081	7·5%
16	...	·064	·00321	·080	"
...	17	·058	·00264	·074	8%
17	...	·056	·00246	·072	8½%
...	18	·049	·00188	·064	9½%
18	...	·048	·0018	·063	10%
...	19	·042	·00138	·057	12%
19	...	·040	·00125	·055	13%
20	...	·036	·00102	·051	15%
...	20	·035	·000962	·050	"
21	21	·032	·000804	·047	16%
22	22	·028	·000615	·043	18%

DOUBLE COTTON-COVERED WIRE (*continued*).

Gauge Numbers.		Diameter of Bare Wire. Inches.	Sectional Area in sq. ins.	Covered Diameter of Wire.	% additional Weight to be allowed for the Insulation.
Standard Gauge. S.W.G.	Birming- ham Gauge. B.W.G.				
...	23	·025	·00049	·04	20%
23	..	·024	·00045	·039	"
24	24	·022	·00038	·037	23%
25	25	·020	·000314	·035	25%
26	26	·018	·000254	·033	27%
27	...	·0164	·000211	·031	30%
...	27	·016	·000201	·030	"
28	...	·0148	·000172	·028	32%
...	28	·014	·000154	·027	"
29	...	·0136	·000145	·0266	33%
...	29	·013	·000132	·025	34%
30	...	·0124	·000121	·0244	35%
...	30	·012	·000113	·024	36%
31	...	·0116	·000106	·0236	37%
...	31	·0115	·000104	·0235	37%
32	...	·0108	·000095	·022	38%
...	32	·0108	·000091	·0218	"
33	33	·01	·000078	·021	39%

Wires smaller than ·035 in. diameter are often double silk- or single cotton-covered. For either of these two coverings the covered diameter is from ·005 to ·007 in. excess of the bare diameter.

And this number divided by the cooling surface (worked as above for the series winding) should lie within the limits given above.

For **compound**-wound machines the shunt and series windings are each carried out separately as above, and the total C^2R watts (shunt + series) per sq. inch of outer cooling surface should have the same values as before. In compound windings it is most economical to work to the same **current density** in the shunt as in the series wire.

In the table on p. 256 some of the more **useful relations** between dimensions of winding space, heating, area of wires, etc., for field windings, are collected.

The proportions given are not accurately true, owing to the fact that the effect of different winding depths on the mean length of the turns, and on the cooling surface, as also the differences in the thickness of insulation on different sizes of wire, have, for simplicity, been neglected, but the accuracy is quite close enough for practical purposes.

TABLE OF WINDING PROPORTIONS.

EFFECT OF THE ALTERATION ON:—									
Required alteration.	Length of Winding Space.	Depth of Winding Space.	Watts per $\square'$ of outer surface of winding (or temp. rise).	Total Watts lost in Winding.	Area of Wire:—		Total number of turns:—		Total Weight of Wire.
					If a Series Winding.	If a Shunt Winding.	If a Series Winding.	If a Shunt Winding.	
To increase the number of ampere turns in a given winding space as $a:1$.	Unaltered.	Unaltered.	Increased as $a^2:1$.	Increased as $a^2:1$.	Reduced as $1:a$ (for the same external current).	Increased as $a:1$ (for the same terminal voltage).	Increased as $a:1$ (for the same external current).	Reduced as $1:a$ (for the same terminal voltage).	Unaltered.
To increase the number of ampere turns as $a:1$, and yet keep the temperature rise the same.	(1) Increased as $a:1$.	Unaltered.	Unaltered.	Increased as $a:1$.	Unaltered (for the same external current).	Increased as $a:1$ (for the same terminal voltage).	Increased as $a:1$ (for the same external current).	Unaltered (for the same terminal voltage).	Increased as $a:1$.
(1) By altering the length of winding space. (2) By altering its depth.	(2) Unaltered.	Increased as $a^2:1$.	Unaltered.	Unaltered.	Increased as $a:1$ (for the same external current).	Increased as $a:1$ (for the same terminal voltage).	Increased as $a:1$ (for the same external current).	Increased as $a:1$ (for the same terminal voltage).	Increased as $a^2:1$.
To reduce the length of winding space as $1:b$, keeping the ampere turns and the temp. rise the same.	Reduced as $1:b$.	Increased as $b^2:1$.	Unaltered.	Reduced as $1:b$.	Increased as $b:1$ (for the same external current).	Unaltered (for the same terminal voltage).	Unaltered (for the same external current).	Increased as $b:1$ (for the same terminal voltage).	Increased as $b:1$.
To reduce the depth of winding space as $1:c$, keeping the ampere turns and the temp. rise unchanged.	Increased as $\sqrt{c}:1$.	Reduced as $1:c$.	Unaltered.	Increased as $\sqrt{c}:1$.	Reduced as $1:\sqrt{c}$ (for the same external current).	Unaltered (for the same terminal voltage).	Unaltered (for the same external current).	Reduced as $1:\sqrt{c}$ (for the same terminal voltage).	Reduced as $1:\sqrt{c}$.
To change the winding to suit a voltage increased as $a:1$, and an external current reduced as $1:d$, keeping the ampere turns and the temp. rise unchanged.	Unaltered.	Unaltered.	Unaltered.	Unaltered.	Reduced as $1:d$.	Reduced as $1:d$.	Increased as $d:1$.	Increased as $d:1$.	Unaltered.

From these considerations it will be seen that the weight of wire required on a machine can be reduced indefinitely by simply increasing the length of winding space and reducing its depth, the heating remaining the same. At the same time the total number of exciting watts increases and also the weight of magnet iron. In practice the rule given above of allowing 1 inch (or a little more) length of winding space for every 1000 ampere turns is found to give the best all-round results.

Efficiencies.

The **electrical efficiency** of a Motor or Generator

$$= \frac{\text{Output watts}}{\text{Output watts} + \frac{C^2 R \text{ watts in armature winding}}{C^2 R \text{ watts in field winding}}}$$

The **commercial efficiency** of a motor or generator

$$= \frac{\text{Output watts}}{\text{Output watts} + \frac{C^2 R \text{ watts in armature winding and brushes}}{C^2 R \text{ watts in field winding}} + \text{"Driving watts"}}$$

where "Driving Watts" include Hysteresis, Eddy Currents, Brush Friction, Windage and bearing Friction—in fact all the losses except the measurable $C^2 R$ watts lost in the windings. The calculation of the combined Hysteresis and Eddy Current losses is explained on p. 244 and the brush friction calculation on p. 264, whilst experiment has shown that a very fair approximation to the combined windage and bearing friction loss in normal well-ventilated multipolar machines is given by the expression—

$$\frac{\text{Watts lost in ventilation and bearing friction}}{\left(\frac{\text{Diameter of armature in inches}}{\text{Length of armature core in inches}} \right)^2 \times \frac{\text{Revolutions per minute}}{1000} \times 0.0003}$$

The **combined or through efficiency** of a direct-coupled engine and dynamo set

$$= \frac{\text{Commercial Efficiency of Dynamo}}{\text{Mechanical Efficiency of Engine.}}$$

The following table gives the approximate average Losses and Efficiencies of Dynamos supplied to English specifications:—

Kilowatts Output.	Exciting watts, as a percentage of full load Output (belt-driven Dynamos).	C^2R watts lost in Armature winding, as a percentage of full load Output (belt-driven Dynamos).	Full load Electrical Efficiency at normal belt speed.	Full load Commercial Efficiency at normal belt speed.	Full load Combined Efficiency of Dynamo and High-Speed Engine.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
1	9	5	86	78	60
10	$4\frac{1}{2}$	$3\frac{1}{2}$	92	85.5	71
100	$1\frac{3}{4}$	$2\frac{1}{4}$	96	93	84
500 and over	1	$1\frac{1}{2}$	$97\frac{1}{2}$	$94\frac{1}{2}$	85

Sparking at the Commutator.

In order that commutation may be sparkless, it is necessary that, by the time the brush ceases to short circuit an armature coil, the current in that coil should have been reduced from its full value, in one direction, down to zero, and then increased again to its full value in the opposite direction. This reversal is accomplished by (*A*) the voltage induced in the coil by its rotation in the main field of the machine (the amount of this voltage depending on the ratio of the armature ampere turns to the field ampere turns and on the amount of lead given to the brushes), and (*B*) the difference in the voltage drop between the brush and the segment just about to leave the brush and that between the brush and the segment under the brush. If reversal did not take place a difference in current density at the two segments in question would occur, and this would result in a definite back voltage tending to produce reversal and so equalize the densities.

This latter voltage (*B*) is only present to any appreciable extent with carbon brushes, and accounts largely for their superiority over metal brushes in preventing sparking.

The forces opposing the reversal of the current in the section and tending to maintain it in its original direction are (*C*) the voltage of self-induction of the short-circuited section, every change in the current round the section causing a corresponding change in the flux looped with the section and so setting up a back voltage opposing the change in current, and (*D*) the voltage induced in the short-circuited section owing to the cutting of the flux (fixed in space) set up by the armature ampere turns.

Practical commutation occurs through the combined effect of

three out of these four voltages (*A*), (*B*), (*C*) and (*D*)—(*A*) and (*D*), of course, never occurring at the same time. Although it is practically impossible to predetermine the absolute value of the resistance voltage (*B*), the reactance volts (*C*), or the voltage induced by the armature field (*D*), yet by making certain assumptions a figure which is approximately proportional to the reactance voltage can be obtained by calculation from the magnetic data and the dimensions of machines, and by experiment the maximum numerical values compatible with good commutation can be arrived at. This has been done by Hobart,* and his rule is as follows—

Reactance †

$$\text{volts per section} = R \times W^2 \times C \times (l + \cdot 1L) \times d \times \frac{s}{b} \times K \times \frac{\cdot 84}{10^8}.$$

where *R* = number of revolutions per minute.

W = number of conductors per section

$$= \frac{\text{total conductors on armature}}{\text{total commutator parts}}.$$

C = total current of machine.

l = length of conductor embedded in slots per turn of armature winding, in inches.

L = length of end connections (*i. e.* part not embedded in slots) per turn of armature winding, in inches.

d = commutator diameter in inches.

s = maximum number of sections short circuited at one time.

$$= \frac{\text{the next greater whole number to the value of width of brush contact arc}}{\text{width of one commutator segment} + \text{one mica plate}}.$$

b = width of brush contact arc in inches.

$$K = \frac{1}{\text{No. of poles}} \quad \text{for parallel coupled (single) armature windings.}$$

$$= \frac{\text{No. of poles}}{4} \quad \text{for series coupled (single) armature windings with 2 sets of brushes.}$$

$$= \frac{\text{No. of poles}}{6} \quad \text{approximately, for series coupled (single) armature windings with as many sets of brushes as poles.}$$

This formula is applicable to slotted drum-wound armatures with normally dimensioned open slots. For smooth-cored

* *Jour. Inst. Elec. Engineers*, vol. xxxi. p. 170.

† This is the maximum instantaneous value of the reactance voltage on the assumption that the current changes sinusoidally. The *average* reactance voltage is the above value divided by 1.57, and this *average* value is independent of any assumption as to current curve shape.

armatures the term in the bracket may be approximately replaced by $\cdot 2l + \cdot 1L$.

For thoroughly satisfactory work with carbon brushes fixed in one position for all loads, the full load reactance voltage calculated as above should not exceed 3 to 4 volts, though good machines with double these values have occasionally been constructed.

Where the machine is intended to run in both directions with brushes fixed at the neutral point, the reactance must, of course, be kept lower than for machines running with brush lead, and probably 3 volts should not be exceeded in this case. When metal brushes are used the value should not exceed about 2 volts, and even then some shifting of the brushes as the load increases will be necessary.

For satisfactory work the stated values for reactance voltage should not usually be exceeded, though when the armature ampere wires per inch of periphery (on which the voltages (D) and (A) depend) are low, or when the carbon brushes used are hard ones giving a high contact resistance (on which the voltage (B) depends) higher values of the reactance voltage are often found permissible. In cases of special difficulty as regards commutation, such as arise in connection with high-speed steam turbo-generators and in motors intended to run with wide variations of speed by means of field regulation, *commutating poles* or *compensating windings* are often employed. The former consist of small intermediate poles fixed to the yoke midway between the main poles of the machine and excited by the main armature current, whilst with the latter the small poles are replaced by a distributed winding carried in slots in the faces of the main poles and having its magnetic axes midway between the main poles. This winding also is connected in series with the armature winding. In a few cases both the commutating poles and the compensating winding have been fitted. It is clear that if this auxiliary polarity is made of the same sign as the forward pole in a dynamo or as the backward pole in a motor, and, further, is produced by a number of ampere turns slightly in excess* of the armature ampere turns the nett

* The flux density under the commutating pole must be such that an average reversing voltage equal to, or rather in excess of, the average reactance voltage is induced in the short-circuited armature coil—

$$\text{Flux density per } \square'' \text{ of commutator pole face} = \frac{\text{average reactance voltage per section} \times 10^8 \times 5}{\text{peripheral speed of armature conductors in ft. per min.} \times \text{length of com. pole parallel to shaft (inches)} \times \text{conductors per section}}$$

[continued opposite.]

polarity in the neutral zone will be in the right direction to produce commutation by increasing the value of the voltage (4) (p. 258). Further, since the compensating poles are in series with the armature winding, the commutating flux will increase proportionately with increasing load (provided saturation of the iron is avoided), so that satisfactory commutation at all loads with the brushes fixed in the neutral zone can be relied upon. The results obtained recently with these devices have been so satisfactory that they have been adopted as standard practice by a number of makers, at least for the larger sizes. Not only is the commutation of ordinary machines improved, but the limits given above for reactance voltage and for ampere wires per inch of armature periphery can safely be exceeded, thus reducing the weight and, to a certain extent, the cost of the machines. The commutating poles are the cheaper form of the device, as the mean length of the turns is much shorter than with the compensating winding. The latter, however, has the considerable additional advantage of preventing distortion of the main field by the armature reaction, and so preventing an abnormal rise in the iron losses. Distributed compensating windings, with or without special commutation poles, are, therefore, generally adopted for turbo-dynamos.*

Leaving out of account the mechanical causes of sparking, the quantities dealt with above are the most important in deciding the sparking qualities of a machine, but the following minor points often also have considerable effect—

Collecting Brushes.—Except for a few special cases for which metal brushes are obviously most suitable, *e. g.* heavy current plating dynamos and low voltage boosters, carbon brushes are almost invariably employed, in spite of the increased size of

In order to produce this flux density the number of ampere turns on 2 commutating poles must =

$$\begin{aligned} & \text{armature A.T. per pair of poles} + \text{A.T. to produce the required F.D. across the double air-gap of the com. pole} \\ & = \frac{\text{total armature conductors}}{\text{No. of poles}} \times \frac{\text{amperes per conductor}}{\text{conductor}} + \left(\frac{\text{flux density required under com. pole}}{\text{under com. pole}} \times 2 \times \frac{\text{air-gap under com. pole}}{\text{com. pole inches}} \times .313 \right). \end{aligned}$$

This is usually equal to about $1\frac{1}{2} \times$ armature A.T. per pair of poles, and its exact value is generally determined by test, the ampere turns being adjusted either by means of a shunt to the winding or by removing turns, until the best commutation is obtained. The peripheral span of the commutating pole face should be equal to about twice the slot pitch of the armature.

* For a full discussion of the use of commutating poles, see I. E. E. Paper by Messrs. Page & Hiss, 1907.

commutator required with them, chiefly because their high contact resistance greatly reduces the tendency to spark. An exception to this general use of carbon brushes occurs in steam turbo-dynamos, because experience has shown that it is practically impossible to construct commutators which will run perfectly true at these high speeds for any length of time. The result is that brushes of a rigid substance like carbon are set in violent vibration and even thrown momentarily off the surface, whilst the greater flexibility of metal wire brushes prevents any such action in their case. This fact renders the use of special commutating devices, such as those described above, indispensable on these turbine generators.

Copper gauze brushes can work at current densities at the contact of from 150 to 250 amperes per square inch, but should not be allowed to cover more than $1\frac{1}{4}$ or $1\frac{1}{2}$ segments of one winding at one time. Carbon brushes vary greatly in hardness and specific resistance and also, though not to so great an extent, in contact resistance. Hard, high resistance, carbons, such as the Le Carbone "S" quality, can work at contact current densities of 30 to 45 amperes, whilst the softest quality, low resistance, carbons (Le Carbone "Z" or "X" quality) will carry 60 or 70 amperes per square inch. Carbon brushes can safely cover two or three segments of one circuit at one time.

Want of symmetry in the armature winding and graduation of the commutating field.—Unless the armature is quite symmetrical, *i. e.* unless all the sections have the same ohmic resistance, and are placed at exactly the same angle relatively to their commutator segments there is likely to be sparking, especially if metal brushes are employed. Slotted armatures are always unsymmetrical when they have more commutator segments than slots, for, whilst the commutator segments progress uniformly round the periphery, the conductors connected to them progress in jumps. This is a point that often gives some trouble with metal brushes, but is of little importance with carbon brushes, it being quite common to employ from two to four commutator plates per slot.

It is also necessary (with certain numbers of slots), in order to obtain a number of conductors suitable for coupling up as a series winding, to place fewer conductors in certain slots than in the rest, but such want of symmetry, again, gives little or no trouble with carbon brushes. The liability to trouble from want of symmetry, or from small inaccuracies in setting the brushes, is especially slight if abrupt changes in field strength at the polar edge are avoided. If the N. and S. pole tips are brought very close together the changes of flux density in this small gap will necessarily be very rapid. In the case of smooth-cored armatures the rule given on p. 248 for the polar arc will allow

a sufficient gap space from pole to pole, but for slotted armatures, however great the distance between the poles may be, the change of density from just outside the pole tip to just under it may be very rapid.

In small slotted machines it is often desirable to bore the magnets eccentrically so as to give a greater length of air-gap at the tips than at the centres of the poles. Sometimes also the edge of each pole, instead of lying parallel to the shaft, is set at an angle to it, or is cut in the shape of a **V**, so that each conductor enters the field very gradually. For the same reason the edges of the pole-pieces are sometimes chamfered off so as to increase the reluctance of the iron and reduce the flux at those points. In the case of laminated poles, this can be conveniently done by providing only half (say) of the plates with the extensions which form the horns of the pole-pieces.

Inequality of pole strength.—In the case of parallel coupled (or lap wound) multipolar armatures the various parallel circuits are under the influence of different poles at a given instant, so that any inequality of the fluxes from the different poles (produced by unequal air-gaps or by bad magnet castings) will lead to unequal current distribution, or even to idle internal currents, and consequent sparking at some of the brushes. A method of wholly avoiding this source of trouble is in common use on the larger multipolar machines, and consists in fitting **balancing** or **equalizing** rings on the armature. These usually take the form of three or four copper rings (each of a cross section about equal to that of an armature conductor) fixed in any convenient position at the back or front of the armature, and each connected to the armature winding at as many equidistant points as the machine has pairs of poles. The connections to the various rings are more or less evenly distributed around the armature. Under normal conditions the rings connect equipotential points and carry no current, but in the case of unequal pole strengths the internal balancing currents follow the low resistance paths through the rings in preference to the higher resistance paths through the brushes, and so cause no sparking.

Commutator Heating.

The heating of the commutator depends (1) on the friction between brushes and commutator, (2) on the C^2R ohmic loss at the brush contact due to the passage of the current, and (3) on the cooling surface of the commutator.

The **coefficient of friction** (copper or carbon brushes and copper commutator) may, as a maximum, be taken as .3.

The usual **pressure** of the brush on the commutator is 1 to

1½ lbs. per sq. inch for gauze brushes and 3 to 3½ lbs. per sq. inch for carbon brushes.

The watts lost in brush friction are—

$$\begin{array}{ccccccc} \text{area of con-} & & \text{peripheral} & & \text{brush} & & \\ \text{tact of one} & & \text{speed of} & & \text{pressure} & & \\ \text{set of brushes} \times & \text{No. of} & \text{commutator} & \times & \text{in lbs. per} & \times & \\ \text{in sq. inches} & \text{sets of} & \text{surface in} & \times & \text{sq. inch} & \times & \\ & \text{brushes} & \text{feet per} & & & & \\ & & \text{minute} & & & & \\ = & \dots & \dots & \dots & \dots & \times & \cdot 0068 \end{array}$$

The brush contact resistance is about .003 ohms. per sq. inch for copper gauze brushes, but carbon brushes have the peculiar property of reducing their contact resistance as the current density increases, so that an almost constant voltage drop occurs across each brush contact.* Within the ordinary range of densities and with almost all the usual grades of carbon this drop per contact may be safely taken as 1 volt.

The ohmic loss with gauze brushes =

$$\frac{\cdot 003}{\begin{array}{c} \text{area of contact} \\ \text{of one set of} \\ \text{brushes in} \\ \text{sq. inches} \end{array}} \times \begin{array}{c} \text{No. of sets} \\ \text{of brushes} \end{array} \times \left(\begin{array}{c} \text{current per set} \\ \text{of brushes} \end{array} \right)^2,$$

whilst with carbon brushes the ohmic loss may be taken as simply

$$= \text{total current} \times 2.$$

The sum of the friction and ohmic losses calculated as above and divided by the peripheral surface of the commutator,

$$i. e. \pi \times \begin{array}{c} \text{diameter of commu-} \\ \text{tator in inches} \end{array} \times \begin{array}{c} \text{length of contact} \\ \text{surface in inches} \end{array}$$

is the number of watts per square inch, and this figure should lie between 2 and 3 for slow-speed unventilated machines, and between 4 and 5 for well-ventilated ones, for a 60° to 70° F. temperature rise.

Eddy Currents.

Besides the eddy currents generated in the iron of the armature cores (see p. 244) there are others which often cause great loss. Thus, eddy currents occur **in the brass or gun-metal spiders** used on gramme armatures, due to the internal magnetic field, produced by the armature ampere turns at full load.

If, in smooth-cored armatures, the *peripheral width of each*

* Arnold, *Die Gleichstrommaschine*, vol. i., 1902, pp. 476-81; Hobart, *Electric Motors*, pp. 102 to 104.

copper conductor exceeds about $\frac{1}{4}$ inch to $\frac{3}{8}$ inch, eddy currents will be produced in them whilst passing through the fringe of magnetic lines at the pole-tips.

To remedy this the conductors should be either made of stranded cable or of a number of separately insulated small wires.

Stranded cable tightly pressed to the required section and having a solid copper area equal to about 80% of the gross area is in common use for such cases.

Eddy currents in the pole-faces.—These occur more or less with all slotted armatures, the amount increasing rapidly with the ratio of slot opening to clearance between armature and magnets due to the difference in flux density at points on the pole-face opposite teeth and points opposite slots. On this account this ratio of slot opening to clearance should not be allowed to exceed 2.*

These eddy currents are always stronger at the trailing horn (in dynamos) than at the leading one, owing to the increased flux density produced by the armature cross ampere turns. This increased density also decidedly increases the total hysteresis and eddy current loss in the armature core, and partly on this account a slot is often cut in the magnets along the lines AB , A_1B_1 , Fig. 72, so as to reduce the distorting effect of the armature ampere turns.

Another way of reducing these eddy currents is to **laminates the poles**. It is becoming a usual practice to make the magnet-cores, of slotted multipolar machines, of sheets of steel or wrought iron, $\frac{1}{16}$ " or less thick, insulated from one another right up to the yoke, into which they are either cast or bolted.†

Magnetic Leakage.

In all magnets there is, besides the useful flux, a certain amount of leakage flux, which passes from points at a higher (magnetic) potential to those at a lower potential, through the air, or other substance, surrounding the main magnetic circuit.

This leakage flux necessitates considerably increased iron sections everywhere, so that, in order to be able to reduce the weight of the machine to a minimum, a fairly accurate determination of the leakage coefficient must be made.

$$\text{The leakage coefficient} = \frac{\text{total useful flux} + \text{total leakage flux}}{\text{total useful flux}}$$

* See R. Goldschmidt, *Electrical Review*, Jan. 3, 1903, for a method of reducing the slot opening by inserting strips of thin sheet iron.

† For a simple formula for calculating the approximate value of the eddy-current loss in solid pole-pieces, see a paper by R. Rüdenburg: *Elektrotechnische Zeitschrift*, Feb. 2, 1905, and *Electrical Review*, June 16, 1905.

For finished machines it can be determined by the test on p. 362.

The leakage coefficient becomes smaller as the size of the machine (keeping to the same type) is increased. Some average figures are given in the following table, but they should only be used for rough preliminary determinations, and, as soon as the approximate dimensions of the magnets and the number of ampere turns have been obtained, the leakage coefficient should be checked by the method given below.

Table of Approximate Leakage Coefficients.

Ordinary 2-pole horseshoe magnets of the type shown in Fig. 69, p. 235, with smooth-cored armature	1.35 to 1.5
Same with slotted armature	1.15 to 1.25
Manchester type magnets, with smooth-cored armature	1.5 to 1.7
Same with slotted armatures	1.25 to 1.35
Multipolar type as in Fig. 72, p. 235, with smooth-cored armature	1.3 to 1.45
Same with slotted armature	1.15 to 1.25

The extra flux due to leakage that has to be carried by the iron is a maximum at the points on the magnetic circuit where the exciting coils are situated, and gets less and less, until the iron just adjacent to the armature carries, practically, only the useful flux. It is usual, however, to determine the leakage coefficient for the worst point only, and to make the section of the whole of the circuit large enough to carry the flux passing at that point.

In the case of such a magnetic circuit as occurs in the dynamo, leakage takes place from all points, and leakage paths exist in all directions. Considered as a complicated network of circuits the flux in various directions could be determined by Kirchhoff's laws,* or some similar rule. But it is quite sufficiently accurate, and at the same time reduces the calculations to reasonable proportions, if the main leakage paths be selected, by inspection of a drawing of the machine, and the flux in each be determined, independently of the others, from the known areas and lengths of these paths and the number of ampere turns acting along each.

Applying the expression for the air-gap ampere turns (p. 249), slightly modified, the four following approximate rules are obtained, in which X represents the number of ampere turns looped

* See p. 106.

with the **leakage** line passing from the centre point of one area to the centre point of the other.

- (1) Leakage in C.G.S. lines between two approximately parallel surfaces (A and A_1 square inches respectively), separated by a distance of L inches

$$= X \times \frac{(A + A_1)}{L} \times 1.6.$$

- (2) Leakage between two surfaces (A and A_1 square inches) approximately at right angles to one another, and whose centre points are separated by a straight line distance of L inches

$$= X \times \frac{(A + A_1)}{L} \times 2.$$

- (3) Leakage between two surfaces (A and A_1 square inches) approximately in the same plane, and whose centre points are separated by a distance of L inches

$$= X \times \frac{(A + A_1)}{L} \times 2.$$

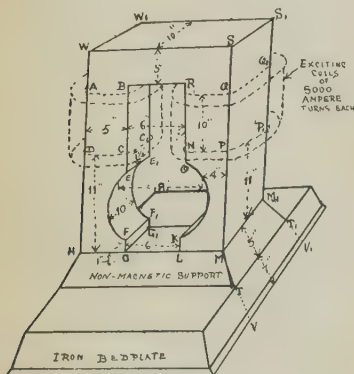


FIG. 84.

- (4) Leakage between two parallel cylinders of diameter D inches $\times$ length M inches, separated by a distance of L inches (distance between axes = $L + D$)

$$= X \times \frac{D \times M}{(L + .3D)} \times 5.$$

Note.—The total leakage from one cylinder placed between two others is twice this amount.

As an **example** of the application of these rules, the complete calculation of the leakage coefficient of the 2-pole dynamo shown in Fig. 84 is given below.

Leakage.		Calculated by Rule No.	Gives in C.G.S. Lines.	
From Area.	To Area.			
BCC_1B_1	RNN_1R_1	1	$5,000 \times \frac{10 \times 10 \times 2}{6} \times 1.6$	$= 266,000$
CEE_1C_1	NOO_1N_1	1	$10,000 \times \frac{10 \times 1\frac{1}{2} \times 2}{6} \times 1.6$	$= 80,000$
FGG_1F_1	KLL_1K_1	1	$10,000 \times \frac{10 \times 1 \times 2}{6} \times 1.6$	$= 53,000$
$ABCD$	$RNPQ$	3	$5,000 \times \frac{10 \times 5 \times 2}{11} \times 2$	$= 182,000$
$A_1B_1C_1D_1$	$R_1N_1P_1Q_1$	3	" " " "	$= 182,000$
$DCEFGH$	$NOKLMP$	3	$10,000 \times \frac{11 \times 4 \times 2}{12} \times 2$	$= 147,000$
$D_1C_1E_1F_1G_1H_1$	$N_1O_1K_1L_1M_1P_1$	3	" " " "	$= 147,000$
QPP_1Q_1	SQQ_1S_1	3	$2,500 \times (10 \times 10) + (5 \times 10)$	$\times 2 = 100,000$
ADD_1A_1	WAA_1W_1	3	" " " "	$= 100,000$
$PM M_1P_1$	TVV_1T_1 and returning thro' the bedplate to area ADD_1A_1	3	$10,000 \times \frac{(11 \times 10) + (7 \times 10)}{28} \times 2$	$= 128,000$
Total Leakage Flux =				1,385,000
and Net Useful Flux through Armature				3,600,000
∴ Total Flux in Magnet Iron				4,985,000
and Leakage Coefficient = $\frac{4,985,000}{3,600,000} =$				1.39

Electrical and Mechanical Stresses.

(1) **Magnetic pull** on an armature placed eccentrically in the field. The following rule is due to Fischer-Hinnen.*

If Z = maximum nett pull in lbs.

x = displacement of armature from centre in inches.

B = density of magnetic lines in air-gap before displacement, in C.G.S. lines per sq. inch.

S = section of polar face in sq. inches.

δ = length of single air-gap before displacement in inches.

* *Continuous Current Dynamos*, p. 262.

α = a multiplier such that ampere turns required for air-gap $\times \alpha$ = total ampere turns on field (α usually lies between 1.1 and 1.5 about).

A = a variable having the following values: In a 4-pole machine, when the displacement occurs in the direction of the axes of two of the poles, $A = 2$.

Ditto when the displacement is in the direction of one of the neutral axes, $A = 2.8$.

In a 6 polar with displacement in direction of pole axes, $A = 6$.

8 " " " " neutral, $A = 9$.

12 " " " " " $A = 15$.

16 " " " " " $A = 20$.

$$\text{Then } Z = \frac{B^2 \times S}{72 \times 10^6} \times \frac{x}{\delta \times \alpha} \times A.$$

(2) **The total peripheral drag on the conductors**, on a smooth-cored armature (tending to break off the driving teeth) in lbs.—

$$= \frac{\text{total H.P. generated in armature winding} \times 33,000}{\text{diameter of armature in feet} \times \pi \times \frac{\text{revolutions}}{\text{per minute}}}$$

or

$$= \frac{\text{total watts generated in armature} \times 169}{\text{diameter of armature in inches (to the centre of the winding depth)} \times \frac{\text{revolutions}}{\text{per minute}}}$$

This total peripheral drag divided by the total number of driving teeth on the armature gives the average shearing force to be withstood by each tooth, neglecting the effect of the binding-bands and of the friction between the winding and the core, both of which reduce the stress on the teeth.

The maximum drag on a single conductor is, in lb.—

$$= \frac{\text{maximum flux density in the air-gap in thousands of C.G.S. lines per sq. inch} \times \text{amperes in each conductor} \times \text{length of a conductor (not including end connectors) in inches}}{11,300}$$

(3) **The centrifugal force on the armature winding** (tending to burst the binding-bands).

The centrifugal force on each conductor tends to make it fly radially outwards. The sum of all these separate little forces can be taken about a diameter, and can then be replaced by two equal and opposite forces tending to tear the bands asunder at two points situated one at each end of this diameter.*

* The method is similar to that employed in calculating the transverse stresses in steam boilers.

The centrifugal force due to the whole winding in lb.—

$$= \frac{\text{weight of winding in lb.}}{\text{mean radius of winding in feet}} \times \left(\frac{\text{revolutions}}{\text{per minute}} \right)^2 \times .00034,$$

and to resolve this into two equal and opposite pulls it must be multiplied by half the average value of the sine of all angles from 0° to 90° , viz. by $\frac{2}{\pi} \times \frac{1}{2}$. The result will be the pull tend-

ing to break two complete sections of the binding-bands, and the force tending to break the whole number of binding-bands on the armature across *one* section in lb. will therefore be—

$$= \frac{\text{weight of winding in lb.}}{\text{mean radius of winding in feet}} \times \left(\frac{\text{revolutions}}{\text{per minute}} \right)^2 \times .000054,$$

and this divided by the total number of wires forming the bands gives the pull on each wire.

The breaking stress for steel piano-wire such as is usually employed for the purpose, is about 45 tons per sq. inch, and the working stress as above calculated should not be allowed to exceed $\frac{1}{10}$ of this figure.

NOTE.—In built-up drum armatures there is no need to include the weight of the end connectors in the calculation.

(4) The **energy stored in the rotating part** of a machine (fly-wheel effect).

For armatures of very large diameters compared with their radial depths the following approximate rule can be used—

$$\text{total stored energy in foot-lb.} = \frac{\text{weight of armature complete, in lb.} \times \left(\frac{\text{mean diameter}}{\text{in feet}} \right)^2 \times \left(\frac{\text{revolutions}}{\text{per minute}} \right)^2}{23,000}$$

the “mean diameter” being taken as the diameter at which (by inspection of a drawing of the armature) the weight can be considered as acting.

For armatures with large radial depth of material, or with heavy internal supporting frames, or for magnets fixed on the periphery of a fly-wheel with thick rim and arms—

$$\text{total stored energy in foot-lb.} = \frac{\text{average weight per cubic foot of material} \times \text{length of material parallel to shaft} \times (d^4 - d_1^4) \times \left(\frac{\text{revolutions}}{\text{per minute}} \right)^2}{60,000}$$

where d is the external diameter in feet, and d_1 is the effective internal diameter in feet, determined from a drawing. The weight of 1 cubic foot of cast iron = 450 lb., wrought iron = 485 lb., cast steel = 500 lb., copper = 550 lb. The stored

energy of the armature worked in this way, together with that of any additional fly-wheels that may be in use, should be about 1500 to 2000 foot-lb. per H.P. developed, in order to secure **sufficiently steady running** with large high-speed direct-coupled compound or triple expansion engines.

(5) The necessary **diameter of the spindle** at its thickest part (*i. e.* inside the armature core) may be considered as a function of the ratio of the maximum h.p. or kw. developed to the speed at which they are developed, and Fischer-Hinnen* gives the rule—

$$\begin{aligned} \text{maximum diameter} \\ \text{of spindle} \\ \text{(mild steel)} \\ \text{in inches} &= 7 \times \sqrt[3]{\frac{\text{h.p. developed}}{\text{revolutions per minute}}} \\ &= 7.7 \times \sqrt[3]{\frac{\text{kw. developed}}{\text{revolutions per minute}}} \end{aligned}$$

The diameter of the spindle in the bearings is usually from 20 % to 25 % less than the above diameter.

These values are very safe ones for most ordinary cases, especially for the larger sizes of machines.

Sizes of Bearings.

(6) The **length of each bearing** is usually made from $2\frac{1}{2}$ (in large machines) to $3\frac{1}{2}$ (in small machines) times the diameter of the spindle in the bearing. The exact value must, however, be chosen so as to avoid excessive pressure per sq. in. of bearing surface due to the combined force of the weight of the armature and the belt pull.

Taking the projection of the spindle in the bearing on a horizontal plane as being the bearing surface, this surface = diameter of spindle in bearing $\times$ length of bearing.

In the case of a two-bearing machine with overhung pulley half the weight of the complete armature may be considered as falling on each bearing whilst the whole of the belt drag may be considered as falling on the pulley end bearing—the two forces on this bearing acting approximately at right angles to one another.

$$\therefore \text{Force on commutator end bearing} = \frac{\text{weight of armature in lb.}}{2}$$

* *Continuous Current Dynamos*, p. 257.

and force on pulley end bearing =

$$\sqrt{\frac{\left(\frac{\text{weight of armature in lb.}}{2}\right)^2 + \left(\frac{\text{total belt drag in lb.}}{2}\right)^2}$$

and total belt drag in lb.

approximately = 3 × difference in tension between the two sides of the belt

$$= 3 \times \frac{\text{h.p. at pulley} \times 33,000}{\frac{\text{diameter of pulley in inches}}{\pi} \times \frac{\text{No. of revs. per minute}}{12}}$$

$$= \frac{\text{h.p. at pulley}}{\frac{\text{diameter of pulley in inches}}{\pi} \times \frac{\text{No. of revs. p.m.}}{12}} \times 380,000.$$

For dynamos or motors with oil ring lubrication and brass or white metal bearing bushes the value of

total force on bearing in lb.

$$\frac{\text{diameter of spindle in bearing in inches} \times \text{length of bearing in inches}}{\pi}$$

should not exceed 100 to 150 per min. in order to allow amply for all contingencies.

(7) The **diameters of the pulleys** of belt-driven dynamos and motors are usually chosen so as to give a peripheral speed of 2000 to 3000 feet per minute and the width of single belting (about $\frac{1}{4}$ " thick) to transmit a given power is given by the approximate rule—

$$\frac{\text{width of single belt in inches}}{\pi} = \frac{\text{h.p. to be transmitted} \times 750}{\text{speed of belt in feet per minute}}$$

Double belts will transmit about 50 % more H.P. for the same width.

The minimum diameter of a belt pulley should not be less than 16 to 18 times the thickness of the belt.

For particulars of rope and toothed gearing, see Unwin, *Elements of Machine Design*, Part I, p. 287 and p. 404.

DYNAMOS AND MOTORS DESIGNED FOR SPECIAL PURPOSES.

Traction Generators.—All machinery used in connection with traction work has to be designed to withstand the large and sudden variations of load which are constantly occurring. On this account all the mechanical details of traction generators are more massive than for lighting machines of the same output. The armatures are almost invariably of the slotted type, so as to remove all strain from the conductors. Large diameter multipolar armatures are preferred, since these, to a certain extent, assist the engine fly-wheel in steadying the speed. In order to reduce the load variations in the generators and engines as far as possible a battery, with or without an automatic reversible booster, is usually used in parallel. Carbon brushes are generally used, as they give sparkless commutation at all loads without any adjustment being necessary. In order to allow for the drop of volts along the feeders, and so to maintain a fairly constant voltage at the terminals of the car motors, at all loads, the generators are overcompounded 10 % or 15 %, *i. e.* the series winding is arranged to increase the terminal voltage at full load 10 % or 15 % above its no load value.

Traction motors.—Owing to the intermittent nature of the work the **rating** of traction motors is somewhat indefinite. Ordinary practice has, however, shown that all reasonable requirements are satisfied if a motor of such a size be selected that, when run for one hour continuously at the maximum horse-power demanded (calculated from the maximum speeds, grades, etc., as on p. 412), the temperature of no part of the machine shall rise more than 100° F. to 150° F. above the atmosphere. This limit, together with the allowable watts per sq. inch of cooling surface mentioned on p. 243, will serve as a rough guide to the correct dimensions for railway and tram motors.

Owing to the confined space usually available for the motors under the framework of the car, all standard traction motors are distinguished from ordinary motors by their very **compact** design. They are generally enclosed, or semi-enclosed, machines, having iron-clad magnets of some such shape as shown in Figs. 70, 71 and 72.

The **armatures** are always slotted, and the collecting brushes are always of carbon. The magnets are usually series wound, as a large torque is required at starting and with overloads, whilst the speed is separately controlled by means of series resistances (reducing the terminal volts).

This method of control is, of course, very uneconomical, and many more or less successful attempts have been made to reduce

the regulation losses by employing compound wound or shunt wound motors, which permit of easy regulation of the field strength independently of the armature current.

Thus the **Johnson-Lundell** system * employs compound wound motors and the **Raworth regenerative control** system † shunt motors. The latter not only permits of economical speed regulation within wide limits but enables the motors to act as generators when the car is descending gradients, thus forming an electric brake and returning the energy, which would otherwise have to be wasted in mechanical braking, into the line.

For heavy railway traction, with speeds ranging from 20 to 30 miles per hour, the motors are sometimes built up on the car axles. For tramcar work, however, with speeds of 8 to 12 miles per hour, this process would lead to excessively large motors, and it is found more convenient to put in smaller machines running at moderate speeds, and **geared** to the car wheels by single or double reduction spur gearing. Single reduction spur gearing gives a ratio of about 1 to 5 conveniently, and double this ratio, or even more, can be got with double reduction, though the latter is not often used.

The outputs of car motors are sometimes specified by the horse-power developed at a certain speed, but usually an additional clause giving either the maximum draw-bar pull or the maximum tractive effort required at the tread of the car wheels is inserted.

$$\begin{aligned}
 & \left. \begin{array}{l} \text{Tractive} \\ \text{effort in lb.} \\ \text{at the tread of} \\ \text{the car wheels} \\ \text{(per motor)} \end{array} \right\} = \frac{\text{Horse-power developed by motor}}{\text{Speed of car in miles per hour}} \times \frac{\begin{array}{l} \% \text{ Mechanical} \\ \text{efficiency of} \\ \text{the bearings} \\ \text{and the re-} \\ \text{duction gear} \\ \text{(if any)} \end{array}}{100} \times 375 \\
 & = \left\{ \frac{2.82}{10^8} \right\} \times \left\{ \begin{array}{l} \text{Total No.} \\ \text{of conduc-} \\ \text{tors on} \\ \text{the arma-} \\ \text{ture} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Current} \\ \text{in each} \\ \text{conduc-} \\ \text{tor} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Total flux} \\ \text{per pole} \\ \text{in C.G.S.} \\ \text{lines} \end{array} \right\} \times \left\{ \begin{array}{l} \text{No. of} \\ \text{poles} \end{array} \right\} \\
 & \quad \times \left\{ \frac{1}{\text{Diameter} \\ \text{of car} \\ \text{wheels in} \\ \text{inches}} \right\} \times \left\{ \frac{\text{Revs. of motor}}{\text{Revs. of car} \\ \text{wheels}} \right\} \times \left\{ \frac{\begin{array}{l} \% \text{ Mechanical} \\ \text{efficiency of} \\ \text{bearings and} \\ \text{reduction gear} \end{array}}{100} \right\}
 \end{aligned}$$

* Hobart, *Electric Motors*, p. 204 et seq. Also *Electrician*, August 4, 1905, p. 628.

† *Electrician*, vol. li. pp. 527 and 585; vol. liii. pp. 41, 46, 106, 249, 330, 537 and 587; vol. liv. p. 68.

It is also convenient to remember that—

$$\left\{ \begin{array}{l} \text{Torque of} \\ \text{each} \\ \text{motor in} \\ \text{pound feet} \\ \text{(i.e. pounds} \\ \text{pull at one} \\ \text{foot} \\ \text{radius)} \end{array} \right\} = \left\{ \begin{array}{l} \text{Tractive} \\ \text{effort at} \\ \text{tread of} \\ \text{car wheels} \\ \text{(per} \\ \text{motor)} \end{array} \right\} \times \left\{ \begin{array}{l} 1 \\ \text{Ratio of} \\ \text{speed} \\ \text{reduc-} \\ \text{tion} \\ \text{gear} \\ \text{(if any)} \end{array} \right\} \times \left\{ \begin{array}{l} 100 \\ \% \text{ Me-} \\ \text{chanical} \\ \text{effici-} \\ \text{ency of} \\ \text{bearings} \\ \text{and re-} \\ \text{duction} \\ \text{gear} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Dia-} \\ \text{meter} \\ \text{of car} \\ \text{wheels} \\ \text{in inches} \\ 24 \end{array} \right\}$$

and that—

$$\left\{ \begin{array}{l} \text{Speed of} \\ \text{car in} \\ \text{miles per} \\ \text{hour} \end{array} \right\} = \left\{ \begin{array}{l} \text{Speed of} \\ \text{motor in} \\ \text{revolutions} \\ \text{per minute} \end{array} \right\} \times \left\{ \begin{array}{l} 1 \\ \text{Ratio of} \\ \text{speed reduc-} \\ \text{tion gear} \\ \text{(if any)} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Diameter of} \\ \text{car wheels} \\ \text{in inches} \end{array} \right\} \times .003$$

Crane motors and **Lift motors** are rated and designed in much the same way as traction motors, as the conditions of work are similar. The magnets are usually series wound, though lift motors are sometimes shunt wound, with a small amount of series winding which is either permanently in use or in use only when starting, and afterwards cut out by the action of the starting switch.

Motors for driving fans or pumps generally work continuously at full load, and are often series wound. If large speed variations are required, *e. g.* in feed-pump motors) a regulating resistance in series with the machine is employed, or armatures with two commutators—coupled in series for low speeds, and in parallel for high speeds—are used.

Battery charging dynamos.—Storage cells are generally charged by passing a current of fairly constant amount through them for the necessary number of hours. The best value for this current is decided by the manufacturers of the cells. As the charging proceeds, the back voltage of the cells gradually rises so that, in order to maintain the correct current, the voltage of the charging dynamo must also be gradually increased. Towards the end of the charge the dynamo voltage may, on this account, have to be increased 30 % to 35 % above its value at starting. Another point to be considered in connection with these dynamos is the fact that series-wound or compound-wound machines are inadmissible, for should the series dynamo voltage by any chance fall below the battery voltage, the current through the armature and the series winding will immediately be reversed, and this will reverse the polarity of the dynamo,

causing it to work as a motor discharging the battery, instead of as a dynamo charging it, and, acting as a series motor, it will pull up, and then run in the reversed direction. To meet these requirements, charging dynamos are always shunt wound, and are supplied with a regulating resistance in the shunt winding to enable the voltage to be adjusted by hand within the required limits (*i. e.* from 110 volts to (say) 140 volts, or from 220 to 280 volts). Sometimes a small amount of series winding is placed on the magnets, to enable the machine to be used as a compound dynamo when lighting the lamps (either alone or in parallel with the battery), but this series winding is always cut out of circuit (or short circuited) when the dynamo is charging the cells. Since the maximum allowable charging current of the cells is often only a small portion of the full output of the main generator in the station, it is usually found preferable to employ a small auxiliary dynamo, or "**booster**," connected in series with the main generator to give the additional volts required as charging proceeds, rather than to make the main generators large enough to give the increased voltage. These "boosters" are arranged to allow their terminal voltages to be varied from the full value (30 or 35 volts in a 110 volt installation) down to zero with the full current passing, by alteration to the exciting current only. It is found that this can be readily done without sparking due to the low voltage.

Heavy current dynamos for plating, electro-chemical, and electro-metallurgical processes.—The generation of heavy currents at comparatively low terminal voltages introduces two difficulties in design. One is the collection of the currents at the commutator without excessive heating, and the other is the prevention of eddy-current losses in the large conductors necessarily employed. The first point is met by the employment of commutators of small diameter but great length, so as to reduce the peripheral speed and friction at the rubbing contact, and yet maintain ample section of contact area. The second leads to the use of slotted armatures with the conductors well embedded in the core, or to multi-circuit (or multiply-re-entrant circuit) smooth-cored armatures, with the separate conductors made up of stranded cable. Plating dynamos are always either shunt wound or separately excited, as series- or compound-wound machines are in danger of having their polarity reversed by the back voltage of the depositing vats, just in the same way as when charging accumulators. The voltage must be under complete control by means of a shunt-regulating resistance.

High-speed steam turbo-dynamos.—Dynamos for direct coupling to steam turbines are gradually coming into more general use, and their high speed involves a quite special design. The sort of speeds met with are—

For 100 kw. dynamos, 3500 revs. per minute.

300 to 400 kw. dynamos, 3000 revs. per minute.

1000 kw. dynamos, 1500 revs. per minute.

2000 " " 1000 " " "

In order to reduce the core losses few poles have to be used, two being found sufficient up to 400 kw. or even larger sizes. Commutation * offers serious difficulties, partly because of the impossibility of keeping a smooth and true commutator surface at the high speeds and partly because of the high commutation frequency and reactance voltage. The rough commutator face

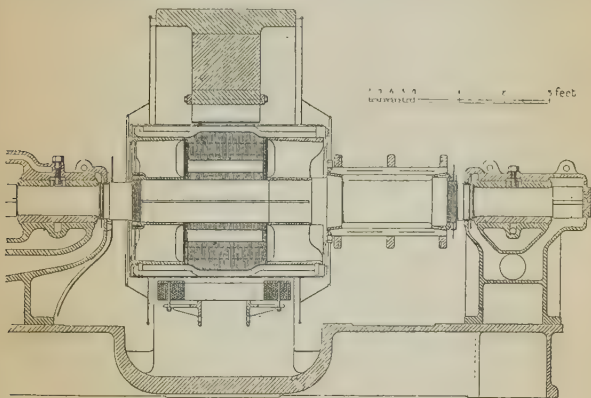


FIG. 85.

makes soft metal brushes almost essential and prevents the general use of carbon brushes, so that in order to stop sparking, either the brushes have to be automatically shifted into a stronger reversing field as the load increases, or, with fixed brushes, the reversing field must be automatically strengthened by the use of a small series-wound reversing pole † or by a series compensating winding, ‡ counteracting the armature reaction. As a further aid to commutation smooth-cored armatures are often employed in place of slotted ones. §

In Figs. 85 and 86 a recent 1000 kw. 500 volt 1200 rev.

* See p. 258.

† See Swinburne's Paper, *Jour. Inst. E. E.*, vol. xix., 1890, p. 105.

‡ Déri's Patent, 4784 of 1900. See also *E. T. Z.*, September 11, 1902, and *Zeitschrift für Elektrotechnik*, 1904, pp. 79 and 96.

§ See also Dr. Pohl's Paper before the *I. E. E.*, Nov. 1907.

Parsons turbo-generator is shown. It is a smooth-cored 4 pole machine with an armature 36" in diameter and 23" long, with 192 conductors. The commutator design is one very commonly adopted for the high-speed machines—being supported against the high centrifugal forces by insulated steel rings shrunk on instead of by the usual cone rings. The compensating winding is held in the deep slots in the pole faces, as shown in Fig. 86.*

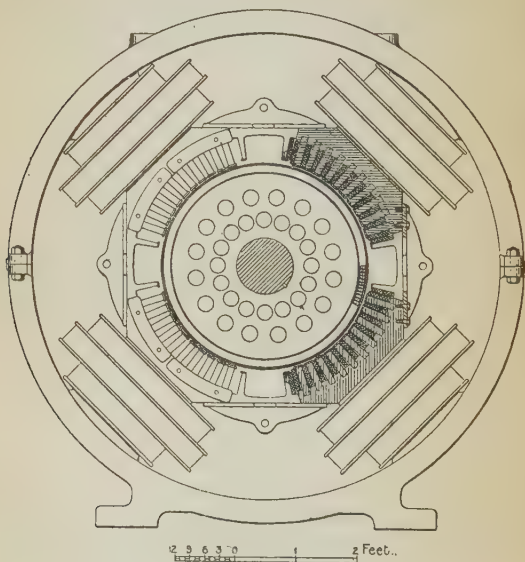


FIG. 86.

Train lighting dynamos.—Dynamoes intended to be driven from the wheel axles of a train and to provide current for the lighting of the train are always used in parallel with a battery, partly in order to steady the light and partly because the battery is useful in providing light whilst the train is at rest. In spite of the presence of a battery, however, some provision has to be made for the wide variations in speed to which the

* Reproduced from Messrs. Stoney and Law's I. E. E. Paper, read April 1908.

dynamo is subjected. An ingenious dynamo for this work has been patented by Professor E. Rosenberg,* and is in use on the Prussian railways and elsewhere. In this machine the working flux is produced by the armature reaction due to a current set up in an auxiliary pair of brushes which are short circuited together. This short-circuit current itself is proportional to the difference between the (constant) ampere turns of the field magnet coil and the ampere turns of the working armature current, so that, supposing the speed rises and the working current tends to rise, the short-circuit current will fall and with it the terminal volts of the machine. The nett result is the practical constancy of the working (output) current over a very wide range of speed, such as 3 or 4 to 1. Another dynamo for the same purpose is that used in connection with the Leitner-Lucas system of train lighting.†

RUNNING DYNAMOS IN PARALLEL.

Shunt-wound dynamos run well in parallel, but unless their characteristic curves are similar in shape the current will not be correctly divided amongst the different machines at all loads without individual field regulation.

Compound machines of the same or of different types and sizes can readily be run in parallel if the resistances of the series windings be adjusted so as to bear approximately inversely the same relation to one another as the outputs of the machines. An "equalizing" bar, as shown in Figs. 150 and 151, must also be used.‡

Series machines are coupled for parallel running just like the armatures and series windings of compound machines.

* See *Electrical Engineer*, April 28, 1905.

† See *Electrician*, July 7, 1905, p. 454. See also *Journal I. E. E.*, vol. xxxix., 1907, p. 132.

‡ See p. 420.

ALTERNATING CURRENT MACHINES.

By P. R. FRIEDLAENDER.

Design.

ALTERNATORS are almost invariably either single-phase, two-phase, or three-phase (with equal voltages in the different phases). Practically the only exception is the **mono-cyclic system** * which is in use to some extent in America. In this system, which is convenient for loads consisting almost entirely of incandescent lamps, but on which a few motors also occur, the great part of the output of the alternator is supplied as an ordinary single-phase current, the small remainder being generated in an auxiliary fine wire winding with a phase difference of 90° (or $\frac{1}{4}$ period) from the main current. This fine wire circuit is only in use when motors are being started up, or when they are in danger of stopping through overloads.†

Single-phase currents are used for lighting loads because of their good voltage regulation and the simplicity of the wiring, and are likely to be increasingly used for traction work in connection with the recently introduced single-phase commutator motors.‡

Two- or three-phase currents are almost equally suitable for power work, but for mixed lighting and power work three-phase supply is generally preferred as securing better pressure regulation than two-phase, and there is also a small reduction in the weight of the mains required.§

There is practically no difference in the design of alternators for one-, two- or three-phases, and they can be considered together.

Knowing the output required in volts and amperes, the number of revolutions per minute, and the periodicity ($\sim$),|| the design can be proceeded with as follows—

* Due to C. P. Steinmetz.

† See Niethammer, *Elektrotechnik und Maschinenbau*, August 12, 1907.

‡ See pp. 343 and 419.

§ See p. 386.

|| The periodicities in common use at the present time are as follows—

For purely motor work (power transmission, etc.), 25 $\sim$ to 40 $\sim$, for combined power and lighting systems 40 to 60 $\sim$, and for purely lighting work 50 and occasionally 100 $\sim$. For traction with single-phase commutator motors as low as 15 $\sim$.

Are lamps will not burn steadily at much below 40 $\sim$, and incandescent lighting, though sometimes successfully carried out at as low as 25 $\sim$ should preferably be kept at not less than 40 $\sim$.

The lower the periodicity is kept, other things being equal, the lower will be the price of alternate current motors and generators, because of the reduction in the number of poles required. With transformers, on the other hand, the cost for a given efficiency increases somewhat as the periodicity is reduced.

I. Decide on the type of machine to be adopted.

- (a) Whether the armature shall be of the **slotted** (wire wound in slots or holes in the iron core), or of the **smooth-cored type**.

For two- and three-phase work the slotted type is invariably used, and for single-phase work also it is in general use because of its good mechanical properties—armatures of the coreless disc type (such as the original Ferranti and the Brush (Mordey) alternators) being practically obsolete.

- (b) Whether the armature or the magnets shall form the **revolving part** of the machine.

On the whole the stationary armature is to be preferred, because (1) on a stationary structure the insulation for high tensions is more easily carried out; and (2) the magnets, being



FIG. 87.



FIG. 88.

as a rule heavier than the armature, give a greater fly-wheel action, and in many cases are actually mounted on the fly-wheel—the rim forming part of the magnetic circuit (so-called “Fly-wheel Alternators”).

- (c) Whether the **magnets** shall have—

- (1) A single ring of alternate north and south poles (Fig. 87).
- (2) A double crown of poles—the poles facing one another being of opposite polarity (Fig. 88, Ferranti).
- (3) A double ring of poles. All those in one ring being of one polarity (Figs. 89, 90 and 91). Such a system (known as an “**Inductor Alternator**”) can be most conveniently excited by a single field coil, which may either be fixed or may rotate with the magnet.

The “alternate pole” type (1) and (2) is usually arranged with a field coil on each polar projection as in Figs. 87 and 88, but type (1) has also occasionally been constructed with a single field coil (“Claw” type), chiefly for machines running at low

speeds, and having a small number of poles—for, unless the latter are spaced well apart, a great deal of leakage takes place.

In any type of alternator the **number of magnet projections of similar polarity** required in each complete crown or ring

$$= \frac{\sim \times 60}{\text{revolutions per minute}} \quad (\text{see p. 291})$$

(where $\sim$ = number of complete periods per second).

The magnet cores of "alternate pole" magnets are usually of laminated iron or steel, cast or bolted into the yoke, though sometimes only the tips of the poles (to a depth of an inch or

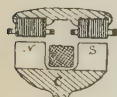


FIG. 89.



FIG. 90.



FIG. 91.

so) are laminated, the rest being cast in one piece with the yoke, and sometimes the poles are not laminated at all.

"Inductor" magnets are usually complete iron or steel castings with the tips of the poles laminated to the depth of a few inches.

In spite of the apparent simplicity of "**Inductor**" machines (type (3) above), alternate pole alternators are much more extensively used, practically all modern alternators being of the type shown in Fig. 87.

The **advantages** of the inductor type are—

(1) Great simplicity of the rotating part, the magnetizing coil being stationary.

(2) The saving in weight of copper and in energy required for excitation effected by the use of a single magnetizing coil.

Against these advantages the following decided **disadvantages** must be set—

(1) Only half the armature conductors are effective at any one instant, for the alternations are produced, not by each armature conductor being cut alternately, by the flux from a

north pole and that from a south pole, but by a flux always in the same direction alternately cutting, first one side, and then the other, of each coil (see Figs. 89 and 91).

On this account, either the number of conductors on the armature must be doubled, making the regulation of pressure very poor, or the flux per pole must be doubled, thus making the whole machine very heavy.

NOTE.—The number of conductors to be used in the flux formula on p. 292 for calculating the total volts of the machine is the total conductors per phase on one armature if the machine has a double armature winding (as in Fig. 89), or simply the total conductors per phase if the machine has only a single armature winding (as in Fig. 92); whilst if only one set of poles is active (as in Fig. 90) only one-half the total conductors per phase must be used.

(2) The leakage coefficient is high, for any leakage that occurs in the intermediate space between two adjacent poles of the same sign (such as at *D*, Fig. 91), not only increases the necessary section of the magnet iron, but also does positive harm by inducing a back voltage in the armature winding.

The flux density in the armatures of inductor alternators may be considerably higher than with alternate pole machines (see table on p. 294), since the iron, instead of being carried through a complete hysteresis cycle, from a maximum flux in one direction to a maximum flux in the opposite direction, only changes from a maximum to (almost) zero and then to a maximum again. On this account the area of the hysteresis loop is reduced by about one-half, and about one-half the value of the "core loss per cubic inch" read off Fig. 81, p. 244, should be used when calculating the iron loss in inductor armatures.

For turbo-alternators two types of revolving magnets are in use: (1) the salient pole type, as in Fig. 93; and (2) the slotted core type, of which two forms are shown in Fig. 94—one with radial slots, and the other with slots perpendicular to the pole axes. The slotted core type enables the winding to be more thoroughly protected owing to its great subdivision, and it can also be worked at a higher current density on account of the greater cooling surface. The arrangement of the end connections, however, offers some difficulties.*

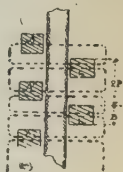


FIG. 92.

* For details of the special types of revolving magnets employed to withstand the high peripheral speeds which occur with steam turbo-alternators, see S. P. Thompson, Howard Lectures, *Electrical Engineer*, September 28, 1906, etc.; A. G. Ellis, *Journal I. E. E.*, vol. xxxvii., 1906, p. 307; Stoney and Law, *I. E. E. Paper*, April 1908; and Miles Walker, *Journal I. E. E.*, vol. xlv., 1910, p. 319.

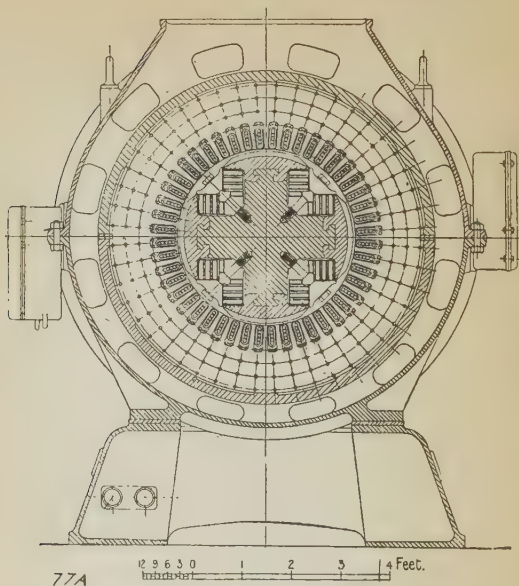


FIG. 93.

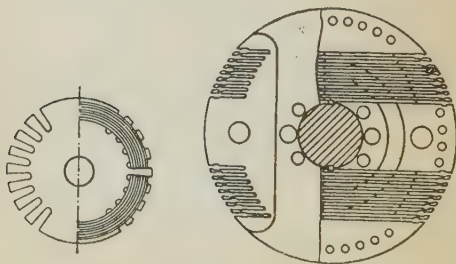


FIG. 94.

II. Determine the approximate **diameter of the armature**, and, if iron cored, its length, or, if of the disc type, the width of the pole-face radially, by the following rule—

$$\left\{ \begin{array}{l} \text{Diameter of} \\ \text{rotating part} \\ \text{(armature or} \\ \text{magnets).} \\ \text{Inches} \end{array} \right\}^2 \times \left\{ \begin{array}{l} \text{Length of magnet-pole} \\ \text{face (parallel to shaft} \\ \text{in iron-cored machines} \\ \text{and radially in disc} \\ \text{alternators).} \\ \text{Inches} \end{array} \right\} = \frac{\text{Volts} \times \text{amperes} \\ \text{per phase} \times \text{No.} \\ \text{of phases} \cdot}{\text{Revolutions} \\ \text{per minute}} \times C,$$

where C has the following approximate values for normal machines built for about 50 ~ :—

Type	C		
	Single-phase	Two-phase or Three-phase	
Iron-cored Alternate Pole type			
Small outputs (up to about 30 kilowatts)	120	80	
Large outputs	to 60	to 40	
Iron-cored Inductor type			
Small	100	75	
Large	to 50	to 40	
Coreless Disc type			
Small	100	—	
Large	to 50	—	

The value of C varies slightly with the periodicity, being greater the higher the periodicity because of the greater number of poles involved.

The **diameter** D should then be determined by the maximum allowable **peripheral speed**.

Usual peripheral speeds are as follows—

Rotating iron-cored armatures { 2000 to 6000 feet per minute according to diameter (see p. 240, under continuous current machines).

Rotating alternate pole type magnets { 4000 to 6000 feet per minute.

Inductor alternators with magnets consisting of solid castings { 6000 to 7000 feet per minute.

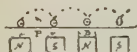
Coreless disc alternators 6000 to 7000 feet per minute.

Steam turbo-generators * 12,000 to 14,000 feet per minute.

Unless overall space is an important consideration it is best, for

* See S. P. Thompson, Howard Lectures, 1906 ; *Electrical Engineer*, September 28, 1906, etc. ; and A. G. Ellis, *Journal I. E. E.*, vol. xxxvii., 1906, p. 307.

good regulation and for minimum weights of active material, to keep the diameter as large and the length of armature as small, as possible. The diameter of armatures should be settled by a compromise between the above figure for peripheral speed, and an allowance (for normal speed machines) of 8 inches to 12 inches peripherally for the pole pitch or distance between the centre of a north pole and that of the adjacent south pole.



Single-phase, 1 hole per pole.

FIG. 95.



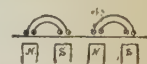
Single-phase, 1 hole per pole.

FIG. 96.



Single-phase, 2 holes per pole.

FIG. 97.



Single-phase, 2 holes per pole.

FIG. 98.

Single-phase inductor,
1 hole per pole.

FIG. 99.

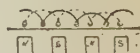
2-phase, 1 hole per pole
per phase.

FIG. 100.

2-phase, 2 holes per pole
per phase.

FIG. 101.

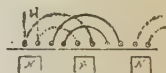
2-phase, 2 holes per pole
per phase.

FIG. 102.

III. Having approximately determined the diameter and length of armature, the **winding** must next be settled.

For slotted armatures the windings consist variously of (1) ordinary solid round or rectangular wire, (2) stranded cable, (3) wide copper strip wound on flat, or (4) for heavy currents, of solid or pressed stranded bars connected at the ends by shaped end connectors with soldered joints.

The number of slots is determined partly by the number of poles, but not absolutely. The greater the number of slots per

pole amongst which the winding is distributed, the better will be the pressure regulation, but at the same time the greater will be the amount of winding space occupied by insulation.

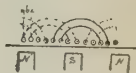
For single-phase armatures there are usually from four to twelve slots per pole. In very high voltage machines in which a great thickness of insulation is necessary a single large slot per pole is sometimes used, but in that case the magnet-cores have to be specially well laminated, and even then there will be considerable noise. The slots can be arranged in various ways as shown in Figs. 95 to 99.

Two-phase and three-phase slotted armatures have generally only two or three slots per pole per phase, though occasionally



3-phase, 1 hole per pole per phase.

FIG. 103.



3-phase, 2 holes per pole per phase.

FIG. 104.

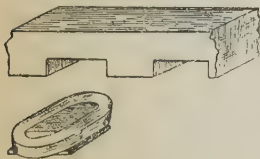


FIG. 105.

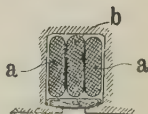


FIG. 106.

four or five slots per pole per phase are employed, especially in large machines with few poles, such as turbo-alternators (see Figs. 100 to 104).

The shape of the slots or tunnels is sometimes circular but more often square or rectangular, the side of the slot facing the field magnets being wholly or partly cut away; otherwise, both the leakage across these points, and also the inductive drop in pressure, would be very great. If, as is usual, rectangular open slots are used, the radial depth of each should not exceed three times its width, and should preferably be much less.

It is sometimes specified that each individual **armature coil** shall be **separately replaceable**, in case of breakdown, in a very few minutes. Two methods of accomplishing this are indicated in Figs. 105 and 106 (Kapp).

Fig. 105 is only suitable for single-phase machines. Each coil is wound in position on a T-shaped core of laminated iron, and

the whole fits into a large slot in the armature core and is held in place by bolts (Ferranti).

Fig. 106 is suitable for one, two, and three phases. The winding in each slot is divided into about three sections, all wound separately on shaped blocks. The outer sections a and α are first slipped in through the cut in the base of the slot, and by a slight stretch are fitted to their respective places in the slot. The third section b is then forced into the slot and wedged in position with a slip of wood. Such special arrangements, however, are not commonly necessary, as quite open parallel-sided slots (b , Fig. 73, p. 236) are now generally employed. These permit former wound armature coils to be used—the finished insulated coil being slipped into the slot through the opening and held in position by wooden wedges driven into the dovetailed recesses at the mouth of the slot.

When there is no stipulation as to removable armature coils, the winding is sometimes threaded into closed or half-closed slots, turn by turn, from the ends.

The **number of turns** of wire in each coil and the actual size of the slots can now be found by the following two rules—

$$\left. \begin{array}{l} \text{The ampere-wires} \\ \text{per 1 inch of periphery} \end{array} \right\} = \frac{\text{total wires in all the slots} \times \text{current in each wire}}{\pi \times \text{diameter of armature in inches}}$$

should lie between 300 and 500—the first figure being used where specially good pressure regulation is required, and the last for ordinary cases.

The second rule is that the allowable **current density** in the conductors is approximately—

For stationary slotted armatures . 1200 to 1800 amps. per sq. in.

For rotating slotted armatures . 2000 to 3000 „ „

IV. The **heating of the armature** can now be determined in exactly the same way as for continuous current machines (see pp. 243-4), viz.—

- (1) Calculate the ohmic C^2R loss in the armature winding (= the total loss in the different phases added together), from the length of each turn measured off a drawing and the known current in the winding.

The relations between the **currents in the armature windings and external circuits** with one-, two-, or three-phase supply are shown in the Figs. 107 to 111, and in the following table. Calling the current per phase of the external circuit (*i. e.* the current through the lamps, etc., connected between each pair of lines), C , and the supply voltage per phase of external circuit (*i. e.* across the lamps), E , the relations are as follows—

No. of Phases.	Value of current in each of the supply lines.	Value of voltage between any two of the supply lines.	Value of current in the winding of each phase on the armature.	Voltage across the winding of each phase on the armature.
Single-phase.	C	E	C	E
Two-phase with 4 wires. See Fig. 107.	C	E	C	E
Two-phase with 3 wires. See Fig. 108.	C in the 2 outer wires $\sqrt{2}C$ in the common return.	$\sqrt{2}E$ from one outer to the other. E from either outer to the common return.	C	E
Three-phase "star" or Υ with 3 wires. See Fig. 109.*	C	E outer to neutral $\sqrt{3}E$ outer to outer.	C	E
Three-phase "star" or Υ coupled with 4 wires. See Fig. 110.*	C	E Outer to neutral or $\sqrt{3}E$ outer to outer.	C	E
Three-phase "mesh" (or "delta" or "triangle") coupling. See Fig. 111.*	$\sqrt{3}C$	E	C	E

The general rule for obtaining the current in each phase from the output is—

* In the case of three-phase systems it is assumed that the lamps, etc., on the different phases are coupled together in the same way as the winding on the alternator armature supplying them. In practice, of course, this is not necessary, a "star" coupled alternator being often used to supply lamps coupled up in "mesh," and *vice versa*.

$$\text{Amps per phase} = \frac{\text{Output in True K.W.} \times 1000}{\text{Power factor} \times \text{No. of phases} \times \text{Volts per phase of winding}} = \frac{\text{Output in Kilo Volt Amperes} \times 1000}{\text{No. of phases} \times \text{Volts per phase of winding}}$$

- (2) Calculate the combined hysteresis and eddy current losses in the laminated armature core, by means of the "equivalent volume" method and Fig. 81 as described for direct current machines, on p. 244, except that for alternators with stationary armatures the values of the losses read

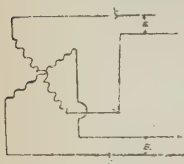


FIG. 107.



FIG. 108.



FIG. 109.

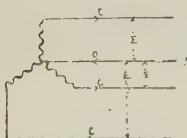


FIG. 110.

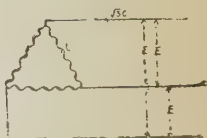


FIG. 111.

off Fig. 81 should be **reduced** some 25 % (*i.e.* should be multiplied by .75) because—

- The larger numbers of poles and smaller radial depth of core as compared with direct current dynamos gives greater uniformity of flux distribution in the core section and so reduces the losses on alternators.
- The fact that the armature of the alternator is stationary, obviates the use of heavy masses of supporting metal and the consequent additional eddy losses which often occur in direct current machines; and
- the common employment of laminated poles or pole tips on alternators avoids all pole-piece eddy current losses.

To find the flux density in the teeth divide the total flux per pole (found as in V. on p. 292) by the nett average iron area of the teeth spanned by a pole-face at any instant.

For the usual flux densities found in practice, see the table on p. 294.

In the case of inductor alternators, as explained on p. 283, the core loss per cubic inch will only be half as great as in the case of an alternate pole machine at the same flux density and, further, this core loss will only occur in the few inches of laminated iron immediately adjacent to the pole face, as the flux through the rest of the magnetic circuit on the armature does not change with the motion of the magnet poles, but is constant in amount and direction. For the same reason it is only necessary to laminate a few inches of the armature core, as shown in Figs. 89 and 90 (p. 282), the rest of the armature being of solid cast iron or cast steel.

(3) Work out the amount of the **cooling surface**.

In the case of iron-cored armatures take the sum of the areas of the outer periphery, the inner periphery, and the two ends of the iron core as effective cooling surface.

For coreless disc armatures there will, of course, be no hysteresis loss and practically no eddy current loss on the armature, so that the only loss to be considered is the ohmic C^2R loss, and the surface is the area of the two flat sides of the winding (not including the area of the non-conducting central core).

Worked as above, the **allowable watts per square inch** for the usual 70° F. temperature rise with ordinary peripheral speeds, and assuming that iron-cored armatures are properly ventilated with not more than 4" between the ventilating gaps and with each ventilating gap about $\frac{1}{2}$ " wide, will be approximately as follows—

Rotating iron-cored armatures . . .	1 to $1\frac{1}{2}$
Stationary iron-cored armatures . . .	$\frac{3}{4}$ to 1
Rotating coreless disc armatures . . .	$1\frac{1}{2}$ to 2
Stationary coreless disc armatures . . .	$\frac{3}{4}$ to $1\frac{1}{4}$.

In iron-cored armatures it is found most economical to make the ohmic loss approximately equal to the iron (hysteresis and eddy current) loss.

V. The Magnets.

The number of poles required is given by—

$$\text{No. of similar poles} = \frac{\text{periodicity } (\sim) \times 60}{\text{per second revolutions per minute}}$$

By "number of similar poles" is meant "the number of north (or south) poles in any one plane," *i.e.* in the case of an

ordinary radial pole magnet with alternate north and south poles in one crown it is the total number of north poles. In the case of an ordinary coreless disc machine with two crowns of poles facing one another, and each crown containing alternate north and south poles, it is the number of north poles in one crown that is meant. In the case of inductor magnets with a crown of north poles and another crown of south poles (or a crown of north poles and a single large south pole (Fig. 90)) the "number of similar poles" is equal to the number of north poles (or south poles) in a complete crown. (See also p. 282.)

The **flux** that must enter the armature from each polar projection at full load

$$= \frac{\text{full load internal E.M.F. per phase} \times 10^8}{\text{periodicity per second } (\sim) \times \frac{\text{No. of conductors in series per phase}}{K}}$$

The "full load internal E.M.F. per phase" is the sum (vectorially added) of the full load terminal voltage per phase, the ohmic ($C \times I$) lost voltage in the winding, and the reactive back voltage due to armature reaction at full load (as explained on p. 297). It must at present, provisionally, be assumed to be 10 % or 15 % greater than the required full load terminal voltage.

The number of conductors in series per phase for single-phase machines is equal to the total number of conductors on the armature. For two-phase it is $\frac{1}{2}$ the total number; and for three-phase $\frac{1}{3}$ the total number of conductors. This is on the assumption that there are no parallel paths on the armature. If an ordinary closed circuit gramme or drum continuous current winding is used to generate single-phase alternating currents (by connecting two collector-rings to two diametrically—in a two-polar machine—opposite points), or two-phase currents (by means of four collector rings connected to points 90° apart), or three-phase currents (with three collector rings placed 120° apart), then the "number of conductors in series per phase" will be respectively $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{1}{3}$ the total number of conductors.

For inductor alternators the note on p. 283 should be consulted. An addition must also be made to the flux per pole given by the formula, to allow for leakage entering the armature at the interpolar gap, as explained on p. 283.

K is a multiplier depending on the relative values of the breadth of the winding forming a coil-side, the span of the polar arc and the pitch of the poles, and is equal to the ratio of the $2 \times \sqrt{MS}$ value of the E.M.F. per conductor to the E.M.F. per conductor of a continuous current armature with the same total flux per pole and running at the same speed.

It is always approximately 2 (2.22 for sine wave of E.M.F.), and its value for any particular alternator can be readily found from the following curves (Fig. 112). These curves embody Kapp's* theoretical figures, which have been found to check closely with experiment.

In order to use the curves, it is only necessary to determine the ratio of peripheral breadth of pole face (B) to pole pitch (P), and the ratio of breadth of winding (b) to pole pitch (P) (see Figs. 87, 92, and 95 to 104).

The corresponding value for K can then be read off.

The values for $\frac{\text{pole breadth}}{\text{pole pitch}}$ of $\frac{4}{3}$ and 1 would only occur with inductor type magnets, whilst $\frac{1}{2}$ and $\frac{2}{3}$ are common with alternate pole magnets.

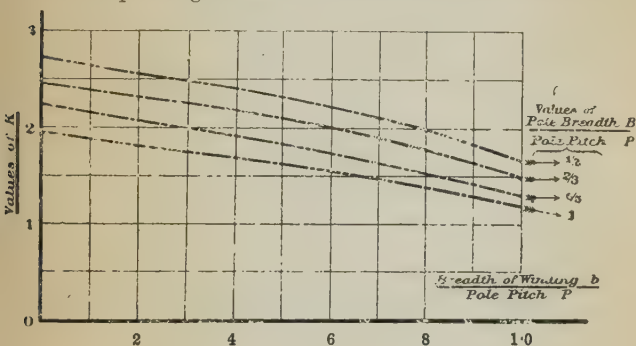


FIG. 112.

By "breadth of winding" is understood the total width of a single sheaf of winding belonging to one phase, lying either between two blank spaces or between coils belonging to other phases. When the winding is in slots this breadth is taken as the distance between the centres of the outermost slots containing such a sheaf of winding. When there is only one slot per pole per phase (as in Figs. 95, 96, 99, 100 and 103) the

$$\frac{\text{breadth of winding}}{\text{pole pitch}}$$

is correctly taken as = 0. When the whole of the armature is uniformly covered with conductors (as in the case of an ordinary continuous current armature used to generate one- or two-phase

* See *Elektromechanische Konstruktionen*.

alternating currents) the $\frac{\text{breadth of widening}}{\text{pole pitch}}$ is 1, whilst for three-phase it is $\frac{2}{3}$.

Having found the flux per pole, the dimensions of the magnets can be approximately determined. For this purpose, and also as a check on the armature dimensions, the following table of **average flux densities** will be useful—

For 40 to 70 ~ Alternators.

	Alternate pole type.		Inductor type.	
	Usual flux densities.		Usual flux densities.	
	C.G.S. lines per sq. inch.	C.G.S. lines per sq. cm.	C.G.S. lines per sq. inch.	C.G.S. lines per sq. cm.
Armature cores.....	52,000	8,000	80,000	12,500
Armature teeth (average) {	65,000	10,000	80,000	12,500
	to	to	to	to
	90,000	14,000	120,000	18,500
Air-gap..... {	32,000	5,000	65,000	10,000
	to	to	to	to
	45,000	7,000	80,000	12,500
Magnet-yokes and cores (wrought iron or cast steel)..... {	90,000	14,000	80,000	12,500
	to	to	to	to
	110,000	17,000	100,000	15,500

The armature densities should be slightly lower for 100 ~

The leakage coefficient should be worked out in the same way as for continuous current machines (p. 266), but can provisionally be taken as 1·3 to 1·5 for the alternate pole type, and 1·4 to 1·7 for the inductor type,

The number of exciting **ampere turns** required on the magnets must next be determined. This is done in exactly the same way as for continuous current machines at no load (see pp. 248 and 252).

Using the flux as obtained on p. 292, the air-gap ampere turns and the ampere turns for the iron part of the circuit are worked out, and added together. This result is then the correct full load excitation without any further allowance for armature reaction. Of course, owing to the interdependence of the inductive drop in volts in the armature, and the magnet excitation, neither can, strictly, be obtained until the other is known, but

in practice an approximate allowance must be made for the total lost volts for use in the flux formula on p. 292, and this allowance must be afterwards checked by the method given on p. 297, and, if necessary, the ampere turn calculation revised.

Knowing the exciting ampere turns, the exciting voltage and the approximate mean length of a magnet turn, the necessary section of wire for the field winding can be obtained exactly as for a continuous current shunt machine (see p. 253).

On alternate pole magnets the allowable **watts for excitation** are usually (owing to the large exposed surface on the numerous coils) determined by considerations of efficiency rather than by the heating. The exciting watts are generally not allowed to exceed 4 % of the full load output in small (10 to 20 kilowatt) alternators, or $1\frac{1}{2}$ % to 2 % in large (200 to 500 kilowatt) ones. The limiting value of the watts per sq. inch of cooling surface of rotating magnet coils, for a temperature rise of about 70° F. at ordinary peripheral speeds, is approximately $1\frac{1}{2}$ to 2, and for stationary magnet windings it is (as for continuous current machines, p. 253) .6 to .9.

VI. To pre-determine the **armature current on short circuit**, and the total **pressure drop** from no load to full load with constant excitation and constant speed.

When an alternator armature running at full speed and volts is short circuited the current will rise until it reaches a point at which the vectorial sum of the ohmic ($C \times R$) lost volts and the reactive back volts in the winding is just equal to the voltage at which the alternator was running on open circuit* (*i. e.* to the voltage generated),

$$\text{or—} \quad \begin{array}{l} \text{No load voltage} \\ \text{generated per phase} \end{array} = \begin{array}{l} \text{Resultant voltage drop with} \\ \text{the short-circuit current} \\ \text{flowing through the armature} \\ \text{winding (per phase).} \end{array}$$

Now (neglecting the effect of saturation of the iron of the magnets) the no-load flux in the magnets is proportional to the exciting ampere turns,

$$\text{or—} \quad \begin{array}{l} \text{No load voltage} \\ \text{generated per phase} \end{array} \propto \frac{\text{Exciting } AT}{\text{Leakage coefficient}},$$

or, practically—

$$\begin{array}{l} \text{No load voltage} \\ \text{generated per phase} \end{array} = \frac{\text{Exciting ampere turns}}{\text{per magnetic circuit}} \times n,$$

where n is a constant.

And the resultant voltage drop (since the permeability of the

* This short-circuit current is practically independent of the speed at which the machine runs, for both the voltage generated and the reactive back voltage (which is the larger part of the total back voltage) increase in proportion to the speed (or $\sim$).

various stray paths is practically constant) can be put down with fair accuracy as proportional to the armature ampere turns with the short-circuit current passing,

$$\text{or—} \quad \begin{array}{l} \text{Resultant} \\ \text{voltage drop} \\ \text{per phase} \end{array} = \begin{array}{l} \text{armature ampere} \\ \text{turns on short} \\ \text{circuit per phase} \end{array} \times n_1$$

where n_1 is a constant.

And since the drop = the no load voltage—

$$\therefore \left\{ \begin{array}{l} \text{Exciting ampere} \\ \text{turns of the field} \\ \text{per magnetic} \\ \text{circuit} \end{array} \right\} \times n = \left\{ \begin{array}{l} \text{Short-circuit} \\ \text{current per} \\ \text{phase} \end{array} \right\} \times \left\{ \begin{array}{l} \text{No. of arm-} \\ \text{ature turns} \\ \text{per phase,} \\ \text{looped with} \\ \text{each mag-} \\ \text{netic circuit} \end{array} \right\} \times n_1,$$

or—

$$\begin{array}{l} \text{Short-circuit} \\ \text{current per} \\ \text{phase} \\ (\sqrt{\text{MS}} \text{ value}) \end{array} = \frac{\begin{array}{l} \text{Field exciting ampere turns per} \\ \text{magnetic circuit} \end{array}}{\begin{array}{l} \text{Maximum number of armature} \\ \text{turns per phase looped with} \\ \text{each magnetic circuit}^* \end{array}} \times \frac{n}{n_1}.$$

The ratio $\frac{n}{n_1}$ involves the leakage coefficient of the magnets, the leakage coefficient of the armature coils, and the ratio of the maximum value to the virtual (or $\sqrt{\text{mean square}}$) value of the short-circuit current. In the case of three-phase machines it also includes an allowance for the interaction of the coils of the different phases.

The value of $\frac{n}{n_1}$ therefore varies considerably for different types of alternator, and should, whenever possible, be determined by a measurement of the actual short-circuit current.

The following average values can, however, be used with good results—

Type of Alternator.	No. of Phases.	Value of $\frac{n}{n_1}$.
Iron-cored armatures with "alternate pole" magnets	{ 1 and 2 3	.6 to .75 .3 to .4
Iron-cored armatures with "inductor" magnets	{ 1 and 2 3	.45 to .55 .3 to .4
Coreless disc armatures	1	.75 to 1

* "The maximum number of armature turns per phase looped with each magnetic circuit" = $\frac{\text{Total number of conductors per phase}}{2 \times \text{Number of north (or south) poles in one plane}}$

The **short-circuit current** per phase with normal no-load excitation should be from three to four times the normal full load current per phase.

Knowing (either from experiment or by calculation) the current on short circuit the **drop in terminal pressure** * can be determined thus—

The resultant lost volts at full load are approximately—

Normal no-load $\times$ $\frac{\text{Full load current per phase}}{\text{Current on short circuit, per phase, with normal no-load excitation.}}$
volts per phase

The $C \times R$ lost volts at full load = Full load current per phase $\times$ Resistance of the winding per phase.

The CR lost volts found in this way should be doubled in order to allow for the effect of eddy currents in the armature and the magnet pole pieces.

To combine the resultant lost voltage, the CR lost voltage and the terminal voltage, set off a horizontal line AC (Fig. 113) to represent the phase of the external current.

From A draw AE to represent, to any scale, the full load terminal voltage per phase, making the angle ϕ such that $\cos \phi =$ the power factor † of the external load (e. g. if the load for which the

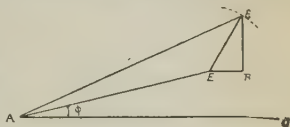


FIG. 113.

terminal pressure drop is required be non-inductive, the line AE will lie along AC . If the power factor of the load is .8 the angle ϕ will be 37° , and so on).

From the point E draw ER horizontally to represent (to the same scale) the $C \times R$ (ohmic) lost volts in the winding, together with the allowance for eddy currents. ER is drawn horizontally because the ohmic (and eddy current) losses are in phase with the current AC .

With centre E and radius (to the same scale as before) equal to the resultant lost volts at full load, as calculated above, describe an arc and find the point (ϵ) in which the vertical line drawn from R cuts it.

Then the line $A\epsilon$ represents (to the same scale) the no load voltage required per phase, in order that AE shall be the full load voltage per phase. $A\epsilon$ should be equal to the "internal E.M.F. per phase" used in the flux formula on p. 292.

$\frac{A\epsilon - AE}{A\epsilon} \times 100$ is the percentage drop in terminal pressure from no load to full load.

* For a discussion of the various ways of estimating alternator drops, see Behn-Eschenburg in *Elektrotechnische Zeitschrift*, May 19, 1904, and *Electrical Engineer*, June 24, 1904. Also Achison, *Journal I. E. E.*, 1904, p. 1064.

† i. e. true watts $\div$ apparent watts.

The vertical line R_e represents the reactive back voltage (which is in quadrature with the current, AC , producing it) to the same scale.

The diagram (Fig. 113) applies equally well to inductive loads (lagging currents) and to capacity loads (leading currents), the only difference being that angles of *lead* are set out *below* the horizontal line AC instead of *above* it. On capacity loads the no load voltage may be *less* than that at full load owing to the leading currents strengthening the magnetic field instead of weakening it.

In place of drawing out the diagram (Fig. 113) for each case, Fig. 119, on p. 312, can be used with advantage, taking the "resultant lost volts at full load," calculated as above, to be equal to the reactive lost volts—a sufficiently accurate assumption in most ordinary cases.

It will readily be seen that the **drop of an alternator** depends greatly on the nature of the load on which it is working—the drop obtained on a lighting circuit (practically non-inductive) being often trebled on a motor load (power factor .75 to .8).

On the Continent drops of 20 % to 25 % on .8 power factor loads are considered satisfactory, and in English practice it is usual to specify a drop of 6 % to 10 % on a non-inductive lighting load.

Many attempts have been made to *compound* alternators, *i.e.* to make them automatically regulate their voltages under all conditions of load, but none have come into general use.* A more common plan for keeping the voltage constant is to fit a *Tirril Regulator* to one or more of the alternators. This regulator consists of a vibrating contact controlled by the bus bar voltage and arranged to cut resistance in or out of the exciter shunt winding according as the bus bar volts are above or below the normal.†

Note on eddy currents in alternators.—When the armature winding is very well subdivided, so as to form a practically uniform winding over the whole periphery, it is unnecessary to laminate any part of the magnets. If, however, the winding is only distributed in two or three slots per pole, it is advisable to laminate at least a few inches of each magnet pole, as the large slots cause periodic changes in the magnetic flux distribution. The armature current also causes periodic changes in the flux, especially with single-phase machines. These changes in the flux produce eddy currents in the pole faces which may easily reduce the efficiency of the machine by several per cent., unless laminations are introduced.

These eddy currents are not, however, an unmixed evil, for their presence is found to give great steadiness to the alternators when running in parallel, owing to the way in which they counteract any tendency to periodic speed variation.

* See Miles Walker, *Journal I. E. E.*, 1905, vol. xxxiv. p. 402, for a discussion of the various methods of compounding.

† See E. T. Z., 1907, p. 1202, and *Elektrotechnik und Maschinenbau*, 1908, p. 421.

Some makers even consider it worth while (especially with single phase machines) to arrange a special winding ("**circuits amortisseurs**" or "**damping circuits**") of copper bars, short circuited on themselves, in holes in the pole faces near the periphery. Such a winding confines the eddy currents to the direction (viz. parallel to the armature conductors) in which they have a maximum useful effect with a minimum waste of energy.* The weight of the copper of this auxiliary winding is usually made approximately equal to, or rather greater than, the weight of the armature winding.

The laminations in the armatures of inductor alternators need only be carried a few inches up from the pole face, as, beyond that, there are no appreciable changes in flux density from point to point, though the flux as a whole rotates as in a unipolar dynamo.

Note on the **running of alternators in parallel**.†—Alternators of all types and sizes, provided they are wound for the same number of phases and the same periodicity, can be run in parallel, though some combinations are far more satisfactory than others.

A great deal depends on the steam-engine, or other source of power used. For, should the governor hunt, or the turning moment vary much at different points in the stroke, large rocking currents may be set up between the different machines in parallel.

Engines driving alternators intended for parallel running, especially when any rotary converters or synchronous motors are to be run from the circuit, should be so balanced and fitted with fly-wheels of such weight that the maximum angular deviation of the rotating part of the alternator either ahead or behind the position which it would occupy if running at a perfectly uniform speed, does not exceed—‡

$$2\frac{1}{2}^{\circ} \frac{\text{No. of } N \text{ poles in one plane}}{\text{i. e. } 2\frac{1}{2}^{\circ} \text{ of phase or "electrical degrees."}}$$

It can be shown that, on the assumption that the engine speed varies in accordance with a straight line law, this corresponds to a maximum permissible *speed* variation above and below the normal of approximately

$$11 \% \frac{\text{No. of } N \text{ poles in one plane}}$$

* See Waters, *Journal A. I. E. E.*, May 1908.

† For a discussion of the problems involved in the running of alternators in parallel, see Gorges, *E. T. Z.*, Part x., 1900, p. 186, and Part xlix., 1902, p. 1053; Rosenberg, *E. T. Z.*, 1902, Parts xx. to xxii., and *Journal I. E. E.* January 1, 1909; Bohll, *Electrician*, vol. lii. p. 784, March 4, 1904.

‡ See Eborall, *Journal I. E. E.*, vol. xxx., May 1901, p. 702; also A. H. Kruesi, *Electrical World*, vol. xxxvii. p. 501.

for double acting single cylinder engines, and—

$$22\%$$

No. of N poles in one plane

for compound engines with two cranks at right angles.

For instance, a single cylinder engine having a speed irregularity of $\frac{1}{180}$ above and below the normal will be suitable for alternators having up to about 40 poles, because

$$\frac{11\%}{20} = .55\% = \frac{1}{180}.$$

Before putting two alternators into parallel it is necessary to—

- (a) Get the periodicity exactly the same in both machines by hand regulation of the speed. This, like (c), is accomplished by the aid of a synchronizer.
- (b) Get the voltages approximately alike by comparison of the voltmeters on the two machines.
- (c) Get the voltages into phase with one another by adjusting the speed until the synchronizer (for description of which see p. 319) indicates that they are in phase.
- (d) In the case of polyphase alternators, it is further necessary to arrange that the rotating field produced by the armature currents in the different phases shall rotate in the same sense, in the two machines, *i. e.* armature coils which occur consecutively in the direction of rotation of the one machine, must be connected in parallel with coils occurring in exactly the same relative positions in the other machine.

(This point can be settled once for all when the machines are first connected to the switch-board, either by following out the windings on the armatures or by connecting three synchronizing lamps (through three transformers if necessary) to the three phases, and noting whether all three go dark together or at different instants—if the former, the connections are correct; but if the latter, two of the terminals of the new machine must be interchanged.)

When these four conditions are fulfilled (or nearly so) the machines can be connected directly together (either when both are running light or both loaded, or when one machine is fully loaded and the other doing nothing), and should then run with a **synchronizing or rocking current** (flowing alternately from one machine to the other), not exceeding one-twentieth of the full load current of either machine.

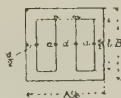
When running in parallel the proportion of the load supplied

by each machine depends only on the steam inlet and not, as in D.C. machines, on the excitation. The effect of variations in excitation is only to cause additional idle currents to circulate in the armatures and compensate for the defects in excitation.

ALTERNATE CURRENT TRANSFORMERS.

Ordinary alternate current transformers require no moving parts, and are consequently called stationary or static transformers, to distinguish them from the various forms of motor-generator. Static transformers are used to raise or lower the supply voltage, and, occasionally, to alter the number of phases.

The essential parts of a transformer are simply a primary and a secondary winding placed close together, so that most of the magnetic flux generated by the primary winding may also pass through the secondary. In order to obtain the best possible magnetic connection between these, all modern transformers have the two windings looped with a closed magnetic circuit consisting of thin sheet-iron stampings of various shapes.



$a = .4 + 1.2d$	.	.	.	.6 to .7d.
$b = .4 + 2.6d$	.	.	.	d
$A = .4 + 3.2d$	.	.	.	3.2 to 3.4d.
$B = .4 + 4.6d$	.	.	.	2.2 to 2.4d.
$L = \text{depth of core} = d \text{ to } 2d.$	.	.	.	$2d \text{ to } 4d.$

All dimensions being in inches.

FIG. 114.

FIG. 115.

The two most usual **shapes of core**, together with the approximately best proportions, as given by Kapp, are shown in Figs. 114 and 115. The proportions are, however, by no means rigid, and large variations from them have but little effect on the efficiency, cost, etc. For instance, a favourite type of recent years has a two-legged core, as in Fig. 114, but with the section of the limbs approximately square (sometimes made to approach a circular section by employing two or three different widths of stamping), so that $d = L$ (about), and with very long limbs placed close together ($b = \text{from } 4d \text{ to } 6d$). This type has the advantage of occupying a small floor space, and the section of the limbs enables cylindrical coils, which are economical in wire and easy and safe to wind, to be employed.

For two-phase work it is usual to use two ordinary single-phase transformers—one connected to each of the two phases. Occasionally, however, a three-legged transformer, as in Fig. 115, is used, but with the area of the central limb $\sqrt{2}$ times the sectional area of either of the outer limbs (as it has to carry the vectorial sum of the fluxes in the two outer limbs). The primary and secondary windings for one phase are placed on each of the two outer limbs.

For three-phase work three separate single-phase transformers are generally used in America and England, but on the Continent special three-legged cores, consisting of three similar limbs placed at the corners of an equilateral triangle, and connected top and bottom by three-armed bundles of stampings, or of three straight limbs in line, connected at the top and the bottom by straight yokes, are common. These have better regulating properties than the three separate transformers, but offer some mechanical difficulties.

The two-legged or “core” type core (Fig. 114) is sometimes built up of straight strips with alternate butt and lap joints at



the corners produced by superposing layers of stampings alternately, as shown in Fig. 116. The whole is held together by the friction between the plates (produced by tightening up the bolts through the holes at the corners). More generally, however,

FIG. 116. plain butt joints are used, and in order to avoid excessive eddy current losses at the joints, a thin sheet of insulation is usually inserted there, thus introducing an air-gap and considerably increasing the magnetic resistance of the circuit.

The three-legged or “shell” type cores (Fig. 115) are sometimes built up of single complete stampings, of the shape shown in the figure, and cut at the line *nm*. Each stamping is slipped over the completed coil of winding, by bending back the central tongue until it is at right angles to the rest of the plate. When in position the tongue is bent back again through the centre of the coil.

More often, however, the core is built up of straight stampings with butt joints, or alternate butt and lap joints.

The thickness of the core laminations varies from .01 inch to .02 inch (.25 mm. to .5 mm.).

The laminations are insulated from one another, either by a layer of varnish or by a sheet of very thin tissue paper.

About 85 % of the completed core depth may be taken as solid iron, the remaining 15 % being occupied by the insulating layers.

Transformers intended for incandescent lighting or for motor driving, should have a low pressure drop at the secondary

terminals between no load and full load, and should also have a smaller watt loss in the iron than in the windings (a usual proportion is: iron watts = $\cdot 7 C^2 R$ watts in the two windings). If, on the one hand, the drop is large, the lamps will not burn with uniform brightness at all times, while if, on the other, the iron loss is large, the "all day efficiency" * of the transformer will not be sufficiently high.

Low pressure drop, especially on inductive loads, cannot be attained unless the magnetic leakage between the primary and the secondary windings is kept down to a minimum (*e. g.* by interspersing the primary and secondary windings, as explained on pp. 312-13).

The following table gives the average losses and efficiencies of good English transformers for incandescent lighting at 50 to 100 ~ per second—

Output in kilowatts.	NO LOAD (IRON) LOSS.		Pressure drop from no load to full non- inductive load.	EFFICIENCY.				
	Watts.	% of full load.		Full load.	$\frac{3}{4}$ load.	$\frac{1}{2}$ load.	$\frac{1}{4}$ load.	$\frac{1}{10}$ load.
1	40	4	2.5 %	93.5 %	93 %	91.2 %	85 %	71 %
5	75	1.5	"	96.3	96.2	95.8	94	86.7
10	100	1	"	96.5	96.5	96.4	95.5	91
20	180	.9	"	97	97	97	95.8	91.5
50	400	.8	"	98	98	98	96.8	92.3
100	700	.7	"	98.2	98.2	98.2	97	93.3

On the other hand, transformers intended for arc lighting need not have a particularly low iron loss, as they are usually worked at full load only. A large pressure drop, also, is a necessity with these transformers, as it increases the sensitiveness of the feeding apparatus of the lamps. This pressure drop is obtained by winding the primary and secondary coils some distance apart on the core or by inserting a few strips of sheet iron between the P. and the S., so that, as the load (or the effective ampere turns of each coil) increases, the leakage may increase also. Sometimes, even, the coils are suspended, so that, as the magnetic repulsion between them increases (due to the increase in the current), they automatically move further apart on the core, and so allow more leakage to occur between them. (Transformers of this "floating coil" type were first used in connection with Ferranti's Rectifiers.)

In order to design a transformer to comply with a given

* The "all day efficiency" = $\frac{\text{kilowatt hours output during the 24 hours}}{\text{kilowatt hours input during the 24 hours}}$

specification as to the input and output, periodicity, pressure drop, no load loss, efficiency, and temperature, proceed as follows—

(1) Decide on the **type of core** to be used, and then assume a tentative value for the sectional area of the magnetic circuit, from the approximate expression—

$$\left. \begin{array}{l} \text{Total nett cross-section} \\ \text{of all the limbs of the} \\ \text{core in square inches} \end{array} \right\} = \text{kw. output} \times C,$$

where C has the values of 10, for 1 kw. size,

7	„	3	„
3	„	10	„
1-1 $\frac{1}{4}$	„	50	„
.75-1	„	100	„

The “total nett cross-section of all the limbs of the core” will, of course, be twice the section of one limb in the case of Fig. 114, will equal the section of all three limbs in Fig. 115, will be three times the area of one limb in the case of a three-phase transformer, and will equal the sum of the areas of the three limbs in a two-phase transformer.

(2) Sketch out a core with the proportions given in Figs. 114 or 115, and determine its nett volume in cubic inches.

Then determine the maximum allowable **flux density** from the expression—

$$\begin{array}{l} \text{Total allowable no} \\ \text{load loss (hysteresis} \\ \text{and eddy currents)} \\ \text{in watts} \end{array} = \begin{array}{l} \text{Volume} \\ \text{of core} \\ \text{in} \\ \text{inches} \end{array} \times \begin{array}{l} \sim \\ \text{per} \\ \text{second} \end{array} \times \begin{array}{l} \text{Hysteresis and eddy} \\ \text{current watts per} \\ \text{cubic inch at 1} \sim \text{per} \\ \text{second.} \end{array}$$

Knowing the maximum allowable no load watts (by specification or otherwise), the allowable “watts per cubic inch” are obtained by the above equation, and the corresponding flux density is read off Fig. 117. This curve is got from wattmeter measurements on numerous transformers with ordinary alternate butt and lap joints between the stampings (Fig. 116), and with ordinary soft iron * at periodicities varying from 40 to 100. To be strictly accurate, separate curves should be used for eddy current loss (which is proportional to the square of the flux density), and for hysteresis (which is proportional to the 1.5th or 1.6th power of the flux density), but in practice the former are so indeterminate that little, if any, increased accuracy can be obtained by this additional complication.

* Of recent years various special brands of transformer iron (usually alloys of iron and silicon or vanadium) have been brought out which show considerably lower losses. Thus Messrs. Sankey's “Lohys” brand gives about 75% of the loss shown in Fig. 117, and their “Stalloy” brand about 50% of it.

NOTE.—The flux density (B) given on the curve is the maximum value and not the average, as the losses depend on the maximum value.

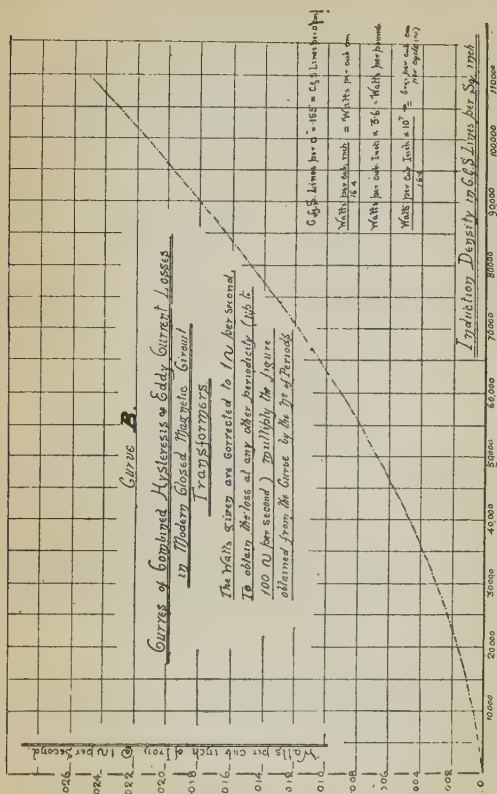


FIG. 117.

The flux density obtained as above, multiplied by the nett cross-section of one limb of the iron core in square inches (taken at right angles to the direction of the flux), gives the **total flux** (maximum value).

The following are usual values for the flux density with ordinary iron (higher values are permissible with the special brands of iron)—

Kilowatts output.	MAXIMUM FLUX DENSITY.			
	40 to 60 ~ per second.		80 to 100 ~ per second.	
	C.G.S. lines per □ cm.	C.G.S. lines per □ inch	C.G.S. lines per □ cm.	C.G.S. lines per □ inch.
Up to 3	10,000	65,000	6,500	42,000
„ 10	8,000	52,000	5,000	32,500
„ 20	7,000	45,000	4,000	26,000
„ 100	5,000	32,000	3,500	22,500

(3) The number of **primary turns** can now be determined thus—

$$\text{Total primary turns} = \frac{\text{Primary E.M.F.} \times 10^8}{\text{Total flux (maximum)} \times \sim \text{per second} \times 4.44}$$

NOTE.—The constant 4.44 is strictly correct only for E.M.F.s of a sine form, but is quite sufficiently near for use in all cases. It is derived thus—

$$\text{Average E.M.F. induced} = \frac{\text{Total flux cut per second} \times \text{Total turns}}{10^4}$$

And since the total flux is cut four times in each period (first, when rising to a maximum; secondly, when falling to 0; thirdly, when rising to a negative maximum; and fourthly, when falling to 0 again)—

$$\text{Average E.M.F.} = \frac{\text{Total flux (maximum value)} \times \sim \text{per second} \times 4 \times \text{turns}}{10^4}$$

but instead of the *average* it is the *virtual* ($\sqrt{\text{mean square}}$) value of the E.M.F. which is required, and for a sine curve—

$$\sqrt{\text{Mean square value}} = \text{Average value} \times \frac{\frac{1}{2} \sqrt{2}}{\frac{\pi}{2}}$$

$$= \text{Average value} \times 1.11,$$

∴ Virtual Total flux

$$\text{E.M.F.} = \frac{\text{Total flux (maximum value)} \times \sim \text{per second} \times \text{Turns} \times 4.44}{10^4}$$

as above.

sec.

$$\text{The number of secondary turns} = \frac{\text{Primary turns} \times \left(\frac{\text{Secondary E.M.F. at full load}}{\text{Primary supply voltage}} \right) + \left(\frac{\text{total drop in volts reduced to the secondary side}}{\text{Primary supply voltage}} \right)}{1}$$

or =

$$\text{Primary turns} \times \frac{\text{No load secondary volts found as on p. 311}}{\text{Primary supply volts}}$$

where "Total drop in volts reduced to the secondary side"

$$= \frac{\text{secondary volts}}{100} \times \frac{\text{Allowable percentage drop in volts from no load to full load.}}{100}$$

(4) Select the **sizes of primary and secondary wires**, making the **current densities** alike in both and approximately as follows—

Output in Kilowatts.	Usual Current Density.
3	1000 Amps. per sq. in.
10	800 " "
20	600 " "
100	500 " "

For transformers up to about 10 kilowatts output it is sufficient to wind the primary and secondary each in a single coil—one being wound entirely over the other, and separated from it by $\frac{1}{16}$ to $\frac{1}{4}$ inch of good insulating material (if for 2000 to 4000 volt supply, and proportionately thicker if for higher voltages). Larger transformers, however, unless the coils are of exceptional length and small depth, have the windings divided into three, four, or five coils per limb, and these are superposed or else placed side by side on the core—a primary coil, then a secondary, then another primary, and so on. As shown on p. 312, this has the effect of reducing the magnetic leakage, which would otherwise produce an excessive drop in secondary volts between no load and full load.

These remarks do not, of course, apply to constant current transformers, or to transformers for use with compound-wound rotary converters (see p. 359), in which leakage is a necessity.

NOTE.—Between each primary and secondary coil an "**earthing shield**" should be placed for the purpose of preventing damage resulting from accidental connection between the high-tension primary circuit and the secondary circuit. This shield usually consists either of a thin sheet of metal or of a layer of thin wire, well embedded in insulation but entirely separating the two windings, and connected directly to earth.

Sometimes an "**earthing device**" is connected between the secondary winding and the earth, either in place of or in addition to an **earthing shield**, in order that, should the pressure between this (low tension) winding and the earth rise above a

certain value (owing to contact between primary and secondary), the latter may be automatically earthed.

Having selected the sizes of wires, calculate the ohmic resistances of the windings, exactly as on p. 253, and determine the total C^2R (ohmic) loss in the two windings at full load—

(5) Work out the **cooling surface of the transformer** which, for convenience, may be taken as—

$1.5 \{ (A \times B \times 2) + (2L(A + B)) \}$ square inches (see Figs. 114 and 115),

but should preferably be determined from a drawing in each case. If, as is sometimes the case, the core is split up into sections, with ventilation-gaps in between, the area of both sides of these gaps must be included in the total cooling surface, and

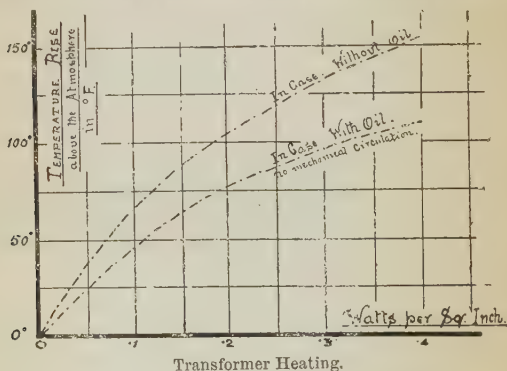


FIG. 118.

if the coil is so arranged that the surrounding air or oil has free access to any internal parts, the surface of these parts should also be included.

The total watts per square inch of transformer surface—

$$\left(\frac{\text{Iron loss} + C^2R \text{ loss in both windings}}{\text{Cooling surface worked as above}} \right)$$

should lie between .1 and .2, as shown by the curves in Fig. 118, which are calculated from figures given by Kapp.* They indicate the probable maximum temperature rise on the core or coils, in degrees Fahr., when a transformer is run continuously, at full load, completely enclosed in a cast-iron case,

* *Elektromechanische Konstruktionen* (G. Kapp, 1898), p. 5.

(1) when the case contains air surrounding the transformer, and (2) when it contains oil. The figures can only be taken as approximate, as the size of the enclosing case and the method of winding have a good deal to do with the temperature rise. If artificial cooling (air blast or water circulation) be employed these figures may be doubled or even trebled for the same temperature rise (the exact value depending on the thoroughness of the circulation).

PREDETERMINATION OF THE PERFORMANCE.

1. The **efficiency** of the transformer at any load is given as a percentage by—

$$\frac{\text{Output watts}}{\text{Output watts} + \text{Iron watts} + \text{C}^2\text{R watts in both windings at the particular load}} \times 100.$$

The iron and copper watts being found as described above, this expression can be worked out for any desired load.

In the case of ordinary constant voltage transformers, with no appreciable magnetic leakage, the iron watts practically do not alter from no load to full load. The only tests required, therefore, in order to obtain the efficiencies of such transformers at all loads, with great accuracy, are a single measurement, by wattmeter, of the watts (iron watts) lost in the core, with the secondary on open circuit, and measurements of the primary and secondary winding resistances, from which the C^2R watts are calculated for each particular load.

NOTE.—An appreciable additional loss is sometimes introduced, especially in the case of heavy current transformers, by the practice of carrying the leads through separate holes in the cast-iron case, eddy currents being produced in the metal of the case. This trouble can be easily avoided by taking both leads through a single large hole in the case.

In the case of constant current transformers and others having considerable magnetic leakage when loaded, this leakage often causes considerable loss in eddy currents, (1) in the iron core, (2) in the copper wire, and (3) in the case or other surrounding metallic objects.

The only safe way to obtain the efficiency of such a transformer is by direct measurement, on a wattmeter, of the combined C^2R losses and eddy current losses by short circuiting one of the transformer windings and applying (through a wattmeter and ammeter) a low voltage (usually from 2% to 10% of the normal voltage) to the other winding, sufficient to cause the

normal full load current to pass. The watts so measured are the combined C^2R and eddy current watts under practically normal full load conditions, whilst the voltage applied is the combined CR and reactive drop of the transformer at full loads. This **short circuit test** is, in fact, a valuable test to apply to any transformer as, in combination with the no load loss test, it gives complete and accurate information as to the transformer efficiency and drop (see p. 364), whilst neither test requires the application of more than a small fraction of the full load power of the transformer. As an alternative, the full load efficiency of the transformer can be obtained by direct watt-meter measurement of both the input and output or (if two exactly similar transformers are available) more accurately, by Sumpner's differential method, as mentioned on p. 370.

2. The **no load current** of a transformer may be considered as made up of two currents, with a phase difference of 90° between them. One of these (the wattless one, lagging 90° behind the E.M.F.) is the magnetizing current which, when multiplied by the primary turns, gives the ampere turns necessary to force the full flux through the magnetic resistance of the iron core and of the joints. The other (in phase with the E.M.F.) is the power current which, when multiplied by the applied (primary) volts, gives the watts absorbed by hysteresis and eddy currents at no load.

Or, **no load current** =

$$\sqrt{(\text{Magnetizing current})^2 + (\text{No load power current})^2}$$

and **magnetizing amps.** (assuming that the joints about double the magnetic resistance) =

$$\frac{\text{Mean length of magnetic circuit in inches} \times \text{Ampere turns per 1 inch at the known maximum flux density}}{\text{Primary turns}} \times 1.5.$$

The ampere turns per 1 inch being read off Fig. 83 (p. 250).

$$\text{Power current, per phase} = \frac{\text{Total iron loss}}{\text{No. of phases} \times \text{Primary voltage per phase of winding}}$$

For multiphase transformers the no load current obtained as above is the no load current per phase, the primary turns per phase being used.

3. The **pressure drop** at the secondary terminals between no load and full load is due to the combined effect of (1) the CR (ohmic) drop in the primary and secondary windings (in phase with the currents), and (2) the reactive lost volts due to the difference (produced by magnetic leakage) between the flux

looped with the primary and that looped with the secondary (this reactive drop lags 90° in phase behind the currents).

The total CR drop reduced to the secondary side =

$$\left(\text{Primary current} \times \text{Primary resistance} \times \frac{\text{Secondary turns}}{\text{Primary turns}} \right) + (\text{Secondary current} \times \text{Secondary resistance}),$$

and the reactive drop can be calculated, as explained below. The resultant of these two components can also be measured by the short-circuit test, as explained on p. 364.

The required full load secondary voltage, the CR drop, and the reactive drop can then be combined in order to get the no load voltage, in exactly the same manner as for alternators (see p. 297 and Fig. 113). Referring to Fig. 113, AE will represent the full load secondary voltage to scale, and the angle ϕ will be such that $\cos \phi =$ the power factor of the secondary load, on which the drop is required. The angle ϕ will be 0° , and AE will be horizontal, for a non-inductive external load, whilst for an inductive load the values of ϕ will be set out *above* and for a condensive load (the external circuit having capacity) *below* the horizontal line. ER (parallel to AC) is the ohmic drop reduced to the secondary side, and $R\epsilon$ (at right angles to the secondary current AC) is the reactive drop, to the same scale, $E\epsilon$ being the resultant of the CR and the reactive drops.

Then $A\epsilon$ is the no load secondary voltage to the same scale, and $A\epsilon - AE$ is the secondary pressure drop from no load to full load. For inductive and non-inductive loads $A\epsilon$ is always greater than AE , and there is an actual drop in secondary terminal voltage from no load to full load with constant primary voltage. There is, however, for every transformer a certain condensive circuit for which the no load and full load voltages are alike. If the capacity of the external circuit is increased beyond this value, the full load voltage AE will be actually greater than the no load voltage $A\epsilon$, and instead of a "drop" the transformer will have a "rise."

Fig. 119 (on the next page) will simplify the above summation. To use it, reduce the two drops to percentages of the full load secondary voltage. From a point on the first circle, vertically above the power factor for which the drop is required, mark off a horizontal equal to the % CR drop. From the point so obtained, set off a vertical equal to the % reactive drop, then the circle on which the end of this vertical falls gives the drop as a percentage of the full load secondary voltage. Thus the dotted line example on Fig. 119 shows that at a power factor of .86 with a copper drop of 10 %, and an inductive or reactive drop of 15 %, the total drop is 16.5 %. One square in either direction represents 2 % of the full load voltage.

The reactive voltage, as a % of the secondary voltage, can be calculated with fair accuracy from the dimensions of the transformer by the following formulæ based on Kapp's figures.*

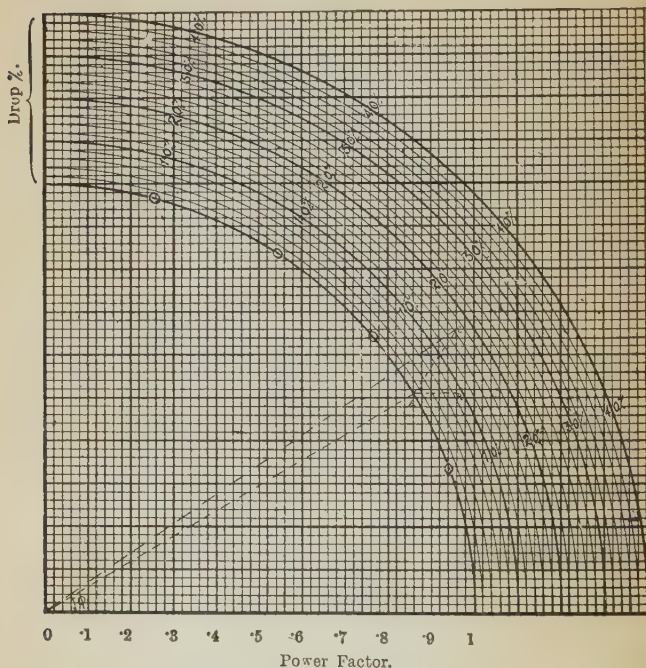


FIG. 119.

$$\left. \begin{array}{l} \text{Reactive drop as a percentage} \\ \text{of the open circuit voltage} \end{array} \right\} = \frac{xp}{NLK^2} (3KS + T)C.$$

Where x = Total ampere turns per limb of the transformer (primary and secondary added together),

p = Mean length of one turn of winding (average of primary and secondary) in inches,

N = No load flux in C.G.S. lines,

* *E. T. Z.*, April 14, 1898, p. 244.

$$= \frac{\text{Volts} \times 10^8}{4.44 \times \text{turns} \times \sim \text{per second}}$$

L = Length of winding space from iron to iron measured parallel to the insulation separating primary and secondary coils, in inches (see Figs. 120 to 123).

K = Number of separate primary or secondary coils per limb (if one winding has one coil more than the other use the larger number). $K = 1$ in Figs. 120 to 123.

S = Thickness of insulation (copper to copper) between each pair of primary and secondary coils, in inches (see Figs. 120 to 123).

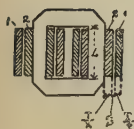


FIG. 120.



FIG. 121.

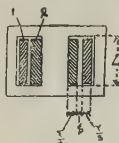


FIG. 122.

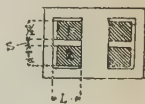


FIG. 123.

T = Depth of winding (primary + secondary) on one limb measured at right angles to the insulation between primary and secondary coils (see Figs. 120 to 123).

C = A constant = 42 for core type transformers (Figs. 120 and 121).
= 55 for shell type transformers (Figs. 122 and 123).

As the full load and no load secondary voltages usually differ by only a slight amount it is sufficiently accurate to employ the reactance voltage as a percentage value of the no load secondary voltage, obtained as above, in connection with Fig. 119.

4. The primary **power factor** of a transformer is the ratio—

$$\frac{\text{True input watts (by wattmeter),}}{\text{Input volts} \times \text{Input amperes}}$$

and its value at any given input (with non-inductive secondary load) is—

$$\frac{\alpha \times V}{A \times V}$$

where A = Input amperes,

V = Input volts (terminals),

and α is got from the expression—

$$A = \sqrt{\alpha^2 + (\text{magnetizing amperes})^2},$$

where the magnetizing amperes are obtained as on p. 310, and v is got from—

$$V = \sqrt{v^2 + (\text{Reactive lost volts due to leakage})^2},$$

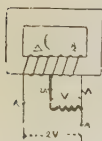
the reactive lost volts being those mentioned on p. 312.

For modern closed magnetic circuit transformers the power factor on non-inductive secondary load is practically unity at quarter load and upwards—the input volts and amperes being very nearly in phase. At no load the figure is about .7.

TRANSFORMERS AND ALLIED APPARATUS FOR SPECIAL PURPOSES.

1. "Auto"-transformers.*

These are often used in cases where the ratio of the two voltages (primary and secondary) is not great—say not more than 4 : 1. In such cases they are much more economical than the ordinary transformer, both in first cost and in efficiency.



Their general design is the same as for ordinary transformers, but the primary and secondary windings, instead of being completely separated from one another by insulation, are connected up to form a single winding, with a connection taken off at the junction of the two (see Fig. 124, which shows diagrammatically the connections and the approximate distribution of the currents in an auto-transformer, transforming from $2V$ volts and A amperes to V volts and $2A$ amperes).

FIG. 124.

Theoretically, if an ordinary transformer is reworked as an auto-transformer for the same voltages with the same core, the same weight of wire (though of different section and with a different number of turns) and the same total losses (C^2R and hysteresis) then—

$$\begin{array}{l} \text{Output} \\ \text{obtainable} \\ \text{as an auto-} \\ \text{transformer} \end{array} = \begin{array}{l} \text{Output} \\ \text{obtainable as} \\ \text{an ordinary} \\ \text{transformer} \end{array} \times \frac{\begin{array}{l} \text{The larger of the two voltages} \\ \text{(primary or secondary)} \end{array}}{\begin{array}{l} \text{The difference between} \\ \text{primary and secondary volts} \end{array}}$$

and in practice an even greater output is obtainable from the auto-transformer, because of the space saved by the absence of insulation between primary and secondary. The chief use of auto-transformers is for starting up alternate current-induction motors—especially those fitted with squirrel-cage rotors—by reducing the motor terminal volts in the ratio of 2 or 3 to 1

* For an examination of the theory of auto-transformers, see *Zeitschrift für Electrotechnik*, September 18, 1904, and September 25, 1904.

for about half-a-minute whilst the motor runs up to speed. The temporary nature of this work makes it possible to neglect the question of heat emission from the surface of the transformer, and to deal only with the heat absorption capacity of the material of the transformer. As has been shown by R. Goldschmidt,* it is safe under these circumstances to work the iron at flux densities approaching saturation (15000–17000 C.G.S. lines per sq. cm.) and the copper at unusually high current densities (7000 to 10000 amps. per sq. inch). Auto transformers are also used a good deal for arc lighting and for supplying low voltage metal filament lamps from 100 volt or 200 volt supply circuits.

2. **Variable ratio pressure regulating transformers** are often used in alternate current central stations, for separately regulating the pressure at the ends of different feeders connected to the same bus-bars. Fig. 125 shows diagrammatically one of the forms in use (that devised by Messrs. Cowan and Still).

The central part *P* can be turned through an angle of 180° by means of a handle, and it is wound with a primary winding connected across the bus-bars. The stationary secondary, *S*, is wound to give the few additional volts required by the feeder with which it is connected in series. When *P* is placed vertically (along *AB*) the secondary is giving its maximum voltage. If *P* is turned through an angle of 90° , the two halves of each section of the secondary winding will neutralize one another, and so give no volts, and if still further turned through another 90° , the secondary voltage will have reached a maximum value again, but will now be in opposition to the bus-bar voltage, so that the resultant feeder voltage is less than the bus-bar voltage.

3. **Stationary phase transformers.**—The only transformer of this nature in common use is one devised by Mr. C. F. Scott† for transforming two-phase currents into three-phase ones and *vice versa*. It consists (see Fig. 126) of an ordinary two-phase transformer, or two separate single-phase transformers. The primaries (say) of

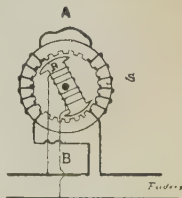


FIG. 125.

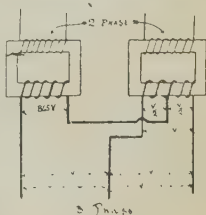


FIG. 126.

* *Journal I. E. E.*, vol. xxxiv., 1905, p. 662.

† *Electrical Review*, April 27, 1894.

these are connected to the two-phase mains. The secondary of one of the transformers is wound to give a voltage equal to the required three-phase line-to-line voltage, and is divided into two equal parts by a middle tapping, connected to one end of the secondary of the other transformer.

This latter transformer's secondary is wound to give $\frac{\sqrt{3}}{2}$ times the three-phase line-to-line voltage. The current in all the secondary wires is equal to the three-phase line current.

Besides the above there are various ways of converting single-phase currents into two-phase or three-phase ones by the use of condensers and choking coils, but none of them are in practical use. A method has also been suggested by Prof. Guido Grassi * of converting three-phase currents into single-phase (this is not a reversible method, and the three-phase load is decidedly unbalanced). An ordinary three-limbed transformer is employed, with a primary and secondary on each limb. All three secondaries are connected in series, and give out a single-phase current when the three primaries are connected to the three-phase mains. The small demand for this sort of transformation has, however, prevented the development of apparatus for the purpose.

4. **Phase shifting transformers.**—Dr. Drysdale † has developed a form of transformer for supplying a voltage at any desired angle of lag or lead to an existing (multiphase) supply voltage. It is the same, in principle, as an ordinary induction motor, but the rotor is only arranged to move through an angle of about 180° by means of a hand wheel. The stator is wound for the available supply, and produces a rotary field in the ordinary way. The rotor may be wound for 1, 2, or 3 phases, and its windings are cut by the rotary field and have voltages induced in them whose *amount* is practically alike for all portions of the rotor, but whose *phase relations* to the stator voltages depend directly on the relative positions of the stator and rotor windings and can be read off on a scale.

5. **Choking, impedance or reactance coils,** as they are variously called, are used largely in alternate current work for pressure regulation. The usual form they take is that of a straight central core of iron wire, or iron stampings, round which (on a wooden or insulation former) a coil of wire is wound, and so arranged that the iron coil can be slipped in or out of the coil so as to vary the number of volts "choked" at will. Sometimes, however, the coil forms a closed or nearly closed magnetic circuit, or one which can be varied from a

* *L'Industrie Electrique*, Jan. 10, 1898, pp. 9, 10.

† *Electrician*, Dec. 11, 1908.

closed iron circuit to a partly air and partly iron circuit, and finally to a complete air circuit.

The current through a choking coil lags approximately 90° in phase behind the voltage at its terminals (unless the C^2R and iron losses are excessive), and on this account the volts choked cannot be directly subtracted from the supply voltage, in order to obtain the resultant effective voltage, but must be subtracted vectorially. Thus if the choking coil is inserted in a non-inductive circuit—

$$\text{Effective remaining voltage} = \sqrt{(\text{Supply volts})^2 - (\text{Volts across choking coil})^2}$$

If the circuit is inductive, having a power factor $= \cos \phi$, the necessary choked volts to give a certain remaining effective voltage, V , can be obtained by the use of Fig. 113. Make AE equal to V . From E draw a horizontal line ER equal to the ohmic lost volts in the choking coil. From R draw a vertical line.* Find the point ϵ on this vertical at which a circle drawn with centre A and radius equal to the total supply voltage cuts it. Then $E\epsilon$ is the voltage required across the choking coil.

The design of a choking coil to allow a given current to pass with a given number of volts across its terminals at a given periodicity is best carried out by a trial and error process to obtain the most economical results in material and in power wasted in the coil itself.

The use of a closed circuit core is inefficient, for, owing to the relation existing between the hysteretic loss and the magnetizing force in normal iron at ordinary flux densities, it is usually impossible to obtain a power factor lower than about .5 to .6 in a choking coil of this type. Since the effectiveness of a choking coil may be said to vary inversely as the power factor, or ratio of true watts absorbed in the coil and core to the apparent watts or product of amps. through coil by volts across coil, it will usually be found advantageous to insert at least a small air-gap in the iron circuit, as by this means the apparent watts can be increased enormously without influencing the true watts appreciably and the power factor can easily be reduced to .05 or less.

In order to calculate the dimensions and winding of an *open circuit* or *partially open circuit choking coil* for a given voltage at a given current and periodicity, the following method may be employed—

- (1) Select a section for the iron core (tentatively).
- (2) Determine the total flux (maximum value) that this core

* Actually ER should represent both the ohmic loss and the iron loss in the choking coil, but as both these are usually negligibly small they can be left out, and the vertical can be drawn directly from E without any appreciable error.

will carry safely, using the same flux densities as for transformers (p. 306).

- (3) Determine the number of turns required in the coil by the ordinary transformer formula (neglecting CR lost volts), and assuming that the whole flux is looped with all the turns. Volts required across coil =

$$\text{Total flux (maximum value)} \times 4.44 \times 10^8 \times \text{total turns} \times \sim.$$

- (4) Having found the turns, and knowing the amperes which the coil is to pass, the two multiplied together and by $\sqrt{2}$ give the total ampere turns (maximum instantaneous value). These ampere turns will practically all be used in forcing the flux across the air-gap between the two ends of the core (at the low flux densities employed in the iron, the ampere turns used in the core itself may be neglected).



If the core is arranged so as to give a well-defined air-gap (as in Fig. 127), the necessary area and length of gap, to suit given ampere turns and flux, are easily calculated by the ordinary formula thus—

FIG. 127.

$$\begin{array}{l} \text{Ampere turns} \\ \text{for air-gap} \\ \text{(maximum} \\ \text{value)} \end{array} = .313 \times \begin{array}{l} \text{Length of} \\ \text{air-gap in} \\ \text{inches} \end{array} \times \begin{array}{l} \text{Total maximum} \\ \text{flux (C.G.S. lines).} \\ \text{Area (effective) of} \\ \text{air-gap (sq. inches)} \end{array}$$

Of course, both the length and area of air-gap being merely tentatively fixed upon, this formula cannot definitely settle either, but, by assuming a convenient value for the one, the other can be obtained, and a few repetitions of the process with different sections of iron core will show whether the most economical core has been selected.

If, however, as is often the case, the core is a simple straight one with an indefinite air-path connecting the two ends, the following empirical formula (arrived at by a method described by Mr. Evershed)* will be found to give good results.

In this type of coil experiment shows that, when the core is placed centrally in the coil, the number of volts choked for a given current and number of turns is almost independent of the *length* of the coil (L), but depends chiefly on the total area of iron core exposed at the two ends beyond the coil. On the other hand, when the iron core is entirely withdrawn from the coil, the volts choked at a given current and with a given number of turns, depend greatly both on the length of the coil and on the section of the hole through it. If, therefore, a choking coil with a large range is required (*i. e.* one giving a maximum range of volts choked at a given current, by varying

* *Electrician*, March 6, 1891, p. 534, and April 17, 1891, p. 724.

the position of the core), the coil should be made as long as possible, with as small a hole through the centre as possible, and with an iron core that projects decidedly (say to a distance of $\frac{1}{2}L$ at each end) beyond the coil, when placed centrally in it.

Calling the length of coil L inches,

„ the length of coil not occupied by the core (when the core is partly withdrawn) l inches,

„ the surface of that part of the iron core which projects beyond the coil at *each* end when placed centrally (or the average of the surfaces projecting at the two ends if not placed centrally) } A square inches.

NOTE.—If the core-end is flush with the coil-end, A will simply be the area of the end face of core.

The average transverse section of the coil (including the central hole) to the centre of the winding depth } a square inches.

Then—

$$\frac{\text{Total ampere turns required}}{(\sqrt{\text{M.S. value}})} = \frac{\text{Total flux in C.G.S. lines}}{\text{Total flux in C.G.S. lines}} \times \left(\frac{.222 l}{a} + \frac{.125}{\sqrt{A}} \right)^*$$

where the “Total flux” is the maximum instantaneous value of the flux worked by the ordinary transformer formula on the assumption that the whole of this flux is looped with all the turns of the coil.

When the whole length of the central hole is occupied by the core, the first term in the bracket (which is proportional to the magnetic resistance of the air-path *inside* the coil) is zero.

When the core is partly withdrawn from the coil, the value of $\sqrt{A}$ to be used in the second term (which is proportional to the magnetic resistance of the *external* air-path) will be =

$$\sqrt{\frac{\text{Area of the projecting iron at the one end} + a}{2}}$$

When the core is entirely removed $\sqrt{A} = \sqrt{a}$.

6. **Synchronizers** for paralleling alternators (see pp. 300, 434) are usually of the form shown diagrammatically in Fig. 128. PP_1 are two primary windings, each designed to work at the full voltage and periodicity of one of the alternators to be paralleled, and S is a small secondary winding designed to give just enough volts to fully light the lamp when both primaries

* The number of ampere turns given by the above formula has been found to check very closely with the experimental results for the case when the core is entirely withdrawn, but will as a rule be found rather higher than the true value when the core is placed centrally.

are acting in unison (*i. e.* both tending to send flux through the central limb of the core in one direction at the same instant). Under these circumstances the lamp will remain dark so long as the two alternators are far from being in phase, and as they get nearer into step, it will flicker more and more slowly until, when they are in exact synchronism, the lamp will burn continuously, and as soon as that is seen to occur, the correct terminals of the two alternators must be immediately connected together, and the alternators will then continue to run in parallel and divide the load between them. The design of a synchronizer is carried out exactly as for a transformer, but, on account of the temporary nature of its work, the current and flux densities can be pressed up much higher. Another arrangement (stroboscopic method) for synchronizing alternators sometimes adopted in central stations is to illuminate the station

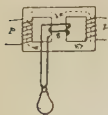


FIG. 128.

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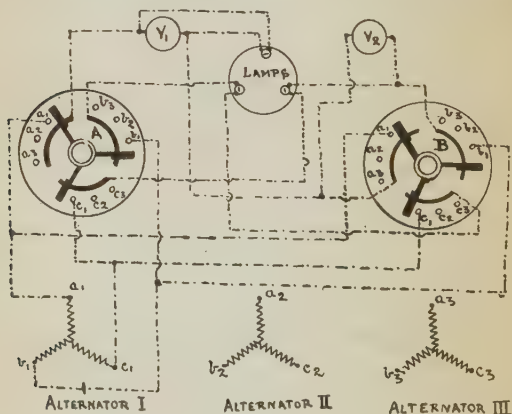


FIG. 129.

by alternate current arc lamps run off the bus-bars (through transformers if necessary). If now the rotating part of each alternator is marked with a number of conspicuous white strokes, equal to the number of poles on the machine, and placed in a circle, any one observing these strokes by the light of the arcs will find that they appear stationary if the alternator is in phase with the bus-bar voltage, but appear to rotate slowly

in one direction or the other, according to whether the speed is too low or too high, if synchronism has not been reached.

For polyphase work exactly the same synchronizer can be used as for single phase, the connections being made to only one phase on each alternator. Sometimes, however, synchronizers of the revolving type are employed with polyphase currents. These indicate whether the incoming alternator is running fast or slow, as well as the actual moment of synchronism. The three-phase revolving light synchronizer devised by Messrs. Siemens and Halske (see *Electrical Review*, February 24, 1899, p. 319) is shown diagrammatically in Fig. 129. The particular one shown is arranged for synchronizing three alternators, but the connection to only one of the alternators is shown in full. In order to synchronize alternators I and II, switch *B* is placed on $a_2b_2c_2$. The alternator voltages are

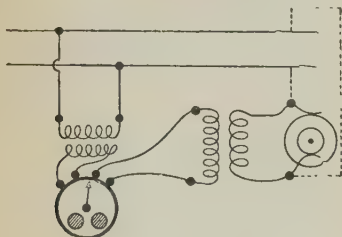


FIG. 130.

then made equal, and switch *A* is placed on $a_1b_1c_1$. The three lamps will then light up in rotation, and the order in which they light up—whether right- or left-handedly—indicates whether the incoming machine is running fast or slow. As synchronism is approached the speed at which the illumination rotates falls, until, finally, the lamps remain permanently alight. The voltmeters V_1 and V_2 are simply to increase the accuracy with which the actual moment of synchronism can be determined. A disadvantage of this system is that for a high-tension circuit a transformer is required for each phase, both of the bus-bars and of the incoming machine. This, with high voltages such as are now becoming usual, adds greatly to the cost of the arrangement, to say nothing of the complication.

A simpler synchronizer* with the same advantages is made by Messrs. Everett, Edgecumbe and Co., and is in extensive

* See *Electrical Review*, June 27, 1902.

use. It is based on the fact that if the stator and rotor of a two-phase induction motor be fed from two sources at the same frequency the rotor will stand still, and if the frequencies differ, the direction of rotation will indicate which of the two is the higher.

A lag of approximately 90° is obtained in the instrument itself by means of a choking coil, so that the device can be used for single-phase circuits also. Fig. 130 shows the connections for a single-phase circuit—for use on polyphase circuits the synchronizer is still only connected to one phase in exactly the same way.

The instrument as usually constructed is fitted in a round case having a dial some 12 inches in diameter, which is often illuminated. The exact moment of synchronism is shown by the pointer coming to rest over a black mark on the dial, and can be determined with absolute accuracy. In order that the direction of rotation may be seen at a distance, *e. g.* at the engine, a red or green lamp is exposed to view according to whether the incoming machine is running too fast or too slow. For high-tension circuits step-down transformers are used.

ALTERNATE CURRENT MOTORS.

Alternate current motors may be broadly divided into two classes—**synchronous** and **non-synchronous** motors—the latter largely predominating in practice.

Synchronous motors are distinguished by having field magnets of unchanging polarity, separately excited either by direct current or by rectified alternate (pulsating) current. The design of these motors is exactly the same as that of alternators, and the rules given on pp. 280 to 301 can be used without alteration. All alternators (single or polyphase) can be used as synchronous motors.

A single-phase synchronous motor has no tendency, and a polyphase machine only a very slight tendency, to start from rest; nor do they give any appreciable torque, except when running at exact synchronous speed (synchronous speed in revolutions per minute = $\frac{60 \times \infty}{\text{number of N (or S) poles in one plane}}$).

On this account they have to be started and brought up to speed by some external means, and so are not at all suitable for work requiring frequent stopping and starting of the motor. If over loaded beyond a certain point they drop out of synchronism and stop.

Since, then, these motors run at one definite speed, irrespective of the supply voltage and of the excitation, it is evident that the balance between supply E.M.F. and armature back E.M.F. must be maintained in a different way from that occurring in continuous current motors. The balance is, in fact, produced by armature reaction, the armature current changing its phase in relation to the back E.M.F. in such a way as either to magnetize or to demagnetize the field, according to the requirements of the case. If a synchronous motor is excited to such an extent that the vectorial sum of the armature back E.M.F. generated at synchronous speed, the ohmic lost voltage and the reactive drop is equal to the terminal supply voltage, the input volts and amperes will be practically in phase with one another and in direct opposition (180°) to the back E.M.F. If, however, the excitation is reduced *below* the above value the armature current (for a given load) will increase and will *lag* (to a greater or less extent) behind the terminal voltage (*i. e.* it will be *ahead* of the armature back E.M.F. and therefore in a position to strengthen the field to the extent required to maintain the balance)—the motor acting, in fact, as an inductive resistance or choking coil in the circuit. If the excitation is raised *above* the normal, the armature current also increases, but it is now a *leading* current (relatively to the supply E.M.F.)—the motor behaving like a condenser in the circuit. An over-excited synchronous motor is, for this reason, sometimes used on supply circuits to counteract the inductive effects of long mains or of lightly loaded transformers, and thus to keep the power factor of the whole circuit approximately equal to 1.

Non-synchronous or asynchronous motors may be subdivided into two classes—

- (1) **Induction motors** (also called rotary field motors, Drehstrom motors or Tesla motors) and
- (2) **Alternate current commutating* motors**, of which several types have been recently developed in connection with single phase alternate current traction schemes.

(1) **Induction motors** consist of a stationary laminated iron structure called the “**stator**,” and a rotating part (generally also laminated) called the “**rotor**.” The stator is slotted on its inner periphery and wound with a number of coils, arranged similarly to an alternator armature and connected directly to the single or polyphase supply mains. The rotor is slotted round its outer periphery and wound with a number of coils, either (1) short circuited on themselves, or (2) arranged as an ordinary two- or three-phase armature winding and connected to slip rings, according to the work for which the motor is intended.

The **rotor winding** is quite independent of the number of phases on which the stator is intended to work. If a short-circuited rotor is used, the winding may either consist of a few thick copper rods passed through the slots and short circuited

* Niethammer, *Schweizerische Elektrotechnische Zeitschrift*, No. 17 et seq., 1904; Steinmetz, *The World and Engineer*, February 6, 1904; Eichberg, *Elektrotechnische Zeitschrift*, January 28, 1904; translated in *Electrical Engineer*, February 26, 1904; Pichelmeyer, *Elektrotechnische Zeitschrift*, June 3, 1904; *Electrical Engineer*, July 29, 1904; M. Osnos, *E. T. Z.*, January 7, 1904; F. Punga, *Single-Phase Commutator Motors*; Görges, *E. T. Z.*, July 25, 1907; R. Goldschmidt, *Alternating Current Commutator Motors*, 1909.

at the ends by copper or brass rings (called a "squirrel-cage" winding), or it may consist of a number of separate coils each short circuited on itself. If a "wound rotor" is used, the commonest arrangement is a three-phase star coupling connected to three slip (collector) rings. The slip rings are only used when starting, so as to allow a starting resistance to be placed in the rotor circuit. When running at full speed the rings are invariably short circuited. With "short-circuited" rotors the starting currents (except for quite small motors) become excessive, so that sizes above 8 or 10 B.H.P. generally have "wound" rotors.

A modified form of induction motor has been patented by Heyland.* The stator is of the ordinary type, but the rotor is fitted with a number of stationary brushes rubbing either on the end rings of the short-circuited winding (if of the squirrel-cage type) or on a special commutator having few segments (if of the wound type). The number of brushes depends on the number of phases for which the motor is designed. Tappings are made in the windings of the different stator phases (one tapping per phase) at points giving a certain small percentage of the supply voltage, and these tappings are connected to the brushes.

In this way all the magnetizing current required by the motor is conveyed from the supply mains direct into the rotor windings, and is there, owing to the nearly synchronous rotation of the rotor, converted into a very slowly alternating current (its periodicity being equal to the "slip" of the motor, or 4 % or 5 % of the supply periodicity). At this low periodicity the choking effect of the current is naturally very small, and the volts required to produce the necessary magnetizing current are very little more than those required to overcome the ohmic resistance of the rotor winding, and the apparent watts are only slightly greater than the true watts. In this way the power factor of the motor can be made practically equal to unity at all loads. This same type of machine can be successfully employed as a self-exciting induction alternator.

Leaving, for the present, single-phase induction motors as being more complicated, the design of a polyphase (two- or three-phase) induction motor can be carried out as follows—

Given the H.P., the voltage, the speed and $\sim$ per second,

(1) Decide on the approximate **diameter and length of the rotor** from the expression—

$$\left(\begin{array}{c} \text{Diameter of rotor} \\ \text{in inches} \end{array} \right)^2 \times \begin{array}{c} \text{Length of} \\ \text{rotor-core} \\ \text{in inches} \end{array} =$$

$$\frac{\text{Output of motor in watts}}{\text{Revolutions per minute}} \times C,$$

* *E. T. Z.*, August 8, 1901, p. 623, and January 9, 1902, p. 28; and *Electrician*, August 16, 1901, p. 623, and January 24, 1902, p. 530.

where C has the following approximate values—

	<i>Small Motors</i>	<i>Large Motors</i>
At 25 ~	100	60
50 ~	125	75
100 ~	170	100

Knowing the (diameter)² × length, the diameter can be selected to give a **peripheral speed** of 3000 to 5000 feet per minute, and the length will then also be known approximately.

(2) Decide the **number of poles** in the stator—

Total number of poles required (North and South) = $\frac{\sim \text{per second} \times 120}{\text{Revolutions per minute at no load}}$

Each of these poles has a span = $\frac{\text{Stator periphery}}{\text{Number of poles}}$ with no actual gaps between adjacent poles.

The speed of an induction motor when loaded is always less than at no load by a small amount (4 % or 5 %), which is known as the “slip” of the motor. The amount of the slip depends almost entirely on the C^2R (ohmic) loss in the rotor windings.

(3) Decide the **flux density** in the air-gap.

Usual flux densities for 40 to 60 period motors are given below

Position.	AVERAGE VALUE OF THE FLUX DENSITY.	
	C.G.S. lines per sq. inch.	C.G.S. lines per sq. cm.
Air-gap	19,500*	3,000*
Stator teeth	52,000*	8,000*
Rest of stator iron	42,000	6,500
Rotor teeth	55,000*	8,500*
Rest of rotor iron.....	65,000	10,000

The **total air-gap flux per pole** is then—

$$= \text{Average flux density} \times \text{Area of each pole face,}$$

$$= \text{,, ,, ,,} \times \frac{\text{Length of rotor in inches}}{\text{pole pitch in inches.}}$$

$$\text{The pole pitch is simply } \frac{\pi \times \text{Inside diameter of stator}}{\text{No. of poles}}$$

* Owing to the shape of the curve of flux distribution over the pole pitch, as explained on p. 329, the maximum values of the flux densities in the air-gap and the teeth are, in a two-phase motor, 1.87 times, and in a three-phase motor, 1.7 times the average values.

(4) Knowing the total flux per pole, the **number of stator conductors** can be found—

$$\left. \begin{array}{l} \text{Total number of stator} \\ \text{conductors per phase} \\ \text{(in series)} \end{array} \right\} = \frac{\text{E.M.F. across each phase} \times 10}{\frac{\text{Total flux per pole}}{(\text{C.G.S.})} \times \frac{\sim}{\text{per second}} \times K}$$

For E.M.F. across each phase of winding, see p. 289.

K is a variable multiplier and is equal to the figure 4.44 (explained in connection with transformers on p. 306) divided by 2 (to allow for the fact that conductors instead of turns are being dealt with in this case) and multiplied by the "breadth coefficient" or ratio of effective flux looped with all the turns per coil to the total flux leaving a pole pitch. When the number of slots per pole per phase is not under 3 the value of this breadth coefficient is .9 for two-phase motors, and .95 for three-phase motors,* so that K can be taken as equal to 2 for all two-phase motors and 2.1 for all three-phase motors.

As a check on the number of stator conductors found as above, the ampere wires per inch of periphery should be worked out—

$$\text{Ampere wires per 1 inch} = \frac{\text{Stator conductors per phase} \times \text{Number of phases} \times \text{Full load current in each conductor}}{\text{Diameter of stator (inside) in inches} \times \pi},$$

and should have a value of about

250 to 300 up to a 12"	diameter stator
400 to 450 from a 12" to a 20"	" "
500 to 550 from a 20" to a 40"	" "

The "full load current in each conductor" can, for this calculation, be assumed to be—

$$\frac{\text{Full load output of motor in watts}}{\text{Voltage across,}} \\ .7 \times \text{Number of phases} \times \text{each phase of the winding}$$

.7 being a fair average value for the product of full load efficiency by full load power factor.

(5) Make the **size of the stator wire** such that the current density at full load is 1500 to 2000 amperes per sq. inch (using the approximate current found as in (4))—the higher figure referring to small motors.

(6) Determine **the size and number of slots in stator** to hold the required number of conductors of the required size of wire. It is usual to put from two to five holes per pole per phase, the larger number being the better. The wire is

* See Heubach, *Der Drehstrom Motor*, p. 181.

commonly threaded through by hand from the end of the core, though occasionally larger motors are built with open slots of the form shown in (b) Fig. 73 and with former-wound coils. The slots or holes are generally rectangular in form, the depth being about three times the width. They must be placed extremely close to the edge of the stator discs, and the narrow bridge of iron left across the mouth of each slot should preferably be cut through so as to reduce the leakage. The teeth should be proportioned so as to give the flux densities shown on p. 325.

(7) The **rotor winding** can be arranged in a large number of different ways without materially affecting the action, the only essential requirement being a number of closed circuits with their effective parts approximately parallel to the length of the core. The total number of rotor slots is preferably made larger than the number of stator slots in the following proportions—

No. of stator slots per pole per phase	2	3	4	5	6	8
No. of rotor slots per pole per phase	3	4 or 5	5 or 6	6 or 7	8 or 10	10 or 12

If the number of rotor slots is equal to the number of stator slots there is likelihood of "dead points" occurring when starting.

As stated above, there are two distinct **types of rotor winding**: the "**short-circuited**" (or "**squirrel-cage**")* and the "**wound**" **rotor**. The former is more especially used for small motors (up to 10 B.H.P.), and in all cases where collector rings are not desirable. The latter usually consists of a regular three-phase or two-phase armature winding such as shown in Figs. 100 to 104, the ends being connected to three or four slip rings respectively.

The actual number of rotor conductors is of no great importance; the more conductors there are, however, the less will the current in each be for a given power, as shown in (b), p. 334. The current density in the rotor at full load (using the current as found in (b), p. 334), may be 3000 to 4000 amperes per sq. inch in the squirrel-cage type, and 2000 to 2500 in the "wound" type.

(8) The **stator current** at no load can now be obtained.

The stator current at no load, per phase =

$$\sqrt{\left(\frac{\text{Magnetizing current per phase (wattless)}}{}\right)^2 + \left(\frac{\text{Power current per phase required for the hysteresis, eddy current and friction losses at no load}}{}\right)^2}$$

* Oerlikon for small sizes use a rotor wound with bare wire in place of the "squirrel-cage."

The "power current" at no load = $\frac{\text{Total watts lost in motor at no load.}}{\text{No. of phases} \times \text{Volts across each phase of the winding (see p. 289)}}$

The magnetizing current for three-phase motors per phase (C_m) = $\left\{ \begin{array}{l} \text{Average value} \\ \text{of the flux} \\ \text{density in air-} \\ \text{gap in C.G.S.} \\ \text{lines per sq.} \\ \text{inch} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Length} \\ \text{of} \\ \text{double} \\ \text{air-gap} \\ \text{in} \\ \text{inches} \end{array} \right\} \times \frac{\left\{ \begin{array}{l} \text{Number} \\ \text{of poles} \end{array} \right\}}{\left\{ \begin{array}{l} \text{Conductors} \\ \text{in series per} \\ \text{phase on} \\ \text{stator} \end{array} \right\}} \times 19.$

Magnetizing current for two-phase motors per phase (C_m) = " " $\times$ " " $\times$ " " $\times 3$,

where the "average flux density" is the density mentioned on p. 325.

When open slots are employed on stator or rotor, an addition to the actual "length of double air-gap" must be made in order to get the effective length for use in the above expression, in the same way as for direct current machines (see p. 249). With the more usual almost closed slots no such addition is necessary.

An addition of about 10 % should be made to the magnetizing current worked as above to allow for the iron path.

NOTE.—The derivation of this expression for magnetizing current may be briefly explained as follows—*

Consider the case of an ordinary distributed stator winding having three or more slots per pole per phase, and assume that only one phase is working. The total flux leaving each pole at a given instant would not be uniformly distributed over the whole pole pitch, but would have a uniform value over the central part of the coil and would then diminish, slot by slot, to zero at the outside edges of the coil.

The maximum value of the flux density occurs, with regard to time, when the magnetizing current is at a maximum. Neglecting the magnetic resistance of the iron part of the circuit the maximum value of flux density (one phase only working)—

$$\begin{aligned} &\text{Stator turns} \\ &\text{per phase} \\ &= \text{looped with} \\ &\text{each mag-} \\ &\text{netic circuit} \end{aligned} \times \sqrt{\text{M.S. value}} \times \sqrt{2} \times \left\{ \begin{array}{l} \frac{1}{\text{Length} \\ \text{of double} \\ \text{air-gap} \\ \text{in} \\ \text{inches}} \end{array} \right\} \times \frac{1}{313}$$

(see p. 249).

Now with one phase only acting this flux density would fluctuate with regard to time between zero and maximum in

* For a more detailed discussion see Hobart's *Electric Motors*, p. 312-328; Heubach, *Der Drehstrom Motor*, p. 170 and 302; De la Tour, *The Induction Motor* p. 56.

step with the current. When, however, all the phases are working, their inter-action has two effects: (1) It makes the curve of flux density with regard to space more peaked, and so increases the maximum value of the flux density above the value produced by one phase acting alone; and (2) instead of allowing the whole flux per pole to fluctuate, with regard to time, between zero and a maximum, it maintains a practically constant total flux (*i. e.* the flux density averaged over the whole pole pitch remains practically constant as the flux as a whole rotates round the stator, though the maximum value of this flux density varies somewhat from instant to instant).

Thus in the case of a two-phase motor the maximum flux density produced by the two phases is 1.41 times that produced by one phase acting alone, and in a three-phase motor the maximum density produced by the three phases acting together is twice as great as that due to one phase acting alone.

Further, with three or more slots per pole per phase, the shape of the curve of flux density with regard to space is such that the ratio of maximum flux density to average flux density is approximately 1.87 in the case of a two-phase motor, and approximately 1.7 in a three-phase motor.

Introducing these figures into the expression for flux density, and substituting the more convenient "average flux density" for the "maximum flux density," this becomes—

$$\begin{array}{ccccccc} \text{Average} & & \text{Stator} & & & & \\ \text{value of} & & \text{turns} & & & & \\ \text{flux density} & = & \text{per phase} & \times & \sqrt{\text{M.S. value}} & \times & \frac{1}{\text{Length}} \times \frac{1}{.313} \times \sqrt{2} \times \frac{1.41}{1.87} \\ \text{(all phases} & & \text{looped} & & \text{of magnet-} & & \\ \text{acting)} & & \text{with each} & & \text{izing} & & \\ & & \text{magnetic} & & \text{current} & & \text{of double} \\ & & \text{circuit} & & & & \text{air-gap} \\ & & & & & & \text{(inches)} \end{array}$$

for two-phase motors, or—

$$\dots \dots \dots \frac{1}{.313} \times \sqrt{2} \times \frac{2}{1.7}$$

for three-phase motors.

From which the expression for magnetizing current given above can be at once obtained.

In order to reduce the wattless currents in these motors to a minimum, the **clearance air-gaps** between stator and rotor have to be made extremely small.

Kapp's rule is—

$$\text{Single air-gap in inches} = .008 + \left\{ \begin{array}{l} \text{diameter of rotor} \\ \text{in inches, } \times .0025 \end{array} \right\}$$

which works out as .02" for 5" diameters.

.033"	„	10"	„
.059"	„	20"	„
.136"	„	50"	„

(9) Current, Power factor, Horse-power, Torque, Slip and Maximum overload for Polyphase motors.

All these points are readily determined, with fair accuracy, by means of the Heyland diagram shown in Fig. 131.

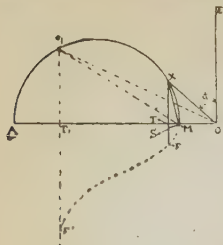


FIG. 131.

Before this can be drawn, however, the value of the **magnetic leakage** between stator and rotor windings must be determined. The flux set up by the stator winding is practically constant in amount and depends simply on the supply voltage, the stator turns and the periodicity, as shown on p. 326. The greater portion of this flux passes into the

rotor windings and is the useful flux of the motor, whilst the rest is leakage flux. As the stator and rotor currents increase with the load on the motor, a greater and greater proportion of the stator flux becomes leakage flux, and less and less flux enters the rotor windings usefully, owing to the increased rotor and stator ampere turns tending to cause leakage. There is, therefore, no constant leakage coefficient in the same sense as in a direct current machine, because, whilst the ampere turns producing leakage increase steadily with the load, the total flux remains practically constant—the increasing stator ampere turns being counterbalanced by the increasing rotor ampere turns. There is, however, a relation, v_1 , between the leakage from the stator winding and the useful flux produced by the stator, **assuming the rotor to be on open circuit**, and simply the magnetizing current to be passing in the stator. In the same way there is a relation, v_2 , between the rotor leakage and the useful flux which would be produced by the rotor if the latter were used for exciting purposes with **the stator on open circuit**. These values are usually about alike, and their amount depends on the slot dimensions, length of air-gap, arrangement of windings, etc., and varies between about .02 and .05. For use in the design of induction motors the two leakages are usually combined into a single coefficient known as the “dispersion coefficient σ ” of the motor.

$$\sigma = \frac{\text{Stator leakage} + \text{Rotor leakage}}{\text{Useful rotor flux}}$$

on the above assumptions as to each member of the motor carrying only a magnetizing current.

The relation existing between v_1 , v_2 , and σ is the following—

Let F = Total (constant) stator flux.

A = "Air-gap flux," *i. e.* that part of F which reaches the air-gap.

f = useful rotor flux.

Then

$$A + v_1 A = F, \text{ and } f + v_2 f = A,$$

$$\therefore f + v_2 f + v_1 f + v_1 v_2 f = F;$$

or

$$f + f(v_1 + v_2 + v_1 v_2) = F;$$

and since

$$\sigma = \frac{F - f}{f}$$

$$\sigma = v_1 + v_2 + v_1 v_2.$$

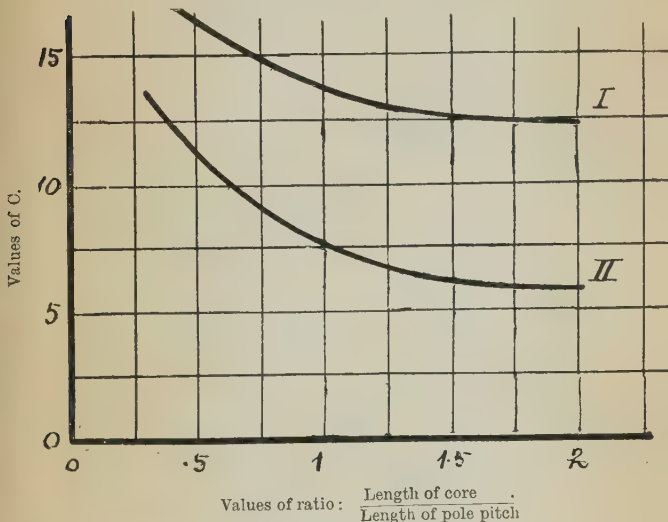


FIG. 132.

A simple way of arriving at the value of the leakage flux in a finished motor consists in taking a short-circuit test on it (see p. 363). If the rotor is short circuited and held at rest, and the normal volts are applied to the stator terminals so that the motor is, what is called, on "short circuit," a heavy current will flow whose value is such that the total leakage flux between stator and rotor is practically equal to the total stator flux, and

only enough volts are induced in the rotor winding to pass the short-circuit rotor current through the ohmic resistance of the rotor winding.

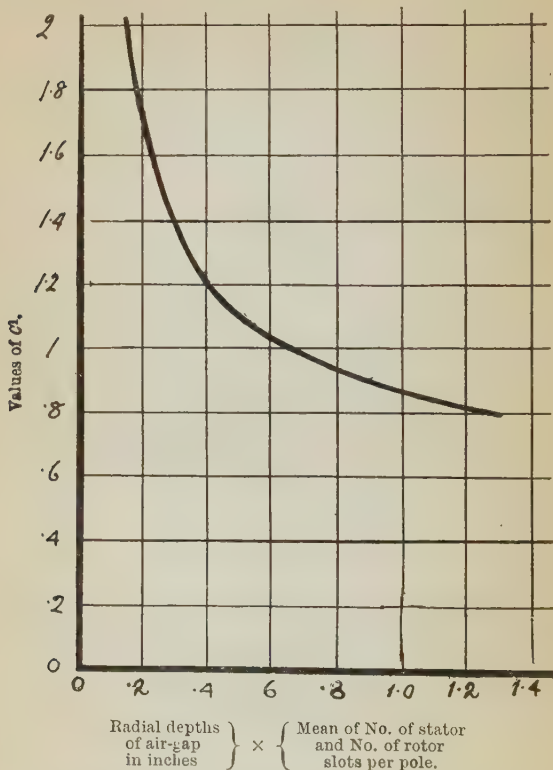


FIG. 133.

Neglecting the rotor resistance, the total leakage flux on short circuit = the total stator flux = $F' = f + \sigma f$, whilst at no load (assuming magnetizing currents in both stator and rotor) the total leakage flux is σf . If the magnetic resistances of both the main magnetic circuit and the leakage circuits are constant

(an allowable assumption so long as saturated iron is absent) the leakage fluxes will be proportional to the currents, and the following relation will hold—

$$\frac{\text{Magnetizing current}}{\text{Theoretical short-circuit current} - \text{Magnetizing current}} = \frac{\sigma f}{(f + \sigma f) - \sigma f} = \sigma.$$

The value of σ usually lies between .04 and .1 for polyphase motors of good modern design, and its value can be estimated with fair accuracy, from the dimensions of the motor, by means of the following expression, due to Hobart—*

$$\sigma = C \times C^1 \times C^{11} \times \frac{\text{Radial depth of single air-gap}}{\text{Pole pitch}}$$

where C is a variable multiplier depending on the ratio of the length of the rotor or stator core (parallel to the shaft) to the pole pitch, and is given in Fig. 132 for various values of this ratio, for both open and closed slots. For the usual partially closed slots a value about midway between the two curves should be used.

C^1 is a second multiplier depending on the product of radial depth of single air-gap by number of slots per pole pitch, and its value is shown in Fig. 133.

C^{11} allows for the smaller inductance of a squirrel-cage rotor as compared with an ordinary wound rotor, and has the value of .75 for the former and 1 for the latter type of rotor.

Whenever possible, of course, σ should be obtained from a test, such as is described on p. 363, made on a motor similar to the one under consideration. Having determined σ either by test or calculation, Fig. 131 can be constructed as follows—Draw the vertical OE to represent (in phase) the stator supply voltage. From O draw OM at right angles to OE to represent the stator magnetizing current per phase (C_m) to some definite scale. Produce OM to A , and make

$$MA = \frac{OM}{\sigma}.$$

Draw a semicircle on MA .

Then it can be shown† that, on the allowable assumptions that the magnetic reluctances of the leakage and main magnetic

* E. T. Z., 1903, p. 934; also Hobart, *Electric Motors*, pp. 386 and 420; *Journal I. E. E.*, vol. xxxiii., April 1904, p. 284. For a detailed investigation of the various causes of leakage, see Behn-Eschenburg, *Journal I. E. E.*, vol. xxxiii., April 1904, p. 392; also R. Goldschmidt, *Electrician*, December 13, 1907.

† S. P. Thompson, *Polyphase Electric Machinery*, p. 225; Heubach, p. 97; De la Tour, p. 123-140.

paths remain constant, and that the rotor and stator windings are resistanceless, the following relations hold good—

(a) All lines such as OX drawn from O to meet the semi-circle represent **stator currents**, per phase to the same scale as that to which OM represents the magnetizing current. The line OA represents (to the same scale) the theoretical short-circuit current which would flow if the windings had no resistance. For the actual short-circuit current see under (f), and also p. 363. Its value is rather less than the theoretical short circuit current.

(b) All lines such as MX , if multiplied by

$$\frac{\text{Total stator conductors (all phases together)}}{\text{Total rotor conductors}} \times \frac{\text{Stator current per conductor}}{\text{Stator current per phase}}$$

represent (to the same scale) the corresponding **rotor-current** in each conductor.

(c) The angle EOX or ϕ for any line OX is the **angle of lag** between stator supply voltage and stator current, and $\cos \phi$ = the **power factor** of the motor at the load corresponding to the current OX . This power factor has its maximum value when OX is a tangent to the semicircle, and this value =

$$\frac{1}{1 + 2\sigma}$$

(d) Vertical lines like XT' drawn from X to the base represent the **torque of the motor** corresponding to the stator current OX . The scale of torque can be determined thus—

Let the stator current represented by OX be C_1 .

Then

$$C_1 \times \frac{\text{Stator supply volts per phase} \times \text{No. of phases} \times \cos \phi}{\text{of winding}} \times \frac{\text{Percentage commercial efficiency of motor}}{100}$$

= the actual power (in watts) obtainable at the motor-

$$\text{pulley ; but torque in pound-feet} = \frac{\text{Watts output}}{\text{Revolutions per minute}} \times 7.$$

Hence

$$\left. \begin{array}{l} \text{Torque} \\ \text{in pound-} \\ \text{feet repre-} \\ \text{sented by} \\ \text{the line} \\ \quad XT' \end{array} \right\} = C_1 \times \frac{\text{Stator volts per phase}}{\text{phase}} \times \text{No. of phases} \times \cos \phi \times \frac{\text{Commer- cial effc.}}{100} \times \frac{7}{\text{Revolutions per minute}}$$

which determines the scale of torque, and the **maximum possible torque** obtainable from the motor is equal to the radius of the semicircle read off to the same scale. This should be at least double the normal full load torque. If called upon to give any larger torque than this the motor will at once stop.

- (e) If a line MS is drawn at right angles to the rotor current line MX till it meets the vertical drawn from X , at S , the length of the line XS represents to some scale the **slip** of the motor for that current. The scale can be found from the fact that the percentage slip

$$= \frac{\left\{ \begin{array}{c} \sim \text{ per second of} \\ \text{supply} \end{array} \right\} - \left\{ \begin{array}{c} \text{Number of poles} \times \text{revolu-} \\ \text{tions of rotor per minute} \\ \hline 120 \end{array} \right\}}{\sim \text{ per second of supply}} \times 100$$

$$= \frac{\text{Total } C^2R \text{ (ohmic) watts lost in the rotor circuits}}{\text{Total } C^2R \text{ watts lost in rotor circuits} + \text{output of motor in watts}} \times 100.$$

- (f) If the slip is worked as in (e) for several points, and the values set off downwards from the base line OA , a curve of slip will be obtained something like the curve MFF^1 shown in the figure. The point F^1 on this curve corresponds to a slip of 100 %, and represents the conditions in the motor at starting. Thus the line OX_1 gives the stator (and MX_1 the rotor) **current per phase at starting** or **short-circuit current** (to the same scales as in (a) and (b)). The line X_1T_1 (to the same scale as in (d)) gives the starting torque. And the angle EOX_1 is the angle of lag at starting.

The effect of the resistances of the windings and the losses in the motor on the accuracy of the Heyland diagram is only small, but can be allowed for when necessary by means of a somewhat more involved procedure. See S. P. Thompson, p. 225 ; De la Tour, pp. 123-140 ; R. Goldschmidt, *Elektrotechnische Zeitschrift*, 1900, No. 33 ; Hugo Grob, *Elektrotechnische Zeitschrift*, 1904, Nos. 22-3.

(10) The **heating of stator and rotor** can now be worked out just as for alternators (see pp. 288-9), that is, by finding the C^2R loss in each winding separately, and calculating the combined hysteresis and eddy current loss in the stator teeth and core by means of the "equivalent volume" method and Fig. 81 as described for direct current machines on p. 244, except that for these induction motors the values read off Fig. 81 should be reduced about 40 % (*i. e.* should be multiplied by '6) for reasons similar to those mentioned on p. 290

under "Alternators." The flux density in the stator (rotor) iron behind the teeth (two-phase or three-phase motors) =

$$\frac{\text{Total air-gap flux (see p. 325)}}{\frac{\text{Radial depth of stator (rotor) discs behind teeth (inches)} \times \frac{\text{Length of stator (rotor) core parallel to shaft less ventilating gaps (inches)}}{\times .9 \times 2.}}$$

And the flux density in the stator (rotor) teeth (two-phase or three-phase motors) =

$$\frac{\text{Total air-gap flux (see p. 325)}}{\frac{\text{Average width of stator (rotor) teeth (inches)} \times \frac{\text{Length of stator (rotor) core parallel to shaft less ventilating gaps (inches)}}{\times .9 \times \frac{\text{No. of stator (rotor) teeth}}{\text{No. of poles}}}} \times 1.6.*$$

There is practically no iron loss in the rotor core, as the periodicity is so low (being in fact merely that due to the "slip," and therefore only 5 % or 6 % of the supply periodicity).

The cooling surface of the stator may be taken as the sum of the areas of inner (bore) periphery, the outer periphery of the stator discs and the two end areas of the stator discs.

The figure obtained for the watts per square inch of stator surface by dividing the total stator C^2R losses + stator core losses by the above cooling surface should not exceed 1 watt per square inch for a 70° F. temperature rise with a well-ventilated protected motor. The watts per square inch of rotor obtained by dividing the rotor C^2R watts† by the rotor surface (worked as for the stator) should not exceed 2 to 2½, and is generally kept below this by efficiency and current density considerations.

Note on Starting Resistances for Induction Motors.

If polyphase induction motors are switched direct on to the supply mains, with the rotor winding short circuited (no matter whether of the "squirrel-cage" or of the "wound" type), the

* This figure of 1.6 is used in place of the ratio of maximum flux to average flux mentioned on p. 325 and p. 329 (viz. 1.87 for two-phase motors and 1.7 for three-phase motors) because only a small proportion of the teeth are ever subjected to that maximum flux density.

† For a simple method of obtaining the total C^2R loss in a squirrel-cage rotor, see Hobart, *Electric Motors*, p. 405.

stator starting current will be excessive. If a starting resistance or a volt-reducing auto-transformer is placed between the mains and the stator, this current rush is reduced, but at the same time the starting torque of the motor is reduced to a still greater extent. This is, however, the only practicable way of starting motors with the squirrel-cage type of rotor. If, however, each phase in the rotor is connected to a slip ring, a far better method consists in placing resistance in the rotor windings, between the rings.

This resistance is gradually cut out as the motor runs up to speed, and the rings are finally short circuited at full speed.

The following is a simple way of finding the correct amount for such a starting resistance. Suppose the rotor to have a three-phase star winding as in Fig. 134.

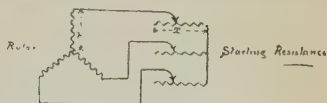


FIG. 134.

Let r = the resistance per phase of motor winding.

x = ,, ,, of the required starting resistance.

c = the full load rotor current per phase found as in (b), p. 334.

Suppose the maximum starting current is not to exceed the full load value (*i. e.* stator starting current = the stator current corresponding to c rotor current).

$\sim_1$ = periodicity of the supply in periods per second

Revs. per minute No. of
 at full load poles.

$$\sim_2 = \frac{\text{Revs. per minute at full load} \times \text{No. of poles}}{120}$$

$\sim_1 - \sim_2$ = slip in periods per second.

Then the volts induced in the rotor windings at starting must equal the resultant of $c(r + x)$ ohmic lost volts and $f \times c \times \sim_1$ inductive back volts (f being some constant).

At normal full load speed the periodicity of the rotor currents is $\sim_1 - \sim_2$, and as the rotor is short circuited, the ohmic loss and the inductive reaction will now be cr and $fc(\sim_1 - \sim_2)$ respectively. Since the total volts induced in the rotor are in each case proportional to the rotor current frequency, and since the part of this voltage absorbed in inductance is also proportional to this frequency, it follows that the part of the voltage absorbed in remittance must be proportional to this frequency, or—

$$\frac{c(r + x)}{cr} = \frac{\sim_1}{\sim_1 - \sim_2}$$

or—

$$x = r \left(\frac{\sim_1}{\sim_1 - \sim_2} - 1 \right).$$

With about a 5 % slip this gives $x = 19r$, and in practice x would be made $= 30$ or $40r$, so as to reduce the starting current (and torque) below the full load values. If the value for x given by the formula is used, the starting torque will be equal to the full load running torque, and this is unnecessarily large for most cases.

The Speed Regulation of Induction Motors.

The speed of an induction motor can be varied in four ways—

- (1) By altering the supply voltage.
- (2) By inserting resistance into the rotor windings (only applicable to motors with slip ring rotors).
- (3) By changing the number of poles formed by the stator windings.
- (4) By connecting the motor in “cascade” (or “tandem” or “concatenation”) with a second induction motor.

Method (1) is only applicable for small speed variations, for as the voltage is reduced though the slip increases approximately as the square of the voltage reduction, yet the maximum torque obtainable from the motor also falls off as the square of the voltage, so that a small reduction is sufficient to prevent even the normal torque being developed.

Method (2) is the one most generally employed, in spite of its inefficiency. As the resistance in the rotor phases is increased the slip necessarily increases in order to allow the current required for a given torque to be induced in the rotor windings. With a given torque the loss in the added resistance is approximately proportional to the speed reduction, so that the efficiency of the method is comparable with that of a direct current motor regulated by the insertion of resistance in series with the armature.

The amount of added resistance per phase required for a given speed reduction at a given torque can be calculated from the fact that—

$$\% \text{ slip} = \frac{\text{Total } C^2R \text{ watts lost in rotor windings + added resistances}}{\left\{ \begin{array}{c} \text{Total } C^2R \text{ watts} \\ \text{lost in rotor} \\ \text{windings + added} \\ \text{resistances} \end{array} \right\} + \left\{ \begin{array}{c} \text{Output of} \\ \text{motor in} \\ \text{watts} \end{array} \right\}} \times 100.$$

See p. 335 (e).

Method (3) complicates the motor windings and switching arrangements, but is fairly commonly employed with motors

intended to run at widely different speeds for long periods, as it is far more economical than method (2).*

The "cascade" arrangement employed for method (4) consists in connecting the stator of one of two motors driving the same spindle to the supply mains and the rotor brushes of this motor to the stator terminals of the second motor, whilst the rotor of this second motor is short circuited in the usual way. Sometimes the rotor and stator of the second motor are interchanged in the above arrangement. Such a combination, if both motors have the same number of poles, runs at half the speed at which either motor alone would run, so that by employing this cascade connection and the ordinary connection (*i. e.* both stators connected to the supply mains and both rotors short circuited) a speed variation of 1 : 2 can be efficiently obtained. The method is employed for traction work with polyphase induction motors.†

Single-Phase Induction Motors.

The general design and calculation of these motors is carried out in the same way as for polyphase machines, but there are one or two important differences. The ordinary type of single-phase induction motor consists of a rotor constructed exactly as for a two- or three-phase machine (see p. 323), and a stator differing from the polyphase stator only in the arrangement of the winding. Such single-phase motors, if the stator is merely wound with a single-phase winding, have no tendency to start from rest, though if given a small initial impulse they will rapidly run up to the normal speed (just short of synchronous speed).

This difficulty is got over in practice by furnishing the stator with an additional "**starting**" winding, in use only when starting, and until a fair speed has been attained. This winding consists of coils of the same span as the main coils, and placed on the stator with their magnetizing axes midway between the axes of two main coils. The starting windings are sometimes wound in the same slots as the main winding and sometimes in separate intermediate slots, as in Fig. 135. With such an additional winding the

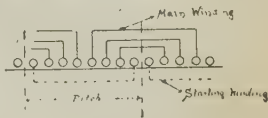


FIG. 135.

* See Belm-Eschenburg, *Electrician*, November 21, 1902; also *Electrical Engineer*, October 7, 1904; and Buckard, *Electrical Engineer*, November 13, 1903.

† See Eborall, *Electrical Review*, January 8, 1904; also Danielson, *Electrical Engineer*, April 1, 1904.

machine can be started up as a two-phase motor, the main winding being connected direct to the supply mains, and the starting winding having a lagging or leading current supplied to it. This lagging or leading current is produced from the ordinary single-phase mains in various ways.

(1) The Maschinen-Fabrik Oerlikon connect the starting winding through a **choking coil**, and the main winding direct to the supply mains.

(2) Messrs. Brown, Boveri and Co. use the main and starting windings in series, with a liquid **condenser** in parallel with the starting winding.

(3) The Langdon Davies motor has considerably more turns in the starting winding than in the main winding. The latter is connected to the mains at starting through a non-inductive **resistance**, and the starting winding is connected direct—the higher inductance of the starting winding giving a lagging current in that branch. Other makers employ modifications of this method, *c. g.* the running and starting windings are sometimes placed in series across the mains at starting, whilst a non-inductive resistance is connected in parallel with the starting winding.

(4) In the Heyland motor no attempt is made to obtain a uniform rotating field. In fact, the starting winding (in which a lagging current is produced owing to its magnetic leakage being greater than that of the running winding) has fewer turns, and is arranged to give a considerably greater flux than the running coil. The two windings are connected, in parallel, direct to the mains. The running coil is divided into sections, which are successively cut into circuit as the motor gets up to speed.

(5) Mono-cyclic motors have special lagging currents supplied to the starting windings by means of a third supply line.

Riccardo Arno uses only one winding, the running winding, and carefully proportions a variable resistance in the rotor circuit so that it is merely necessary to give the motor a slight impulse by hand in order to make it run up to speed. The Allgemeine Elek. Gesell. employ a small "pilot motor" fixed to the main motor spindle and started as in (3).

The *number of ampere turns* is made approximately alike in the starting and the main coils, but since the starting winding is only in use for a few moments the current density in that winding can be made very high without danger, and so only about one-third of the slots need be given up to the starting winding, the remaining two-thirds being occupied by the main winding.

With the following reservations the calculations for a **single-phase motor** can be carried out by the rules 1 to 10 for polyphase motors given on pp. 324–336—

(1) A rotor and stator of a given diameter and length will give about a 40 % or 50 % greater output, as a polyphase, than as a single-phase motor.

(2) The **magnetizing current** of a single-phase motor is much greater than the magnetizing current per phase of a polyphase machine. For, when a polyphase motor is running at full speed and no load, there is practically no current in the rotor winding, and the rotating field is produced by the currents of different phases on the stator acting together. In a single-phase motor, on the other hand, running at no load, there is always a considerable (almost wattless) rotor current—enough, in fact, to produce a flux at right angles to the stator flux, and almost equal in amount to it. The resultant of these two fluxes forms the rotating field. This rotor current has to be counterbalanced by an increased stator current, just as in the case of the primary and secondary of a transformer. On this account the magnetizing current found by means of the expression for two-phase motors given in (8), p. 328, must be multiplied by $\frac{2\sigma + 2}{2\sigma + 1}$ in order to get the true stator magnetizing current (where σ has the meaning explained on p. 330).

This alteration in the magnetizing current requires the diagram in Fig. 131 to be amended as in Fig. 136, before it can be applied to single-phase motors.

In Fig. 136, OM is the stator magnetizing current given by the two-phase formula in (8), p. 327,

$$\text{and } ON = OM \times \frac{2\sigma + 2}{2\sigma + 1} = \text{approximately } 2 \times OM$$

$$MA = \frac{OM}{\sigma}$$

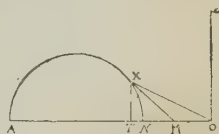


FIG. 136.

Draw the semicircle on NA . Then all such lines as OX represent stator currents, and all lines such as MX rotor currents.

Then the same rules as are given in (9) (pp. 330-6) for polyphase motors, apply to single-phase motors, using Fig. 136 in place of Fig. 131.

By a comparison of the two figures it is easily seen that a given single-phase motor can always be improved as to power factor, no load current, starting torque and maximum torque, by converting it into a polyphase motor.

The following table gives some average figures of the **performances of single and three-phase induction motors** as supplied by English and Continental firms—

SINGLE-PHASE.					THREE-PHASE				
B.H.P.	1	5	20	100	1	5	20	100	
Type of Rotor	Short Circuited	Wound	Wound	Wound	Short Circuited	Wound	Wound	Wound	
Speed { at No Load at Full Load	1500 1400	1500 1420	1000 970	750 725	1500 1400	1500 1420	1000 970	750 725	
Power factor { at No Load at Full Load...	.2 .7	.2 .75	.2 .8	.3 .85	.2 .78	.2 .8	.2 .85	.3 .9	
Commercial Efficiency (Full Load)	65 %	76 %	84 %	91 %	76 %	80 %	89 %	92 %	
Starting Current in terms of the normal Full Load Current	2	$\frac{3}{2}$ -1	$\frac{1}{2}$ - $\frac{3}{4}$	$\frac{1}{2}$	3 to 4	1	1	1	
Starting Torque {	Running Light (Belt on a Loose Pulley)				Normal Full Load Torque				

(2) ALTERNATE CURRENT COMMUTATING MOTORS. (For references see p. 323.)

Owing to the poor performance of the ordinary single-phase induction motor at starting and as regards overloads, together with the many advantages of a single-phase supply for long-distance traction work, a strong inducement exists for the development of a single-phase motor having characteristics more or less similar to those of the direct-current series-wound traction motor. On this account experiments have been carried out on a large scale during the last few years in connection with various types of single-phase motors fitted with commutators. These motors may be broadly divided into two classes, viz. —

- (a) Plain series motors, and
- (b) Repulsion motors.

(a) The *series alternating motor*, shown diagrammatically in Fig. 137, works on exactly the same principle as the direct-current series motor, the full supply current passing through both armature and field winding, and since both the direction of the armature current and the direction of the field flux reverse (owing to the alternations of the current) at the same instant, the torque exerted by the motor is always in one direction, though it varies periodically from zero to a maximum value.

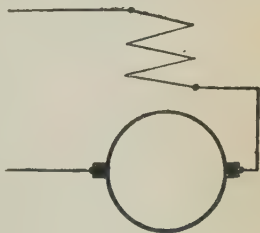


FIG. 137.

Owing to the alternating nature of the flux the magnet iron has, of course, to be thoroughly laminated, but otherwise the machine is very similar in construction to a direct current motor.

Since, for a given flux, the greater the number of field turns the greater will be the inductive back voltage set up in them, the number of field ampere turns must be kept down to a minimum if the power factor of the motor is to be reasonably high. This is accomplished by working with a small air-gap length and a low flux density in the air-gap. For the same

reason (*i. e.* to keep the power factor up) the cross flux set up by the armature ampere turns must be kept down, and this is sometimes accomplished by inserting gaps in the centre of each pole (*e. g.* at AB , A^1B^1 , Fig. 72) or, in what are called "compensated series motors," by arranging an additional opposing winding on the magnets midway between each pair of poles.

This opposing winding may either carry the main supply current, or may be simply a short-circuited winding in which the currents induced by the armature cross flux tend to reduce that flux.

In these A.C. machines the commutation is liable to give trouble, for, besides the ordinary reactance voltage set up by the reversal of the current in the commutated section (see p. 258), a voltage is set up in the section by the transformer action of

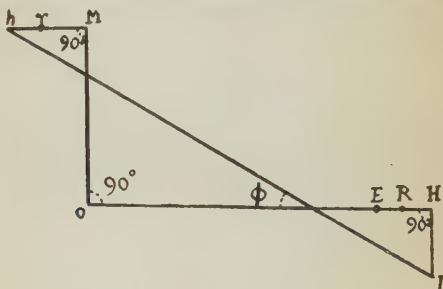


FIG. 138.

the main alternating flux. This latter voltage is especially troublesome at starting, and, in order to prevent it setting up an excessive current in the coils short circuited by the brushes, the connections of the winding to the commutator are usually made of high resistance material. At the same time the reactance voltage has to be kept lower than for a direct current machine.

The approximate vector diagram of a plain series alternating motor is shown in Fig 138, on the assumption that the brushes are set on the neutral line, as would, of course, be the case in a reversible motor.

The horizontal direction represents the phase of the supply current or the flux. OE is the back voltage ($\sqrt{MS}$ value) induced in the armature by its rotation in the main field, and is equal to—

$$\frac{\text{Revs. per minute}}{60} \times \frac{\text{Total conductors on face of armature}}{\text{No. of parallel paths from brush to brush}} \times \frac{\text{Maxm. value of flux per pole in C.G.S. lines}}{10^8} \times \frac{1}{\sqrt{2}}$$

(See p. 246.)

ER is the resistance drop in the armature winding = resistance of armature $\times$ supply current; RH is the voltage drop corresponding to the armature core * and friction losses

$$= \frac{\text{armre. core loss} + \text{friction loss in watts}}{\text{supply current}}$$

HI is the inductive drop in the armature winding, due to magnetic leakage. This is equal to about 5 % of OE if the magnet poles have a slot down the centre, and is practically zero if a compensating winding is fitted.

OM is the back voltage self induced in the magnet winding by the main flux alternating through it =

$$\frac{\text{Maxm. value of flux per pole}}{10^8} \times \frac{\text{periodicity of supply}}{2\pi} \times \frac{\text{Total field winding turns in series (all poles together)}}{4.44} \times \frac{\text{magnet leakage co-efficient}}{1}$$

(See p. 306.)

Mr is the resistance drop in the magnet winding =

$$\frac{\text{Resistance of magnet winding}}{\text{Supply current}}$$

rh is the voltage drop corresponding to the hysteresis and eddy current loss in the magnet iron =

$$\frac{\text{Hysteresis and eddy current watts}}{\text{Supply current}}$$

Then hI is the total supply voltage required at the particular current and speed chosen, and the power factor of the motor is equal to the cosine of the angle between the supply volts and the supply current = $\cos \phi$.

Since the ratio of armature ampere turns per pair of poles to field ampere turns per pair of poles cannot, in an uncompensated machine, be allowed to exceed a certain value (from 1.5 to 2) if the machine is not to have excessive field distortion and is to be able to run sparklessly in both directions with a fixed brush position, it can be shown that, in order to allow a reasonable power factor (from .85 to .95) to be obtained with such a motor, the number of poles employed must, with usual

* For a method of calculating the core loss in these motors see Goldschmidt, *Alternating Current Commutator Motors*, p. 59.

supply periodicities, be large as compared with direct current work. For satisfactory work the armature periodicity ($= \text{Revs. per minute} \times \text{No. of poles} \div 120$) when rotating at the normal full load speed should be from two to three times as great as the supply periodicity. On this account such motors are difficult to wind for higher supply periodicities than about 25.

(b) The simple **repulsion motor** (shown diagrammatically in Fig. 139) consists of a laminated stator, without salient poles, carrying a distributed winding arranged, exactly as for an induction motor, in slots on the inner periphery, and of a rotor fitted with a commutator and wound like an ordinary direct

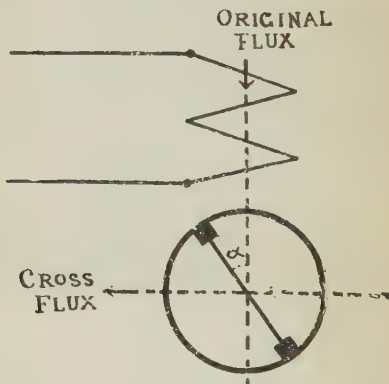


FIG. 139.

current armature. The stator winding is connected to the supply circuit, and the brushes on the commutator are short circuited together and have no electrical connection with the supply circuit.

The brushes are set at a small angle (usually from 15° to 25° assuming that the distance from pole centre to pole centre is called 180°) to the field pole axis.

When a current is passed through the stator winding, an alternating flux (called here the "original flux") is sent through the stator and rotor windings, and sets up an opposing voltage in the stator winding and a current in the rotor winding (which practically forms the secondary of a transformer of which the stator winding forms the primary). This rotor current (whose amount depends on the brush position), acting in

those rotor conductors which lie within twice the angle of brush displacement from the pole axis produces a "cross field" at right angles (in a two-pole machine) to the original stator field. This cross field is the main magnetic flux of the motor, and the torque is principally due to the interaction of this cross flux and the current in the rotor conductors.

In such a motor six voltages may be distinguished—

- (1) The stator supply voltage.
- (2) The back voltage induced in the stator winding by the original flux alternating through it.
- (3) The voltage induced in the rotor turns lying within the angle $180^\circ - 2\alpha$ (in a two-pole machine) by the original flux alternating through them.
- (4) The voltage induced in the rotor turns lying within the angle 2α by their rotation in the original flux.
- (5) The voltage induced in the rotor turns lying within the angle 2α by the cross flux alternating through them.
- (6) The voltage induced in the rotor turns lying within the angle $180^\circ - 2\alpha$ (in a two-pole machine) by their rotation in the cross flux.

Assuming that the distribution of each of the fluxes is sinusoidal, *i. e.* that the curve of voltage induced in a single conductor rotating at uniform speed in either field is of sine form, the values of these different voltages may be stated as follows—

Voltage (1) is equal and opposite to voltage (2) if the stator resistance and leakage drops, which can be kept quite small, are neglected.

Voltage (2) =

$$\begin{array}{ccccccc} \text{Max. value} & & & & \text{Total} & & \\ \text{of original} & & & & \text{stator} & & \\ \text{flux in} & \times & \text{Periodicity} & \times & \text{turns} & \times & \\ \text{C.G.S. lines} & \times & \text{of} & \times & \text{in series} & \times & \\ \text{per pole} & \times & \text{supply} & \times & \text{(taking} & \times & \\ & & & & \text{all poles} & & \\ & & & & \text{together)} & & \\ & & & & & \times & \frac{4.44}{10^8} \times \frac{2}{\pi} \text{ (see p. 306).} \end{array}$$

This voltage lags 90° in phase behind the original flux.

In calculating voltages (2), (3) and (5) it must be remembered that with a sinusoidal flux distribution and a uniform winding the maximum instantaneous value of the flux is only looped with those turns which have their planes exactly at 90° (in a two-pole motor) to the direction of the flux, whilst those turns whose plane is parallel to the direction of the flux are looped with no flux. The *average* flux looped with each turn between these two extreme positions is, in fact—

$$\begin{array}{c} \text{Average} \\ \text{value of} \\ \text{the sine of} \\ \text{angles between} \\ 0^\circ \text{ and } 90^\circ \end{array} \times \begin{array}{c} \text{Max.} \\ \text{flux} \end{array} = \begin{array}{c} \text{Max.} \\ \text{flux} \end{array} \times \frac{2}{\pi}, \text{ hence the presence} \\ \text{of } \frac{2}{\pi} \text{ in the above} \\ \text{expression.} \end{array}$$

Further, if the brushes are displaced from the axis of the flux, as they must be in a repulsion motor, the rotor turns effectively looped with this average flux are reduced in proportion to the cosine of the angle of displacement between the brush axis and the axis of the particular flux dealt with.

∴ voltage (3) =

$$\frac{\text{Max. value of original flux per pole in C.G.S. lines}}{\times \frac{2}{\pi} \times \text{periodicity of supply}} \times \frac{\text{Total conductors on face of rotor}}{2 \times \frac{\text{No. of parallel paths between brushes}}{\times \cos a \times \frac{4.44}{10^8}}}$$

This voltage lags 90° behind the original flux. In the same way voltage (5) =

$$\frac{\text{Maximum value of cross flux per pole in C.G.S. lines}}{\times \frac{2}{\pi} \times \text{Periodicity of supply}} \times \frac{\text{Total conductor on face of rotor}}{2 \times \frac{\text{No. of parallel paths between brushes}}{\times \sin a \times \frac{4.44}{10^8}}}$$

This voltage lags 90° behind the cross flux.

Voltage (4) =

$$\frac{\text{Revs. per minute} \times \text{Total conductors on face of rotor}}{\times \frac{\text{No. of poles}}{\text{No. of parallel paths between brushes}}} \times \sin a \times \frac{\text{Maximum value of original flux per pole}}{\times \frac{1}{6 \times 10^{10} \times \sqrt{2}}}$$

(See p. 246.)

This voltage is in phase with the original flux.

Voltage (6) =

$$\frac{\text{Revs. per minute} \times \text{Total conductors on face of rotor}}{\times \frac{\text{No. of poles}}{\text{No. of parallel paths between brushes}}} \times \cos a \times \frac{\text{Maximum value of cross flux per pole}}{\times \frac{1}{6 \times 10^{10} \times \sqrt{2}}}$$

This voltage is in phase with the cross flux.

In order to draw the approximate vector diagram (Fig. 140) of a repulsion motor at a given supply current and speed, the following process can be employed—

Neglecting the magnetizing current, for the original flux the value of the rotor current corresponding to the given stator current will be such that the rotor magnetizing effect in the direction of the original flux will be equal and opposite to the stator magnetizing effect, *i. e.*

$$\frac{\text{Rotor current} \times \text{Rotor face conductors}}{\text{No. of poles} \times \text{No. of parallel paths between brushes}} \times \cos \alpha = \frac{\text{Stator current}}{\text{turns per pair of poles}}$$

Knowing the rotor turns and the angle of brush displacement, the approximate value of the rotor current can therefore be

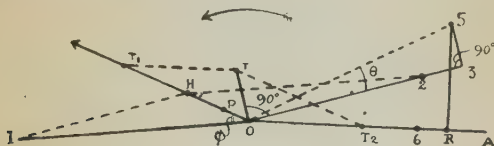


FIG. 140.

obtained. From the rotor current, the rotor turns, the angle of brush displacement and the dimensions of the magnetic circuit the value of the cross flux can be obtained for—

$$\begin{aligned} \text{Rotor current} & \times \sqrt{-} \times \frac{\text{Total conductors on face of rotor}}{\text{No. of poles} \times \text{No. of parallel paths between brushes}} \times \frac{2}{\pi} \times \sin \alpha \\ & = \frac{\text{Maximum value of cross flux per pole}}{\text{Magnetic resistance to the cross flux, per pair of poles}} \end{aligned}$$

where "magnetic resistance" stands for the ratio of
 ampere turns required
 flux produced and is calculated from the air-gap and
 iron dimensions by means of the rules given for direct current
 machines on p. 249.

Knowing the cross flux, the values of voltages (5) and (6) can be calculated. Then draw OA (Fig. 140) horizontally, to represent the phase of the rotor current.

Set off $O6$ along OA to represent voltage (6) in amount, to any scale.

Make $6R$ equal to the voltage drop in the rotor winding resistance.

From R draw $R5$ at right angles to OA , and equal to voltage (5) to the same scale.

Since the brushes are short circuited, no nett voltage can exist in the rotor, so that the resultant voltage $O5$ must be

equal and opposite to the resultant of the remaining two rotor voltages, viz. voltages (3) and (4).

The values of (3) and (4) are not yet known, but from the fact that they are at right angles to one another and that their resultant is equal to $O5$ they can now be arrived at thus—From O draw a line $O3$ at an angle θ to $O5$ such that $\tan \theta = \frac{\text{Voltage (4)}}{\text{Voltage (3)}}$ which, from the expressions given above for voltages (4) and (3) will be seen to equal

$$\frac{\text{Revs. per second}}{\text{periodicity of supply}} \times \tan \alpha,$$

$$\text{or } \tan \theta = \frac{\text{Revs. per second}}{\text{periodicity}} \times \tan \alpha.$$

From the point 5 drop a perpendicular on to $O3$. Then $O3$ is the voltage (3) and the line 3 to 5 is the voltage (4) to the same scale as $O6$, etc. From the value of voltage (3) or voltage (4) so obtained the value of the original flux can be calculated and the value of voltage (2), which is shown at $O2$. Further, the nett magnetizing ampere turns for the original flux =

$$\frac{\text{Maximum value of original flux per pole}}{\text{original flux per pair of poles}} \times \text{Magnetic resistance to}$$

Draw OT leading $O3$ by 90° and equal to the above nett magnetizing ampere turns, to any scale. From T draw TT_1 parallel to OA and of such a length that OT_1 = maximum effective value of the stator ampere turns per pair of poles =

$$\text{Stator current} \times \frac{\text{Stator turns per pair of poles}}{\text{pair of poles}} \times \sqrt{2} \times \frac{2}{\pi}$$

to the same scale as OT .

Completing the parallelogram of which OT is the diagonal and OT_1 one of the sides, the point T_2 is obtained such that OT_2 = maximum effective value of the rotor ampere turns in the direction of the original flux

$$= \text{Rotor current} \times \frac{\text{Total rotor conductors}}{\text{No. of poles} \times \frac{\text{No. of parallel paths between brushes}}{\text{No. of parallel paths between brushes}}} \times \frac{2}{\pi} \times \cos \alpha \times \sqrt{2},$$

from which the true value of the rotor current can be obtained.

The phase of the stator current is given by the direction of the line OT_1 .

The **stator supply voltage** (neglecting magnetic leakage from the winding) is the resultant of voltage (2) shown at $O2$, the

ohmic drop in the resistance of the stator winding and the voltage corresponding to the core and friction losses in the motor. Set off OP along OT_1 to represent the ohmic drop (to the same scale as $O2$) and PH to represent the other losses—

$PH = \frac{\text{losses}}{\text{supply current } OT_1}$ and complete the parallelogram, obtaining Ol as the value of the stator supply voltage, *i. e.* voltage (1). The cosine of the angle ϕ between the stator supply voltage Ol and the line OT_1 representing the stator magnetizing effect (or current) in phase, is the power factor of the motor at the particular load dealt with. It will be seen that the supply current lags somewhat behind the supply voltage, but that this lag would have been much greater if voltage (4) represented by the line 5 to 3 were absent, *i. e.* the voltage induced in the rotor by its rotation in the original flux is a compensating voltage and tends to improve the power factor of the motor. This is a characteristic of all repulsion motors.

Another characteristic is the fact that the section short circuited under the brush is subjected to two opposing voltages—one due to the rotation of the section in the original flux (this corresponds to voltage (4)) and the other due to the inducing action of the cross flux (this corresponds to voltage (5)). At stand-still (starting) only the latter is present, and a heavy short-circuit current may be set up, just as in the plain series motor, but as the speed increases the rotation voltage rises and more or less neutralizes the induced voltage, until, at a speed which is usually about synchronous speed (*i. e.* the number of

revs. per minute which is equal to $\frac{\text{Supply periodicity} \times 120}{\text{No. of poles}}$), complete neutralization occurs, and the repulsion motor is considerably better off at about this speed than the plain series motor at its normal speed.

Both the above compensating effects are most complete at synchronous speed, and on this account the number of poles used for a repulsion motor should be chosen, so that under normal conditions it is running at approximately synchronous speed.

In the plain series motor the whole power developed has to be conveyed through the commutator, whilst in the repulsion motor little more than the exciting energy has to be carried by the commutator, so that for the same output the commutator for the latter can be somewhat smaller than that for the former.

The number of **rotor turns** on a repulsion motor is a matter of choice, as it is quite independent of the supply voltage. If very few turns are put on, the current to be carried by the commutator becomes excessive, whilst, on the other hand, if very many turns are employed, the number of turns per section

and the reactance voltage per section become excessive and sparking may occur.

Although the brushes are short circuited together and no voltage exists between the points of the commutator under the brushes, yet, as explained above, considerable voltages are induced in the rotor winding and actually exist between certain points on the commutator. On this account the insulation on the rotor must safely withstand the maximum value of any one of the voltages (3), (4), (5) or (6).

The **section of the iron** in the stator yoke and in the rotor behind the slots has to be rather greater in a repulsion motor than in a plain series motor with the same flux per pole, because these parts have to carry the resultant of the "original flux" and the "cross flux" in a repulsion motor. These two fluxes are at approximately 90° to one another in phase and, at synchronous speed, they are approximately equal, so that their

$$\text{resultant} = \sqrt{\left(\text{original flux}\right)^2 + \left(\text{cross flux}\right)^2} = 1.41 \times \text{original flux.}$$

Many **special types of repulsion** motor have been suggested and experimented with from time to time. The most important modification is the **Winter-Eichberg** * (or **Latour**) motor, in which, besides the short-circuited brushes (set in this case with the angle $\alpha = 0^\circ$), additional brushes at right angles (in a two-pole motor) to these are employed. By means of a variable-voltage current-transformer connected to these additional brushes any required exciting current can be passed into the rotor, and the value of the cross flux can be regulated independently of the stator supply current. In this way the good points of the repulsion motor, viz. high power factor and low voltage induced in the sections short circuited under the brushes, can be attained over a wide range of speed instead of being confined to approximately synchronous speed. At the same time the arrangement permits the torque of the motor to be economically regulated independently of the speed, which is an important advantage in connection with railway work.

Schüler † has patented a combined repulsion and induction motor for single-phase work. The rotor consists of a direct current armature fitted with collector rings at one end and a commutator at the other. At starting the machine acts as a repulsion motor, with the commutator brushes short circuited and the collector ring brushes open or only connected through a high resistance. As the speed increases the resistance is gradually cut out of the collector rings, until at full speed (just short of synchronous speed as in an induction motor) the rings are

* *Loc. cit.* (see p. 278).

† *Electrician*, Jan. 15, 1904, p. 481.

short circuited and the motor runs as an ordinary induction motor with the commutator brushes carrying little or no current.

In this way the high starting torque of the repulsion motor and the good speed regulation of the induction motor are combined.

MOTOR-GENERATORS AND ROTARY CONVERTERS.

1. **Motor-generators to convert continuous current at one voltage into continuous current at a higher or lower voltage.**—This is accomplished either by two entirely separate machines—a motor and a generator—mechanically coupled together, or by a single machine with two entirely separate windings on the armature—each connected to its own commutator. The first type is used when a variable ratio of transformation is required, as, by varying the excitation of one or both fields a large variation in the ratio can be obtained. In this way motor-generators (or **motor-boosters** as they are sometimes called) are wound to assist main generators in central stations when charging batteries—the main generator giving (say) 100 volts and the booster 30 or 35 volts reducible to 5 or 10 volts (at any current from the full value to zero) by merely weakening the booster field (the booster field would in such a case be arranged for separate excitation from the main generator).

Such motor-generators are also used as “**balancers**” or “**equalizers**” for three-wire systems. For this purpose the two machines have exactly similar armatures and are usually shunt wound, and the shunt of each machine is connected to the opposite side of the system to that across which its armature is connected. In this way very close regulation of the voltages on the two sides of the system can be maintained with little or no hand regulation. In order to avoid all hand regulation with a large difference of current on the two sides of the system, the fields are sometimes compound wound and can thus be arranged either to maintain equal volts on the two sides, or actually to raise the voltage on the more heavily loaded side automatically—the series winding on the motor (for the time being) raising the speed at the same time that the other series winding strengthens the dynamo field.

The second type, with double-wound armature and only one field, allows of no variation in the ratio of transformation, and the only way of reducing the generator terminal volts is the wasteful one of reducing the motor terminal volts by means of resistance in the main circuit. Such machines are, however, sometimes used on small three-wire installations as “**balancers**” to help automatically to keep the voltages on the two sides of

the system alike, independently of the difference of load on the two sides.

The **winding** of such motor-generators is carried out exactly as for ordinary continuous current machines; the output at one side plus the input at the other being, of course, about equal to the output that the same machine would give as an ordinary generator.

The **armature reaction** of such double-wound armatures is negligibly small, as the ampere turns of the generator winding cancel those of the motor winding to a very great extent, with the result that practically no change in the brush position is required for different loads, even with metal brushes.

The closeness with which such a machine used as a **balancer** can equalize the two sides of the three-wire system depends entirely on the armature resistance—the difference in volts between the two sides at a given current being equal to the sum of the lost volts in the two armature windings at that current. This difference will vary from zero at no load to 5 % or 6 % at full load. Balancers or equalizers also occasionally consist of an ordinary direct current generator with a single commutator at one end and two or three slip rings (connected to equidistant points of the armature winding) at the other end. The outer mains of the supply system are connected to the two sets of brushes on the commutator, whilst the middle wire is connected to an artificial neutral point formed at the centre of a choking coil whose ends are connected to the armature through the slip rings.*

There is also another type of motor-booster which is occasionally used, where only a small **variation** (say 20 %) in the **generator volts** is required. It consists of a long armature carrying two independent windings, one of which only encloses part of the core length (one set of end connections being carried across in a gap left in the core some distance from the end) whilst the other winding runs the whole length of the armature. There

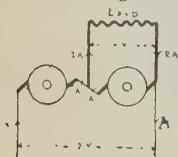


FIG. 141.

are two independent field magnets, each with a distinct winding—one is small, and acts only on that part of the armature which carries the single winding, whilst the larger magnet embraces the rest of the armature. By varying the field of the small magnet the ratio of transformation can be varied.

“**Auto**” motor-generators (such as shown diagrammatically in Fig. 141) can

* See F. H. Jeannin, *Electrical World and Engineer*, October 22, 1904, p. 683, for a description of numerous “static balancers” of this sort; also C. C. Garrard, *Electrical Review*, June 14, 1907; and Hawkins, *Journal I. E. E.*, 1910, vol. xlv, p. 704.

be used with advantage for small ratios of transformation, such as 1:2 or 1:3. They can consist either of two separate machines or of a single machine with a double wound armature.* Both armature windings in series are connected across the higher of the two voltages. It will be seen that the action is exactly the same as in a three-wire balancer.

The output of an auto motor-generator will be rather less than—

$$\left\{ \begin{array}{l} \text{Output of same} \\ \text{machine as an} \\ \text{ordinary motor-} \\ \text{generator} \end{array} \right\} \times \frac{\text{The higher of the two voltages} \\ \text{(input or output)}}{\text{The difference between the} \\ \text{input and the output voltage.}}$$

Motor-boosters of various kinds are largely employed in connection with **electric traction stations**. Thus "**Track Boosters**" (or "Return feeder Boosters," or "Negative Boosters") are used to reduce the large earth return currents on traction systems, as explained on p. 434. These usually consist of a shunt-wound motor running at practically constant speed and direct-connected to a series-wound dynamo which is placed in series with the return feeder, or with its armature in series with the return feeder and its field coils in series with the supply feeder for the corresponding part of the line. The volts given by the dynamo are then approximately proportional to the current, and so make up for the ohmic drop in the feeder at all loads.

When storage batteries are used in connection with traction loads or other fluctuating loads a motor-booster is often inserted in series with the battery across the bus-bars, as shown in Fig. 142. The generator in this case

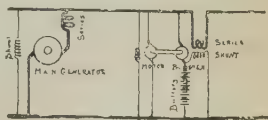


FIG. 142.

series winding is in series, not with the booster armature, but with the main feeder. When the feeder is carrying the normal load, the series winding will be equal and opposed to the shunt, and the booster will give no volts. If the feeder current increases, the series winding overpowers the shunt and the booster adds a few volts to the battery, and so enables it, more or less automatically, to supply most of the extra current. If the

* See *Electrician*, July 1909, for a description of Messrs. Crompton's "C.M.B." auto converter consisting of a single machine with a single commutator fitted with two distinct pairs of brushes and enabling a variable output voltage to be obtained economically.

feeder current fall below the normal, the shunt overpowers the series winding, and the booster therefore gives a few volts in the opposite direction and so charges the battery from the main generators, which thus continue to work at about full load. Modified forms of such **automatic reversible boosters** have been patented by Mr. J. S. Highfield,* Mr. McIntosh,† Mr. Turnbull,‡ and Messrs. Bruce Peebles.§

Plain, separately excited, hand-reversed boosters are also sometimes used in series with lighting batteries.

An important advantage of these reversible boosters is the fact that they do away with the need for the expensive and somewhat troublesome battery charging and discharging switches, all "end cells" and the large number of heavy mains between the battery room and the switchboard—the whole of the cells being permanently connected to the mains through the booster.

2. **Alternate current voltage transformation** is produced by static transformers, described on pp. 301–322.

Transformations in number of phases (one-phase to two-phase, three-phase, etc.) can be produced by stationary apparatus, but none of these arrangements are much used (except the Scott connection for two-phase to three-phase conversion, see p. 315). More usually a motor of one number of phases is coupled to a generator of the other number of phases. Frequency conversions are also usually produced by coupling motors and generators of different numbers of poles together. Alterations in number of phases or in frequency can also be effected by using the stator of an induction motor for one supply and its rotor for the other supply.

3. Conversion of alternating currents into direct currents or *vice versa*.

(a) **Rotary converters** || are very commonly used for converting single-, two-, three- or other phase **alternating** currents into direct current, or *vice versa*.

This conversion can, of course, be obtained by simply coupling

* *Journal of Institute of Electrical Engineers*, vol. xxx. No. 152, 1901.
Patent number, 12972, 1900.

† Patent number, 25522, 1902. See also *Electrician*, Dec. 9, 1904.

‡ Patent number, 12290, 1902.

§ See also S. P. Thompson's *Dynamo Electric Machinery*, 7th edition, pp. 780–786.

|| For further information on rotary converters the following papers may be consulted: *Electrical World of New York*, 1. 1. 98, p. 12, and 12. 2. 98, p. 216, by Woodbridge and Child; *Journal of Institute of Electrical Engineers* for Nov. 1898, Prof. S. P. Thompson; *L'Electricien*, No. 17, 1899, G. Kapp; *Electrical World of New York*, vol. xxxii. 1898, p. 650, etc., C. P. Steinmetz; *Journal of Institute of Electrical Engineers*, vol. xxx. No. 150, 1901, Eborall; *Electrical World and Engineer*, April 9, 1904, F. G. Baum; *Electrical Engineer*, April 29, 1904, E. Elsässer.

a continuous and an independent alternate current machine together, but a large reduction in price and increase in efficiency can be obtained by using a rotary converter, having a single continuous current armature and commutator, fitted in addition with a set of collector rings (two for single-phase, four for two-phase, three for three-phase, and six for six-phase supply) connected to the winding at symmetrical points.

The design of such a machine differs but little from that of an ordinary continuous current dynamo, and it may have a smooth or slotted armature. The number of poles is decided, in the same way as for an alternator, by the periodicity and speed—

$$\text{Number of poles} = \frac{\text{Periodicity per second} \times 120}{\text{Speed in revolutions per minute}} \quad (\text{see p. 291}).$$

The magnet cores, for some inches at least, should be laminated, to prevent eddy currents. The air-gap flux density should be kept high, so as to reduce distortion due to armature reaction.

The **ratio of transformation** has a definite value depending on the number of phases, and the relation of pole breadth to pole pitch. The following table is taken from Prof. Thompson's paper (*loc. cit.*) and gives the theoretical values of the transformation ratio.* In practice the figures differ a little from those given, owing to the fringe of the field—

Voltage for :	Pole pieces arranged to give sine wave of E.M.F.	Pole pieces covering 66 % of the pole pitch.	Pole pieces covering 50 % of the pole pitch.
Continuous current	100	100	100
Single-phase alternating ...	70·7	75	82
Two-phase	70·7	75	82
Three-phase	61·2	64	71
Four-phase	50·	53	58
Six-phase	35·3	37	42

The **heating** of the armatures of these machines from C^2R loss in the windings is peculiar, in that it is not evenly distributed over the armature. Those parts of the winding which lie on either side of the points to which the collector rings are connected have to carry much larger currents than parts midway between two such points. If we neglect the effect of this uneven

* Kapp gives the same values (*loc. cit.*).

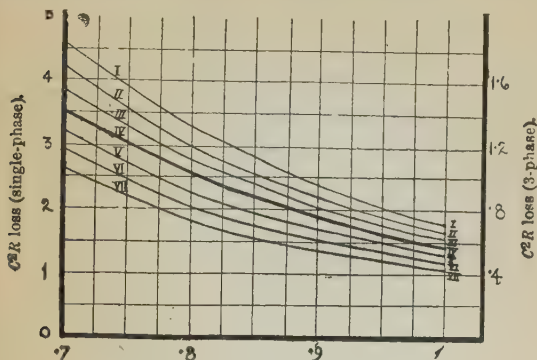
heating (in well-ventilated armatures this may safely be done) and consider only the total C^2R watts lost on the armature, the following figures give the **relative outputs** (amperes $\times$ volts) of a given machine when used as a rotary converter, and when used as a simple continuous current generator—

No. of Phases in the alternate current side.	Power Factor.	Output as a Rotary Converter.
		Output as a Continuous Current Generator for equal C^2R loss.
1	{ 1	·85
	{ ·8	·63
2	{ 1	1·62
	{ ·8	1·0
3	{ 1	1·34
	{ ·8	·77

These figures assume a sine wave of E.M.F. and an efficiency of 100 % (*i. e.* the true input and the output watts are considered equal). With ordinary pole spans the E.M.F. wave is not of sine form and the ratios in the last column are usually somewhat higher. The following curves (Fig. 143) show the effect of various **power factors and efficiencies** upon the C^2R losses in a rotary converter; they are calculated on the assumption that about half the total losses (armature and bearings) are C^2R losses in the armature winding, and that the E.M.F. wave is of sine form. The left-hand scale applies to single-phase converters and the right-hand one to three-phase ones (approximately).

Rotary converters, whether used to convert from continuous into alternate current ("inverted rotaries") or *vice versa*, are usually started up on the continuous current side, and only connected to the alternating mains when up to full speed. Polyphase converters are, however, sometimes designed to start up as ordinary asynchronous motors, the exciting winding being either disconnected or left connected to the D.C. brushes, and the necessary field induced in the magnets by armature reaction alone. Sometimes a separate small induction motor is mounted on the spindle of the rotary converter simply for starting purposes. The running of rotary converters in parallel on the alternating side is a somewhat troublesome operation on account of the "surging" currents which are liable to be set up between the different armatures, and which, if not checked, may cause one or more

of the machines to fall out of step. In practice it has been found possible to reduce this difficulty greatly by the intro-



Power factor (either leading or lagging currents).

Curve showing relative C^2R losses in a machine used as a rotary converter and as a direct current machine (C^2R losses with the direct current alone taken as unity in each case).

- Curve I. Converting from alternating to direct current, and assuming 30 % of the direct current power lost in armature and bearings.
- Curve II. Converting from alternating to direct current, and assuming 20 % of the direct current power lost in armature and bearings.
- Curve III. Converting from alternating to direct current, and assuming 10 % of the direct current power lost in armature and bearings.
- Curve IV. Converting either from alternating to direct, or direct to alternating, and assuming 0 % of the direct current power lost in armature and bearings.
- Curve V. Converting from direct to alternating current, and assuming 10 % of the direct current power lost in armature and bearings.
- Curve VI. Converting from direct to alternating current, and assuming 20 % of the direct current power lost in armature and bearings.
- Curve VII. Converting from direct to alternating current, and assuming 30 % of the direct current power lost in armature and bearings.

FIG. 143.

duction of a series of low resistance damping circuits, similar to the Leblanc "amortisseur" (see p. 298), in slots in the magnet faces.

The fields of converters are usually excited by simple shunt windings connected across the continuous current terminals. Though variations of the field strength have no direct effect on the ratio of continuous to alternating voltage, yet it is possible in certain cases (when converting from alternating to continuous current), by means of a properly proportioned compound winding,

to obtain a constant or even a rising output voltage with increase of load. This effect, however, is only attained through the over-excited converter acting as a condenser in the circuit at full load, and so, in combination with a certain amount of reactance in the alternate current supply mains or transformers, actually raising the terminal voltage on that side as the load increases.* In the case of "Split-pole" converters† the D.C. voltage is regulated by taking advantage of the fact that variations of polar arc affect the ratio of the D.C. to the A.C. voltage.

(b) **Bragstadt and La Cour**‡ have recently patented a new type of rotary converter, to which they have given the name of "cascade converter."

It consists of an ordinary single or polyphase induction motor mounted on the same spindle as an ordinary direct current dynamo. The stator of the induction motor is connected to the alternating supply, and its rotor is connected (by means of a number of cables passing through the hollow spindle between the two machines) to symmetrically spaced points on the armature winding of the direct current machine.

The combination is started up like any ordinary induction motor, but, owing to the cascade connection, it runs at only half the synchronous speed if, as is usual, the two machines have the same number of poles. In this way half the power is transmitted from the alternating side to the direct current side mechanically through the spindle, and the other half electrically from rotor to rotor.

The cost of such a machine is less than that of a motor generator but more than that of an ordinary rotary converter of equal output. The advantages are—

- (1) For a given supply periodicity and speed the number of poles on the direct current machine is only half as great as for the corresponding rotary converter, and this gives more satisfactory proportions for direct current work.
- (2) The value of the direct current voltage is independent of the alternate current supply, and by suitably winding the rotors any required voltage can be obtained. The direct current voltage can also be varied 10 % or 15 % either by hand regulation or by compounding the direct current field.

* See A. C. Eborall, "Polyphase Substation Working," *Jour. Inst. E. E.*, vol. xxx., 1901, p. 718; and Parshall and Hobart, *Electric Generators*, p. 324.

† See *A.I.E.E.*, 1908, vol. xxvii., pp. 899, 949; also *Electrical World*, 1907, p. 892.

‡ See *Elektrotechnische Zeitschrift*, June 9, 1904, p. 480; and *Zeitschrift für Elektrotechnik*, No. 15, 1904, p. 221; also Hallo's Paper before the I. E. E., Dec. 21, 1908.

(c) Motor generator sets, either in the form of a synchronous A.C. motor coupled to a D.C. dynamo or of an A.C. induction motor coupled to a D.C. dynamo, are also much used for converting A.C. to D.C. The latter have the advantages of easy starting on the A.C. side and of greater stability under overloads, but they have bad power factors and efficiencies as compared with the synchronous sets.

(d) Besides the above mechanical methods of converting A.C. into D.C. there are a number of electrochemical rectifiers in use for small current work. Of these the best known are the Nodon Aluminium "Valve" and the Cooper-Hewitt Mercury Vapour rectifier (see Moss, *Journal I. E. E.*, 1907, vol. xxxix. p. 692, and *E. T. Z.*, January 5, 1911).

TESTING CONTINUOUS AND ALTERNATE CURRENT GENERATORS, MOTORS, AND TRANSFORMERS.*

By P. R. FRIEDLAENDER.

1. MAGNETIC LEAKAGE TESTS.

(a) On continuous current generators, and motors, and on alternators by means of a *ballistic* galvanometer.

Wind an exploring coil of a few turns of fine wire on the armature outside the winding proper, and in such a position as to embrace all the lines entering the armature from one pole face (*i. e.* the coil will be wound across a diameter for two-pole machines, and across an arc corresponding to each pole for multipolars). Wind a second exploring coil of about the same number of turns, or rather less, on the magnets, close to one of the exciting coils, and on that side of it furthest removed from the armature.

Pass the normal exciting current round the magnet-exciting coils and connect first one and then the other exploring coil to the ballistic galvanometer, and note the deflection in each case when the exciting current is suddenly broken or reversed by means of a quick-break switch.

Then the leakage coefficient of the machine =

$$\frac{\text{Deflection with magnet exploring coil}}{\text{Deflection with armature exploring coil}} \times \frac{\text{Number of turns in armature exploring coil}^\dagger}{\text{Number of turns in magnet exploring coil}}$$

* Without attempting to deal with the subject at all fully, a few of the less usual tests are described here. For detailed descriptions of the testing of electrical machinery, see S. P. Thompson's *Dynamo Electric Machinery*, p. 913; Kinzbrunner, *The Testing of Continuous Current Machines*; G. D. Aspinall Parr, *Electrical Engineering Testing*; J. A. Fleming, *Handbook of Electrical Laboratory and Testing Room*, pp. 495-616, vol. ii.; R. H. Smith, *Alternate Current Testing*.

† For a simple differential method of carrying out the leakage test without the use of a ballistic galvanometer, see R. Goldschmidt, *E. T. Z.* 1902, p. 15, and also *Electrician*, May 24, 1907.

(b) On **alternate current induction motors**.—The leakage coefficient or “dispersion coefficient” of an induction motor is the ratio of the leakage flux (or difference between rotor and stator flux) to the total rotor flux when the magnetizing current only is passing in both stator and rotor (see p. 329).

The simplest and most accurate method of arriving at this coefficient consists in measuring the magnetizing current of the motor and the current taken on short circuit with the rotor at rest when full volts are applied to the stator terminals.

The magnetizing current is obtained by running the motor unloaded at the correct terminal volts and periodicity, and measuring both the input current per phase and the input watts per phase (by wattmeter).

Then the magnetizing current per phase =

$$\sqrt{\left(\frac{\text{Input current}}{\text{per phase}}\right)^2 - \left(\frac{\text{Power current}}{\text{per phase}}\right)^2}$$

the “power current per phase” being the input watts per phase divided by the volts per phase. This power current is usually so small compared with the magnetizing current that it may be neglected and the wattmeter reading dispensed with, *i. e.* the magnetizing current may, without any great error, be taken as equal to the total no load current.

The short-circuit current per phase is obtained by holding the rotor at rest or, preferably, rotating it slowly by hand with its slip rings (if any) short circuited and applying a sufficient voltage to the stator terminals to cause about the normal full load stator current to pass into the motor. The actual short-circuit current with full terminal volts can then be taken as

$$\left\{ \frac{\text{Current per phase}}{\text{passing during test}} \right\} \times \frac{\text{Full terminal volts}}{\text{Terminal volts during test}}$$

The leakage or dispersion coefficient σ of the motor is then given with sufficient accuracy for all practical purposes, by*—

Magnetizing current per phase

$$\left\{ \frac{\text{Short-circuit current per phase}}{\text{at full volts}} \right\}$$

$$\text{for, as shown on p. 333, } \sigma = \frac{\text{Magnetizing current}}{\text{Theoretical short-circuit current} - \text{Magnetizing current}}$$

* See Behn Eschenburg on “Magnetic Dispersion in Induction Motors,” *Jour. Inst. E. E.*, vol. xxxiii. No. 165, 1904, p. 239.

and $\left(\begin{array}{c} \text{Theoretical} \\ \text{short-circuit} - \text{Magnetizing} \\ \text{current} \quad \quad \text{current} \end{array} \right)$ is usually very nearly equal to actual short-circuit current.

The actual short-circuit current line OX_1 (Fig. 131, p. 330) can also be conveniently decided by this test.

(c) Alternate current **transformer leakage and pressure drop measurement.**

The drop in the secondary terminal pressure of a transformer between no load and full load depends partly on the ohmic (CR) lost volts in the primary and secondary windings when the full load current is passing, and partly on the reactive back volts induced in both the primary and secondary windings by magnetic leakage flux which is looped with one of the two windings to a greater extent than with the other.

The terminal voltage, the ohmic drop, and the reactive drop on a transformer can be combined in order to get the total drop by means of a vector diagram, as described on p. 310.

The most accurate method of measuring the drop is the following. It entails the use of a wattmeter (not essential, but advisable), a voltmeter and an ammeter. The secondary winding of the transformer is short circuited on itself. The ammeter, voltmeter and wattmeter are connected to the primary, and sufficient voltage e (usually 2 % to 10 % of the normal primary E.M.F.) to give the full load current in the primary is then applied to the primary. The total watts lost in the circuit are at the same time read on the wattmeter.

Then, referring to the diagram, Fig. 113, p. 297, make the horizontal line ER equal to—

$$\frac{\text{Watts read on wattmeter}}{\text{Current read on ammeter}} \times \frac{\text{Secondary turns}}{\text{Primary turns}}$$

With E as centre and $e \times \frac{\text{Secondary turns}}{\text{Primary turns}}$ (where e is measured on the voltmeter as above) as radius, draw a circular arc and find the point ϵ where this arc cuts the vertical $R\epsilon$ drawn from the point R . Then $A\epsilon$ is the no load secondary voltage required for a full load voltage equal to AE . And $\frac{A\epsilon - AE}{A\epsilon} \times 100$ is the actual drop as a percentage of the no load terminal pressure. The vertical line $R\epsilon$ represents (to the same scale) the reactive voltage due to the total magnetic leakage (reduced to the secondary side).

NOTE.—In the case of small transformers with little leakage, and in which there are, therefore, practically no eddy current

losses due to leakage into the case, etc., the wattmeter can be dispensed with, for practically the only watts lost will then be ohmic (the hysteresis and eddy current losses in the iron core being negligible at the low primary voltage). These C^2R watts can be accurately calculated for any given current from the measured resistances of the windings. In this case ER could with fair accuracy be made equal to—

$$\left(\text{Resistance of primary} \times \frac{\text{Primary current}}{\text{Secondary turns}} \times \frac{\text{Primary turns}}{\text{Primary turns}} \right) + \left(\text{Resistance of secondary} \times \frac{\text{Primary current}}{\text{Secondary turns}} \times \frac{\text{Primary turns}}{\text{Secondary turns}} \right)$$

When no great accuracy is required, the drop in a transformer can, of course, be directly measured by noting the secondary terminal pressure on open circuit and at full load, the applied primary pressure being alike in both cases. But since the usual drop with good transformers is only 2 % or $2\frac{1}{2}$ % of the terminal pressure, large errors would be almost inevitable in reading this on a voltmeter measuring the whole terminal pressure.

2. ALTERNATOR "DROP" OR PRESSURE REGULATION TESTS.

The ratio (stated as a percentage) of the difference between the full load and no load terminal voltage of an alternator running at constant speed and with the normal full load exciting current to the no load voltage, is known as the "drop" of the alternator. Sometimes it is the ratio of the voltage difference to the normal *full* load voltage which is specified, and this figure is known as the "*rise*" of the machine.

The values of these ratios for a given alternator depend on the power factor of the load taken from the machine, being three or four times as great with a zero power factor (wholly inductive load) as with a power factor of unity (wholly non-inductive load), and it is usual to specify the permissible value both with a power factor of 1 and also with one of about .8.

The value of the drop can, of course, be obtained by direct measurement of the terminal voltage, first at the correct full load with the specified power factor, and then, with the same excitation, at no load. When, however, it is impossible or inconvenient to arrange for the machine to be fully loaded on the test, a very fair approximation to the drop on any required load can be obtained if the short-circuit current of the machine

is known, as explained on p. 297. To arrive at the short-circuit current by experiment it is simply necessary to short circuit the alternator armature through an ammeter, run the machine at any convenient speed (which may be well below or above the normal speed, since the short-circuit current keeps practically constant over a wide range of speed) and then pass a sufficient exciting current round the magnets to produce the normal full load current in the short-circuited armature.

Then with the same excitation find the voltage which the machine will give on open circuit when running at its normal speed.

This voltage may then, with fair accuracy, be considered as the resultant lost voltage at full load, and the length of the line $E\epsilon$ in Fig. 113, p. 297, may be made equal to this voltage. The effective voltage drop (or rise) for any required power factor can then be obtained, as on p. 297.

3. INDUCTION MOTOR SLIP.

The slip of an induction motor at any given load, stated as a percentage,

$$= \frac{\left\{ \begin{array}{l} \text{Speed of rotating} \\ \text{field set up by the} \\ \text{stator windings} \end{array} \right\} - \left\{ \begin{array}{l} \text{Speed of} \\ \text{rotor at the} \\ \text{given load} \end{array} \right\}}{\text{Speed of rotating field set up} \\ \text{by the stator windings}} \times 100$$

$$= \text{Periodicity of rotor currents} \times 100 \div \text{Periodicity of supply.}$$

The speed of the rotating field is known directly from the periodicity of the supply and the number of poles for which the motor is wound, and is equal to—

$$\frac{\text{Periodicity} \times 120}{\text{No. of poles}} \quad \begin{array}{l} \text{in revolutions} \\ \text{per minute.} \end{array}$$

The speed of the rotor can, of course, be measured directly by means of a speed counter, but since the slip only reaches a value of 3 % or 4 % at full load, no great accuracy in determining the slip can be attained by this means.

The slip can, however, be measured directly in a simple manner and with great accuracy by means of any ordinary galvanometer or millivolt meter. If the motor has a "wound" rotor the galvanometer terminals should be connected across a few inches of one of the cables leading from the rotor collecting rings to the starting switch. This will cause the needle to be deflected first one way and then the other way with a frequency equal to that of the currents in the rotor, and by counting the number of complete (double) swings in a minute,

multiplying by 100 and dividing by the number of complete cycles of the supply current in one minute, the percentage slip is obtained directly.

In the case of squirrel-cage rotors and other rotors without collector rings, the galvanometer terminals should be connected to the opposite ends of the spindle of the motor. A stray field due to the rotor currents is usually found to exist, and this is sufficient to cause the galvanometer needle to swing backwards and forwards in unison with the rotor periodicity, just as in the previous case.

4. EFFICIENCY TESTS.

Continuous and Alternate Current Motors or Generators.

Method (a): **Measure each loss separately.**

The ohmic loss in the armature and field windings can be calculated from the known currents and the measured resistances. What may be called the "driving watts" * are obtained by running the machine, entirely unloaded, as a motor with the correct normal speed and excitation, and measuring the input watts. The driving watts obtained in this way should be increased 15 % to 20 % (and in alternators even more) to allow for the increase in eddy current and other losses which always occur when the machine is fully loaded.

Method (b): **The Cardew through efficiency method.**

For this method a second machine similar to the machines under test is required (mechanically similar as to size, number of bearings, etc., though the outputs or windings of both machines need not be alike).

Couple the two machines end to end by means of a flexible coupling. Connect up one of the machines as a motor to drive the other as a dynamo, and measure the input watts on motor and output watts on dynamo when the dynamo is giving its correct output at normal speed. Then—

$$\frac{\text{Input at motor} - \left(\text{Output at generator} + C^2R \text{ watts in both armatures and both fields} \right)}{2}$$

can be taken as the "driving watts" of each machine under test, from which, together with the C^2R watts calculated from the known currents and resistances, the efficiency can be obtained. The objection to this method is that any inaccuracy in the measurement of either input or output occurs as an error of equal percentage value in the deduced efficiency.

* *i. e.* all losses except C^2R watts.

Method (c): The **Hopkinson** or **differential efficiency test**.

This method requires the use of a second machine, mechanically and electrically the same as the one under test, and also some auxiliary source of power equal to 15 % or 20 % of the power of the machine whose efficiency is to be measured. This source of power may be either a motor, a dynamo, or a storage battery.

Couple the two main machines together end to end by means of a flexible or other coupling.

If the source of power is a motor this must be mechanically connected to the two main machines by belt or otherwise.

If an auxiliary dynamo or battery is used (and this is, as a rule, the most convenient and accurate) the electrical connections may be either as in Fig. 144 or as in Fig. 145.

Fig. 144 requires an auxiliary dynamo capable of carrying the

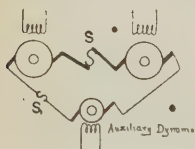


FIG. 144.

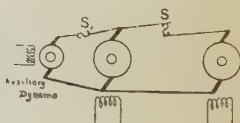


FIG. 145.

full current of the main generators, whilst Fig. 145 requires one giving a much smaller current, but equal in voltage to either of the main generators.

It is most convenient to separately excite the fields of all the machines, and so have them completely under control.

Suppose the "series" Hopkinson test shown in Fig. 144 is adopted, with the auxiliary dynamo (or battery) in series with the main machines. To start the arrangement close the switch S which connects poles of similar sign on the two machines. Separately excite the fields, weakening the field of one of the main generators relatively to the other (the one with the weak field will act as the generator of the combination). Start up the auxiliary dynamo, having connected it in such a way that its voltage assists the voltage of the machine with the weakened field (generator). Close switch S_1 . The large machines will then start, and their speed and the current passing can be brought up to the normal full load figure by adjusting the relative strengths of the two fields and the voltage of the auxiliary machine. The motor voltage should be adjusted to have a value as much above the normal as the dynamo voltage is below the normal, so that the average of the flux densities in the two machines has about

the normal value. Both machines will then be running at full load, and yet the power supplied from the outside source will only be equal to the total losses in both machines.

If measurements of the main current, of the volts across the main generator's terminals, of the volts across the auxiliary dynamo's terminals, and of the watts expended in both (separately excited) main machine field circuits are then made, the total losses in each machine can be obtained. They are equal to—

$$\frac{\left\{ \begin{array}{c} \text{Auxiliary dynamo} \\ \text{volts} \end{array} \right\} \times \left\{ \begin{array}{c} \text{Main} \\ \text{amperes} \end{array} \right\} + C^2R \text{ loss in both fields}}{2}$$

and the commercial efficiency of either of the two main machines at an average output equal to—

$$\left(\frac{\text{Auxiliary volts}}{4} + \begin{array}{c} \text{Main dynamo} \\ \text{volts} \end{array} \right) \times \text{Main amperes}$$

is—

$$\left(\frac{\text{Auxiliary volts}}{4} + \begin{array}{c} \text{Main dynamo} \\ \text{volts} \end{array} \right) \times \text{Main amperes}$$

$$\frac{\left\{ \left(\frac{\text{Auxiliary volts}}{4} + \begin{array}{c} \text{Main dynamo} \\ \text{volts} \end{array} \right) \times \text{Main amperes} + \begin{array}{c} \text{Total losses} \\ \text{in one} \\ \text{machine as} \\ \text{above.} \end{array} \right\}}{2}$$

In the same way for the "parallel" Hopkinson test shown in Fig. 145, the efficiency of each of the main machines at an average output of

$$\text{Terminal voltage} \times \left(\begin{array}{c} \text{Main} \\ \text{dynamo} \end{array} \times \text{current} + \frac{\text{Auxiliary current}}{4} \right)$$

is equal to

$$\text{Terminal volts} \left(\begin{array}{c} \text{Main} \\ \text{dynamo} \end{array} \times \text{current} + \frac{\text{Auxiliary current}}{4} \right)$$

$$\frac{\text{Terminal volts} \left(\begin{array}{c} \text{Main} \\ \text{dynamo} \end{array} \times \text{current} + \frac{\text{Auxiliary current}}{4} \right)}{2} + \begin{array}{c} \text{Total losses} \\ \text{in one} \\ \text{machine} \end{array}$$

the "total losses" in one machine being

$$\frac{\left(\text{Terminal volts} \times \begin{array}{c} \text{Auxiliary} \\ \text{current} \end{array} \right) + C^2R \text{ loss in both main fields}}{2}$$

The Hopkinson test (especially the last arrangement) is superior to any of the other methods of determining the efficiency both in accuracy and in the small amount of energy wasted during the test. Its application to alternate current generators and motors is not so convenient, although, in the case of synchronous machines, the generator and motor parts can be the two halves of one and the same armature. This result is best attained by dividing the magnet poles up into two equal sets, each distributed approximately uniformly around the machine, and sending the normal exciting current round one set and a weakened and reversed current round the other set, so as to make half the armature act as a generator and the other half as a motor with a reduced back voltage.*

In the case of induction motors a difference of 10 % or 15 % between the speeds of the two similar machines under test must exist at full load, and this speed difference must be diminished for lower loads, so that a belt drive must be used with carefully graduated pulleys, and this complicates the test.†

(d) The Prony Brake is often used for testing the efficiency of electric motors in the same way as it is used for engine efficiencies.‡

(e) Alternate current **transformer efficiency tests.**

An analogous method to the Hopkinson efficiency test given above, for obtaining the efficiency of a transformer, is due to Sumpner.||

The efficiency of a transformer is, however, most commonly obtained, as explained on p. 309, by the separate measurement of each loss.

5. INSULATION TESTS.

The most satisfactory way of determining the quality of the **insulation** is to subject the machine to a test at an E.M.F. well in excess of its working voltage. Thus it is common to specify that the machine shall stand a pressure equal to twice the working voltage, if applied between the windings and the frame for some minutes, when the machine is warm (immediately after a long run at full load).

Sometimes, however, the actual insulation resistance is specified. This is not, in itself, a safe test, as a machine may have a very high insulation resistance and yet may readily break

* S. P. Smith, *Electrician*, July 14, 1905.

† See Weekes and Sumpner, *Electrical Engineer*, June 17, 1904.

‡ For a description of numerous ways of applying this test, see Kinsbrunner, *The Testing of Continuous Current Machines*.

|| See Fleming's *Handbook of Electrical Laboratory and Testing Room*, vol. ii. p. 602.

down when subjected to a pressure little in excess of its working E.M.F.

A common and handy instrument for measuring insulation resistances up to fifty megohms is the direct reading "**ohmeter**."

For **higher insulations** the following method can be used. It requires a primary or secondary battery of (say) 100 volts† or more, an ordinary D'Arsonval or mirror galvanometer, and a standard resistance of (say) 100,000 ohms or so. A part of the battery (say one or two cells) is connected to the galvanometer through the standard resistance and the number of cells adjusted to give a convenient deflection d . Then the standard resistance is replaced by the insulation resistance to be measured, and the number of cells in use is increased until approximately the same deflection is obtained as before = d^1 . Then the insulation resistance will approximately equal—

$$\text{Standard resistance} \times \frac{d}{d^1} \times \frac{\text{Number of cells used in second measurement}}{\text{Number of cells-used in first measurement}}$$

(Instead of varying the number of cells used, the necessary range can be got by shunting the galvanometer to a greater or less extent.)

There are no generally accepted rules as to the **necessary insulation** resistances of machines of various sizes, but many of the electric supply companies have their own specifications. Thus, one company requires all motors installed to have an insulation resistance between the whole of the windings and

earth of not less than $\frac{20}{\text{Normal input current in amperes}}$ megohms.

Another gives the following figures—

Motors of 1 H.P. not less than 5 megohms.

2	"	"	"	2½	"
4	"	"	"	1½	"
8	"	"	"	1	"
16	"	"	"	¾	"
25	"	"	"	½	"

* See p. 165.

† If no very great accuracy is required, and if a high resistance standard (e.g. a megohm) is at hand, the ordinary supply mains can be used.

ELECTRICAL TRANSMISSION OF POWER.

By D. K. MORRIS, PH.D.

LONG distance power transmission is used—

(1) Because of the greater economy of generating power on a large than on a small scale, viz. when a large amount of power is wanted in small units distributed over a considerable area, as in industrial centres and in extended traction and lighting schemes.

(2) When the cost of generation is considerably less at some moderate distance than it is at the place where the power is wanted. This may arise from local conditions such as high ground value, cost of carrying fuel (as in mining districts); or else from a neighbouring supply of water-power, available at moderate outlay.

Most Economical Size of Conductor, and Working Pressure.*

The condition of greatest economy of transmission when the amount of power to be transmitted is fixed is that the total annual expenditure arising directly or indirectly from the transmission line should be a minimum.

This total yearly cost of the transmission is made up of—

- (1) Annual cost of generating the units wasted in the line.
- (2) Annual cost of plant for generating these units.
- (3) Annual cost of line.

The first item

= kw. lost in line
× hours per ann. during which the normal current flows
× actual cost of generating one unit.

The second item

= cost of generating station per kw. capacity
× kw. lost in line
× percentage to cover (a) the annual interest on capital outlay on station, and (b) the depreciation and maintenance of station.

* For a full discussion, see Kilgour's *Practical Electrical Engineering*, and a paper by Teichmüller, *Elektrotechnische Zeitschrift*, March 6, 1902.

And the last item

= cost of transmission line per mile
 × length of line in miles (including return)
 × percentage to cover interest on capital expended on
 line and the maintenance and depreciation of the
 line.

The most economical transmission voltage and cross sectional area of copper is only to be obtained from a knowledge of how each of these quantities varies with alteration of the voltage and with the size of the cable. And the solution depends also upon whether the voltage at the generator or at the receiver end of the line is taken as constant.

The loss in the line itself

$$= \left(\begin{array}{c} \text{Generator} \\ \text{volts} \end{array} \right) \times \left(\begin{array}{c} \text{Current} \\ \text{in line} \end{array} \right) - \left(\begin{array}{c} \text{Power to be supplied} \\ \text{at end of line} \end{array} \right);$$

and both generator voltage and line current are capable of variation independently for a given power to be transmitted.

Assuming that the voltage at the receiving end of the line is fixed (viz. for given power, a constant current),

(a) **Kelvin's Law** (1881), holds

$$\left\{ \begin{array}{l} \text{Annual cost} \\ \text{of } C^2R \text{ losses} \\ \text{in line} \end{array} \right\} = \left\{ \begin{array}{l} \text{Annual interest on, and depreciation of,} \\ \text{that part of the line whose cost varies} \\ \text{with the section of the conductor.} \end{array} \right\}$$

In applying this principle, it will be seen that account must be taken of the high cost of the **insulation** which will be incurred where underground cables are used, and also, though to a less degree, in the case of overhead wires where the pressure is large, *e. g.* 10,000 volts.

Kelvin's law gives the most economical section of conductors to employ and (indirectly) the pressure of the generator, when the pressure at the other end of the line, and therefore the maximum working current, is fixed beforehand (by requirements of load, etc.).

It appears also only to hold when capital can be borrowed at a fixed rate of interest (as by local authorities, etc.).

Modifications of Kelvin's law are—

(b) **Ayrton**, 1886.

$$\left\{ \begin{array}{l} \text{Gross annual revenue derived from sale} \\ \text{of energy transmitted by conductor} \end{array} \right\} = \text{a maximum.}$$

$$\left\{ \begin{array}{l} \text{Generation and distribution costs} \\ \text{including charges for depreciation and} \\ \text{interest on all capital invested.} \end{array} \right\}$$

This rule, again, only holds when the capital can be borrowed at a fixed rate of interest, according to Kilgour, who gives the following more general condition of maximum economy—

(c) Kilgour.

$$\frac{\left\{ \begin{array}{l} \text{Gross annual} \\ \text{revenue from} \\ \text{sale of energy} \\ \text{transmitted} \\ \text{by conductor} \end{array} \right\} - \left\{ \begin{array}{l} \text{Generation and distribution} \\ \text{costs including depreciation} \\ \text{charges and interest on that} \\ \text{part only of capital which is} \\ \text{borrowed at a fixed rate of} \\ \text{interest (debentures)} \end{array} \right\}}{\left\{ \begin{array}{l} \text{Total capital involved excepting that} \\ \text{borrowed at a fixed rate of interest.} \end{array} \right\}} = \text{a maximum.}$$

Relative cost of overhead and underground conductors.—The difference in cost is so great that in many cases transmission of power to long distances with underground conductors becomes financially impossible. The Board of Trade Regulations relating to high tension conductors, thickness of insulation, etc., do not now necessarily hold for power transmission schemes.

The approximate **cost of the transmitting line** is made up of two parts, one ($B \times s$) proportional to, and the other (A) independent of the cross sectional area of the conductor. When the cost can be expressed strictly in this way, Kelvin's law holds. But with high tension insulated cables such estimates cannot be made with accuracy; and the law is of little use except for overhead bare conductors.

Stuart Russell* gives estimates for the cost of a mile of transmission line on various systems. And Abbott† has published a careful set of curves of the cost of cables from which the above constants may be estimated.

The following expression of Kelvin's law‡ for the **most economical section** of conductor in such cases is applicable also to three-phase lines—

Let $A + Bs$ = cost of line in £ per mile as above; s being taken in sq. in., and both cost and section relating to one lead only.

e = rate of interest and depreciation on capital expenditure per cent.

c = average current in amperes (effective current in one lead if three-phase).

* *Electric Light Cables*, by Stuart Russell, London, 1901.

† *The Electrical Transmission of Energy*, by Abbott, New York, 1898. See also Herzog and Feldmann, *Handbuch der Elektrischen Beleuchtung*, Berlin, 1901.

‡ Unwin, *The Development and Transmission of Power*, 1894, p. 272.

k = cost of generating 1 E.H.P.-annum (including *all* annual charges) in £ per annum [viz. the product of the cost of generating 1 E.H.P.-hour into the number of hours per annum during which the energy is applied].

$$\left. \begin{array}{l} \text{Most economical} \\ \text{section in sq. in.} \\ \text{(one lead only)} \end{array} \right\} = \begin{array}{l} \cdot 0755 C \sqrt{\frac{k}{eB}} \text{ for copper conductors.} \\ \cdot 097 C \sqrt{\frac{k}{eB}} \text{ for aluminium conductors.} \end{array}$$

These correspond to the following **current densities**—

$$\left. \begin{array}{l} \text{Most economical} \\ \text{current density} \\ \text{in amperes per} \\ \text{sq. in.} \end{array} \right\} = \begin{array}{l} 13 \cdot 2 \sqrt{\frac{eB}{k}} \text{ for copper conductors.} \\ 10 \cdot 3 \sqrt{\frac{eB}{k}} \text{ for aluminium conductors.} \end{array}$$

And the **drop in volts** per mile can be obtained at once from the formula on p. 390. (The constants are calculated for the specific resistances given on p. 389.)

If the rate of interest + the depreciation rate for the line amount to 10 %, which for overhead conductors may be regarded as high as the conductor itself hardly depreciates at all in value, the above expressions become—

$$\left. \begin{array}{l} \text{Most economical} \\ \text{section in sq. in.} \end{array} \right\} = \begin{array}{l} \cdot 0238 C \sqrt{\frac{k}{B}} \text{ for copper.} \\ \cdot 0306 C \sqrt{\frac{k}{B}} \text{ for aluminium.} \end{array}$$

$$\left. \begin{array}{l} \text{Most economical} \\ \text{current density} \\ \text{in amperes per} \\ \text{sq. in.} \end{array} \right\} = \begin{array}{l} 42 \sqrt{\frac{B}{k}} \text{ for copper conductors.} \\ 33 \sqrt{\frac{B}{k}} \text{ for aluminium conductors.} \end{array}$$

These formulæ contain no assumptions except those mentioned, and give strictly that conductor whose annual cost (C^2R losses and capital charges, etc.) is least at a given working pressure. The **difficulty of applying** them arises chiefly from the difficulty of estimating the constant B .^{*} Multiplied by the sectional area (sq. in.) of one conductor, it gives that part of the cost (in £) which varies with the cross sectional area of the conductor. The modifications of Kelvin's law, above given, are due to the fact that when this area is altered, not only does the

^{*} The cost of a single conductor per mile is £546 $\times s$ for copper at £60 a ton, and £372 $\times s$ for aluminium at £140 per ton. B is therefore about 600 and 400 respectively for bare conductors of these metals.

cost of the line alter, but also that of the whole installation. With the magnitude of the capital also, the conditions (rate of interest, etc.) of obtaining it may alter.

In many practical cases, the volts lost along the line of most economical section (and in extreme cases even its rise of temperature) will be found excessive. The section is then really determined by the change in pressure allowable for a given change of load; this amount depending on the character of the load. In the case of **tramway feeders**, a fluctuation, which would not be excessive for the motors, may give rise to too great a variation in the brightness of the car-lamps.

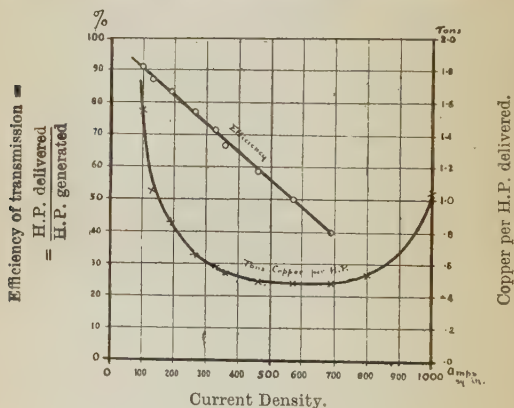


FIG. 146.

A curve (Fig. 146) is given by Forbes,* showing how the **efficiency of distribution** varies with the weight of copper used per H.P. delivered at 100 miles distant, by a 10,000 volt **continuous current transmission**. The figures (p. 377) are taken from the curve.

For electrical pressures (V) other than 10,000 volts and distances (D) differing from 100 miles—multiply the current density by $\frac{V}{100D}$, and divide the "tons per mile" by the

* "The Long Distance Transmission of Power," by Prof. G. Forbes, *Society of Arts Journal*, Nov. 25, 1898, p. 25.

$\frac{\text{H.P. delivered}}{\text{H.P. generated}}$ = efficiency.	$\frac{\text{H.P. generated}}{\text{H.P. delivered}}$ = "inefficiency."	Tons of copper per H.P. delivered.	Current Density amps. per sq. in. (approx.).
90.9%	1.1	1.55	100
87	1.15	1.05	130
83.3	1.2	.86	190
77.0	1.3	.66	266
71.5	1.4	.58	330
66.7	1.5	.54	360
58.8	1.7	.49	460
50	2.0	.48	570
40	2.5	.48	690
—	—	.53	800
—	—	1.04	1000

square of this factor. These modified figures will then correspond to the efficiency given in the first column.

The above table is true also for **single-phase** and separate circuit **two-phase** systems (four-wire) at 10,000 effective volts. For **three-phase**, the figures correspond to a voltage of 8600 effective volts between any pair of wires. Correction must, however, be made for power factor, see p. 386.

Detailed rules and tables are given by Forbes,* showing also a simple way of finding the most economical current density.

Cost of a H.P.-year.

The financial success of a power transmission scheme depends on the excess of the selling value of a H.P.-year over the cost of generating it (including capital charges).

The cost of generation depends on the **load factor**, while the selling value may not.

Where coal is the source of power, the margin of profit of a large power transmission plant will depend much on the excess of cost of coal (due to carriage) at the point where the power is utilized over that at the generating station.

In isolated places where power is a *necessity* (as in gold mines, etc.), and the cost of coal is prohibitive, an enormous outlay on the transmitting line and the hydraulic works may still bring in a good return.

* In using these, the corrections given on p. 626 of the same volume of the *Society of Arts Journal* must be noted.

TABLE OF EQUIVALENTS AND COST OF ONE HORSE-POWER FOR A YEAR.

H.P. hours.	Kilowatt-hours, or Board of Trade Units.	Cost at			
		$\frac{1}{2}d.$	1d.	2d.	4d. per unit.
1-H.P. hour.	746 units	37d.	75d.	1 49d.	2 98d.
One H.P. for a year of 1000 hours	746 units	£ 1 55	£ 3 11	£ 6 22	£ 12 43 per year
" " " 2000 "	1492 "	3 11	6 22	12 43	24 9 "
" " " 3000 "	2238 "	4 66	9 32	18 65	37 3 "
" " " 4000 "	2984 "	6 22	12 43	24 9	49 7 "
" " " 6000 "	4476 "	9 32	18 65	37 3	74 6 "
" " " 8760 "	6535 "	13 61	27 2	54 5	108 9 "
(i.e. without intermission)					" "

In **England**, a H.P.-year of 3000 hours can be generated on a large scale for from £5 to £10. But power sells in many districts at 2*d.* a unit, or £18·61 per H.P.-annum of 3000 hours.

In **South African** mines, from £45 to £100 is paid for 1 H.P.-annum continuous load. In the Cripple Creek mines, **Colorado**, £15 to £26 was paid (1899) per H.P. of motor connected (for continuous use) for motors ranging from 30 H.P. down to 5 H.P. respectively.

Some prices per H.P.-annum in **Switzerland** (1899) are given below *—

Locality.	Price per H.P.-annum.	No. of hrs. in year (daylight hours).	Size of Motors.
	£ s.	hrs.	H.P.
Zürich	40 0	2000	1
"	32 0	2000	5
Olten-Aarberg	4 7	2000	1
"	8 7	3000	1
"	3 11	2000	5
Sihl works	3 19	1000	1
"	3 4	1000	5
"	2 19	1000	10
Rheinfelden	4 7	2000	1
Chaux de Fond and Val de Travers	£7 to £12	according to amount and character of load.	

At Rheinfelden (1899), $\frac{1}{2}$ *d.* per H.P.-hour ($\frac{2}{3}$ *d.* per unit) was charged to consumers taking 500 H.P.

At Schaffhausen the H.P.-year costs £4 to £6, according to quantity and to whether high-tension current is accepted or not.

In **Italy** the average operative cost of 1 H.P. for a year, as estimated in 1898 from all available figures, was £2 14*s.* from water-power (24 hours daily load) and £7 14*s.* from steam-power (3000 hours per annum).

Some details of a few well-known **high-tension power transmission** installations are given on p. 380. The selection includes examples of the larger and more recent power schemes, and, besides showing the practice as regards system, frequency and generator voltage, illustrates the transmission voltages which have been found suitable for various lengths of power line.

Power Transmission Line.	Date of Opening.	Maxm. Distance, miles.	H.P. trans- mitted.	Trans- mission voltage.	Generator volts.	Periods.	System.
Willesden (M. E. Supply Co.).....	1900	6	10000	10000	500	60	2-phase
Greenwich (L.C.C.).....	1903	10	16000	6600	6600	50	3-phase
Shepherd's Bush (C.L.R.).....	1900	6	5500	5000	5000	25	3-phase
Chelsea (Met. Dist. Ry.).....	1906	9	48000	11000	11000	33	3-phase
Neasden (Met. Ry.).....	1905	12	16000	11000	11000	33	3-phase
L.B. and S.C. Ry.	1909	20	7500	6000	6000	25	1-phase
Lancaster—Hewsham (M. Ry.).....	1908	7	500	6600	6600	25	1-phase
Newcastle-on-Tyne Elec. Power Co.....	1904	16	40000	30000	5750	40	3-phase
Glasgow Corp'n. Trams.....	1902	10	15000	6500	6500	25	3-phase
Liverpool—Southport.....	1904	18	11000	7500	7500	25	3-phase
Spiez (Burgdorf—Thun—Berne).....	1899	30	2700	16000	4000	40	3-phase
Jungfrau Ry.....	1900	10	1500	7000	7000	38	3-phase
Albula—Zurich.....	1908	85	20000	16000	7000	50	3-phase
St. Maurice—Lausanne.....	1902	35	5000	22500	4500	D.C.	series
Moutier—Lyons.....	1907	80	5000	57000	7200	D.C.	series
Cie. El. du Nord (canal haulage).....	1907	52	10000	45000	...	50	3-phase
Paderno—Milan.....	1898	32	13000	13500	13500	40	3-phase
Caffaro, Lombardy.....	1906	32	12000	40000	...	50	3-phase
Madrid—Molinár.....	1909	155	30000	66000	6600	50	3-phase
Soc. Iberica, Spain.....	1905	125	12000	33000	...	50	3-phase
Moosberg—Munich.....	1907	34	5500	50000	5000	50	3-phase
Dalmatia (Kerka) Carbide Works.....	1909	22	24000	30000	30000	50	3-phase
Hamburg—Altona.....	1907	9	7000	30000	...	50	1-phase
Kykkelsrud, Norway.....	1908	54	12000	50000	5000	50	3-phase
Bay County—San Francisco.....	1904	154	12000	60000	2200	60	3-phase
Telluride—Salt Lake City.....	1903	85	12000	40000	5000	...	3-phase
Snoqualmie—Tacoma—Seattle.....	1904	90	50000	50000	...	...	3-phase
Spokane Falls.....	1905	100	8000	60000	4000	60	3-phase
Central Illinois Ry.	1907	80	10000	3300	3300	25	1-phase
Niagara—Buffalo.....	1900	30	60000	22000	2200	25	2-phase
Niagara—Toronto.....	1906	70	100000	60000	12000	25	3-phase
Shawinigan—Montreal.....	1903	84	10500	50000	2200	30	2-phase

Working Pressure on Line.

The **efficiency** of transmission (omitting effects of low power factor) = $\frac{\text{power transmitted}}{\text{power generated}} = \frac{(E-e)C}{EC} = 1 - \frac{e}{E}$ where E and C stand for pressure at, and current from, the generator terminals, and e = volts lost through resistance along the line.

For the *same efficiency*, the ratio of lost volts to generated volts must be the same. The volts which may be lost increase therefore directly with the pressure of transmission. But, for a given power, the current causing this loss of volts varies inversely with the generator pressure. Thus—

The **weight of conductor** required is inversely as the square of the generator volts, or—

The **economy of transmission** with a given conductor is proportional to the square of the transmitting pressure.

The **cost of transforming** precludes the use of high-tension except for long lines. The cost of high-tension transmissions may also become excessive from local conditions requiring the use of an underground cable, which costs from 4 to 10 times as much as an overhead line.

The **cost of** almost every part of a power transmission **plant**—the copper (in machinery, line, etc.) alone excepted—increases rapidly with increasing pressure.

The **working pressure** most suitable for a given line depends in many ways on the **character of the load**. Thus if the load consists solely of polyphase induction motors, a relatively high working-pressure will be safe. (For efficiencies, see p. 342.)

Comparison of Direct and Alternating Currents for Power Transmission.

Although each case must be taken by itself, the following summary may be of use.

1. Line and Generators.

(a) **In favour of direct currents** are,—simplicity of system and controlling devices, superior *regulation*, greater accuracy and simplicity of *power measurements*. The power factor cannot be other than unity. A battery can be used, which is advantageous with certain kinds of loads. Direct currents will transmit full twice as much power along a given conductor as single-phase alternating currents of the same *maximum* pressure even with unity power factor and a favourable wave form. The disturbance in neighbouring telephone wires is relatively small. There is no capacity current, or danger of rise in pressure from the effects of resonance.

(b) **In favour of alternating currents.**—The practicability of using high tension, and thus greatly reducing the cost of long lines. As the high-tension windings of alternators is fixed, alternators can be built for pressures up to 15,000 volts; while 2000 or 3000 volts must be taken as the extreme limit of direct current machines.* D.c. Transformers (motor-generators) are expensive and similarly limited on the h.-t. side to 2000 or 3000 volts, while a.c. can be economically transformed up to or down from more than ten times this pressure (see pp. 397-8, "Limits of pressure"). In connection with high-tension switches and lightning arresters, it is found that the a.c. are is more easily extinguished than is the direct current. The alternating current gives rise to less electrolytic action, and to no magnetic disturbances except to telephones.

2. Motors.

(a) **In favour of the use of direct current** motors as compared with a.c. motors.—D.c. motors are more easily and efficiently regulated in speed. The a.c. Induction motor has a definite speed at a given frequency, and any attempt to regulate the speed by altering the slip means reduced efficiency.† Regulating by inserting resistance into the rotor of an a.c. motor is equivalent to controlling a d.c. motor by insertion of resistance into its armature circuit, instead of controlling the field. The method is not only inefficient, but results in a variation of speed with every change of load. With direct current motors the wiring and the power measurements are simple, and the power factor is unity. There is a slightly higher starting-torque. The clearance in a.c. motors has to be extremely small.

(b) **For alternating currents** Polyphase Induction motors are much more mechanical in construction, having no commutator and none but *fixed* windings connected to the mains, and the revolving windings being for low voltage only. Much higher pressures are practicable (even with small motors) than with Direct Currents. They need very little skilled attention.

For traction work, *single-phase* series motors constructed like d.c. series motors, but with compensating winding and laminated throughout, are being used with success at very low frequencies, as 15 periods. Such motors give efficient speed regulation. Repulsion motors (Elihu Thomson) and other single-phase motors combining the principles of the series, repulsion and induction types have been much developed of late for single-phase traction work (pp. 343 and 419).

* Unless connected in series as recommended by Thury and installed at Lausanne, Lyons, etc. (see p. 385).

† See p. 330.

3. Lamps, etc.

Direct current is preferable for arc lighting. And for glow-lamps, a.c. is impracticable below 25 or 30 periods per second owing to flickering (see also p. 388).

Some of the advantages, chiefly regulation and improvement of load factor, of direct currents are obtainable for alternate current circuits by the use of **rotary converters** and **batteries**.

Power Factor.

A drawback to the use of alternating currents for power transmission is their liability to get out of step, either in advance of the E.M.F. or behind it (most commonly the latter).

The **power supplied** from the line (assuming single phase) is then = line volts $\times$ current taken $\times$ power factor. And power factor = cosine of effective angle of phase difference between E.M.F. and current; or $\frac{\text{real watts}}{\text{apparent watts}}$. This factor

must be less than unity, and may in practice be even as low as .6. The current, and therefore line losses, are then correspondingly increased, the latter with the square of the current.

(a) *Inductance* in the circuit—such as (1) choking or reactance coils; (2) unloaded induction motors and transformers; and, in a lesser degree, (3) small motors at any load;—causes *lagging* idle component of current.

(b) *Capacity* in line, particularly in underground cables, or in the motors, etc., working on the circuit, causes a *leading* but equally idle component of current.

Lagging current decreases the effective excitation of the generators and so tends to drop the line voltage if not counteracted. Leading currents have an opposite effect.

When these two effects exist together, they tend to counteract each other (see below).

The **increase in the current required** for a given power due to lagging currents is about 10 % where the load consists mainly of large induction motors (over 20 H.P.), and over 30 % where the chief load consists of small motors, say under 10 H.P. The corresponding increases in line losses are 21 % and 70 %.

Methods of raising the power factor.—(1) The increased losses due to low power factor may be diminished by the effects of **capacity** in the line or specially inserted. But complete compensation by means of line capacity brings about the conditions for resonance of the circuit with a periodicity equal to that of the mains, and is accompanied by dangerous rises in the maximum pressure in parts of the circuit (see p. 126). Morley

(*Jour. I. E. E.*, vol. 43, 1909, p. 618) gives comparative estimated cost of obtaining leading current by (a) Mansbridge self-sealing paper condensers grouped in series, (b) Moscieki glass condensers, and (c) synchronous motors. This estimate favours the use of condensers.

(2) The most important and practical mode of regulating the power factor occurs in the use of **over-excited synchronous** machinery, *i. e.* synchronous motors and rotary converters, the latter being compounded on the direct current side to give the required excitation (see theory of synchronous motors, pp. 322 *et seq.*)* The power factor of such machines can be brought up almost to unity by suitable excitation of the field magnets of the synchronous motors.

(3) Heyland's device† for improving the P.F. of polyphase induction motors consists in introducing into the rotor winding suitable E.M.F.s obtained from the mains by means of a kind of commutator slip-ring, and thus bringing the rotor nearly into synchronism, thereby increasing the power factor. This or any other device which tends to make the motor approximate to the straight series type tends at the same time to bring the P.F. up to unity.

A power factor actually equal to unity is, however, only possible when E.M.F. and current have the same **wave form**. Apparatus and machinery tending to distort the form of current-wave (such as arc-lamps, reactance coils with unsaturated cores, unloaded transformers, etc.) will lower the maximum efficiency of any phase-correcting device.

Relative size of Conductors for various systems of distribution.

1. Continuous current and single-phase alternating current systems.

If the maximum transmitting pressure is fixed, the two-wire system is the most economical in conducting material, for all distances.

With equal working pressure, and a load which can be subdivided, a three-wire system permits double the transmitting pressure, and therefore double the "drop" for the same percentage loss in the line; while the current to be carried by the "outers" is only half. These require thus only $\frac{1}{4}$ the section of the conductors for a two-wire system.

With a middle wire for balancing of half or equal section to that of outers, the above 25% becomes increased to 31.3% and 37.5% respectively.

* Eborall, A. C., "Polyphase Substation Working," *Jour. Inst. E. E.*, 1901; also Pashall, H. F., "Electric Generators," 1901.

† See p. 324.

The following **relative weights** of copper or other conductors are required for different systems in order to transmit the same amount of power over the same distance with equal loss per cent. and equal working pressure—

System.	Relative Weight of Conductor.	Relative Transmitting Pressure.
2-Wire	100	1
3-Wire { All three of equal section...	37.5	} 2
{ Middle wire of half ,, ...	31.3	
4-Wire { All four of equal section ...	22.2	} 3
{ Inner wires of half ,, ...	16.7	
5-Wire { All five of equal section....	15.6	} 4
{ Inner wires of half ,,	10.9	

With four- or five-wire, or even three-wire distribution the P.D. between one terminal of a motor or lamp and the earth may exceed the working pressure of the motor or lamp.

Thury continuous current high-tension system.*—This is a simple two-wire system. A number of series-wound direct-current generators of 2000 to 2500 volts are connected electrically in series, and maintain a *constant current* in a long line. Motor-transformers placed in various sub-stations have their motor-windings connected in series with the line. Variations in load are met at the generating station by changes in the pressure of transmission. This system is employed by the Compagnie de l'Industrie Electrique of Geneva (see table, p. 380).

For local distribution the generator side of the motor-transformer consists either of an alternator of, say, 5000 volts (for distribution within a radius of some miles), or else of a D.C. generator of any required pressure.

The *machines are insulated* by mounting the bed-plate on porcelain insulators, and the coupling with the turbine or other prime mover consists of stout rubber links. The floor of the station near the machines is covered with an insulating layer of asphalte half-an-inch thick.

For successful working at constant current, the *motors*, which are series-wound, must run at *constant speed* whatever the load. This is effected by a governor or regulator which controls the value of a resistance shunting the field winding, and at the same time adjusts the position of the brushes. The resistance is attached to the motor and not placed on a separate switch-board.

* See Cuénod, *Journal Soc. Int. des Electriciens*, 1900, and *Elektrotechnische Zeit.*, Aug. 13, 20 and 27, 1902.

2. Polyphase current systems.

The table below shows the relative weight of conductor needed in order to transmit a given power to a certain distance with the same loss in every case.

The last column gives the relative cost of the different systems as far as the conducting material in the line is concerned, *when the P.D. is limited solely by the difficulty of proper insulation.*

The next table shows concisely the relative **weight of conductor required** by the various systems in order that each may transmit a given H.P. with equal economy and equal strain on the insulation. The figures must be increased by dividing by the square of the power factor when this is not unity.

System.	Weight with same max. P.D. between any Wires.
2-phase { Separate pairs of conductors all of same section }	100
2-phase { Common return having 1.41 times the section of others }	145
2-phase { Common return, but all three of equal section }	150
3-phase. Three equal conductors	75
3-phase { Four conductors, neutral of half area—star }	87.5
3-phase { Four conductors, but all four of same area—star }	100
“ Monocyclic ” with power-conductor of half section of others }	125

System.	Relative weights of conductors.	
	D.C. (two-wire) = 1.	
	P. f. = 1.	P. f. = .9.
Continuous current (2-wire)	1.0	1.0
3-phase	1.5	1.86
3-phase, with $\frac{1}{2}$ section neutral	1.75	2.17
Alternate current single-phase	2.0	2.46
2-phase, separate circuits	2.0	2.46
3-phase with equal section neutral	2.0	2.46
Monocyclic	2.5	3.08
2-phase, common return, 1.41 $\times$ section	2.9	3.58
2-phase, common return, equal to others	3.0	3.7

The star or **Y**-connection for windings of three-phase machinery is most used. In high tension transmissions the middle point is frequently earthed.

The "mesh" (or Δ) winding offers **difficulties of regulation** in distribution circuits in which lamps occur.

With rotary converters, and a continuous current motor or lighting load, regulation becomes easy and secondary batteries may be employed.

Three-phase transmission is more economical than two-phase. The inductive drop and lag in lines on this system is also much less ($\frac{1}{3}$ of that of a two-phase four-wire line of same length). Two-phase four-wire systems seem to admit of better regulation than three-phase. This is of importance with mixed loads, such as motors and lamps.

Single-phase (ordinary alternating current) systems are simple and cheaper than polyphase; but self-starting motors to run on such circuits are more complicated and less efficient and convenient than motors for polyphase circuits.

A **polyphase system** admitting of good regulation has been proposed by **Steinmetz**.^{*} Three ordinary single-phase generators running at the same frequency and connected in series by special switching arrangements will take up such a relative phase that the total E.M.F. through the system at every instant = 0. The three points of junction will furnish three-phase currents; while each machine may be loaded and regulated separately for lighting or other single-phase load. Modifications of this system are possible (*e. g.* separate boosters on each line).

A **polycyclic system** having many advantages has been proposed[†] in which, on ordinary alternating or three-phase current, is superposed a current of three times the frequency. Direct current may also be superposed.

Frequency.

Those most generally employed are

50 ~ for H.T. long distance distribution.

25 ~ for transmission to converter sub-stations for D.C. traction.

15 ~ for single-phase traction with a.c. motors.

The frequency should not be greater than 60 periods per sec. for power distribution, nor less than 25 or 30 ~ for lighting. The frequency chosen for mixed loads is usually between these limits. (See also table on p. 380.)

^{*} *Electrician*, 1899, p. 236.

[†] *Arnold, E. T. Z.*, 1902, p. 569.

The use has been suggested of a **special three-phase lamp** having three filaments joined across the phase (**Y-fashion**). By this means it is possible to use 16 and even 8 C.P. glow-lamps with no appreciable fluctuation at a frequency of 25 ~ or even less.

Rotary converters regulate better and are more reliable at low frequencies. Transformers cost more for low frequencies than for high.

A drawback to the use of low frequency currents, apart from the difficulties with lamps, is the limit which it places on the **speed of small motors**. Thus, 25 periods per sec. corresponds to 3000 poles per min. This, with a four-pole winding, means only about 700 revs. per min., which is extremely slow for a small motor (see table below). Induction motors having less than four poles are rarely used for constructive reasons.

From most other points of view, however (reduced induction drop and capacity currents in line, and less interference with telephone lines, and improved regulation), a low frequency is desirable, and 25 ~ is now the standard for long distance power transmission schemes involving traction work.

The standard frequency in Germany is 50 ~ for all but the very largest schemes.

TABLE OF FREQUENCY AND SPEED OF SMALL MOTORS.

Frequency.		Speed.*		
English and Continental.	American.	4-pole winding.	6-pole winding.	8-pole winding.
Periods per second.	Poles per minute.	Revs. per minute.		
~				
15	1,800	450	300	—
20	2,400	600	400	—
25	3,000	750	500	375
30	3,600	900	600	—
40	4,800	1,200	800	600
50	6,000	1,500	1,000	—
60	7,200	1,800	1,200	900
70	8,400	—	1,400	—
80	9,600	—	1,600	—
83 $\frac{1}{3}$	10,000	—	1,670	1,250
100	12,000	—	—	—
125	15,000	—	—	—

* These figures give the speed of synchronous running. A deduction amounting to 2 % with large motors, and 5 % or even more with small motors, must be made from these figures in order to allow for the slip of the loaded motor.

Weight and Resistance of Power Transmission Lines and Feeders.

For the same number of volts lost, the weight of metal required for power transmission varies with the square of the distance the power has to be transmitted.

The **weight** of a conductor a sq. in. in cross section is—

For copper . . .	9.10a	tons per mile, or	11.58a	lbs. per yd.
aluminium . . .	2.66a	“ “	3.39a	“ “
iron . . .	7.98a	“ “	10.15a	“ “

The **formulae** below relate to **good commercial copper and aluminium**, and are based upon the values given for **specific resistance** at 60° F. in the following table—

Material.	Microhms per cm. cube.	C.G.S. units per cm. cube.	Microhms per inch cube.	C.G.S. units per inch cube.
Copper	1.7	1700*	.669	669
Aluminium	2.8	2800	1.102	1102

Good hard drawn copper conductors will have a 2% higher resistance than figures given above; and very pure soft copper a specific resistance at least $1\frac{1}{2}\%$ lower.

The densities of copper and aluminium are taken in the formulae at 8.9 and 2.6 respectively.

$$\therefore \text{Relative density} \frac{\text{Copper}}{\text{Aluminium}} = 3.42 = 10:3 \text{ nearly.}$$

$$\text{Relative conductivity} \left(\frac{\text{Copper}}{\text{Aluminium}} \right) = 1.65 = 5:3 \text{ nearly.}$$

Conductivity of aluminium = 60.7% that of copper.

Some commercial aluminium has conductivity higher than this; Wilson † gives the figure 61.5 to the same standard; and the $1\frac{1}{2}$ sq. ins. aluminium feeder cables for the N.W. Elevated Railway, Chicago, are stated to have a conductivity of 62% of fine copper, or 62.5% of above figure for copper.

Lost volts or drop.—This is equal to (current density) $\times$

* This corresponds to 99.5% conductivity according to the standard for soft copper recommended by a joint committee representing the Inst. E. E., the G. P. O., and cable manufacturers (*Electrician*, December 22, 1899, p. 302), and does not differ by more than $\frac{1}{2}\%$ from Matthiessen's standard of soft copper as variously given by different authorities. It is the same as the figure 1.696 microhms per cm. cube at 15° C. frequently quoted as Matthiessen's standard.

† *Jour. Inst. E. E.*, 1902.

(specific resistance) $\times$ length; the factors being expressed either all in inch or all in centimetre measure.

The allowable current density varies inversely with the specific resistance. The product of these two is therefore independent of the kind of metal used for the line.

One thousand amps. per sq. inch for copper being taken as basis, the current densities for copper and aluminium are

		Current densities.	
		Amps. per sq. inch.	Amps. per sq. cm.
Copper	.	1000	155
Aluminium	.	607	94

For conductors of any material in which the current density is taken as stated above,

$$\text{Lost volts} = \begin{cases} .000669 \times \text{length in inches.} \\ .000264 \times \text{length in cms.} \end{cases}$$

Also,

		Volts.
Volts lost per 100 yards of single lead	=	2.41
" " " 1000 feet	" "	= 8.03
" " " mile	" "	= 42.4
" " " kilometre	" "	= 26.4
and 1 volt every 41½ yards.		

These figures must be increased by 4 % for every 10° C. (18° F.) above 15½° C. (60° F.).

Resistance of a single cable

$$= \frac{.0424}{a} \text{ ohms per mile.}$$

The above figures apply to all conductors in which the current density corresponds to 1000 amps. per sq. inch in copper.

For different current densities, these figures may be altered in the same proportion.

A current density of 415 amps. per sq. inch in copper (and corresponding densities in other materials) gives a drop of 1 volt for every 100 yards.

The weight of conductors of different materials having the same length and giving the same "drop" with a given current varies directly with the product of specific resistance $\times$ specific gravity.

Watts lost.—The ohmic loss per unit volume of any conductor

$$= (\text{current density})^2 \times (\text{sp. res.}).$$

In copper at 1000 a sq. in., the loss is

2.1 watts per lb. (cold), or

2.5 watts per lb. (50° C. rise).

Relative weight of electrically equivalent conductors of

$$\text{copper and aluminium} = 2.08 = \frac{1}{0.48}.$$

For overhead lines and feeders, the two metals are equally economical when the price of aluminium per ton = 2.08 $\times$ price of copper per ton; thus, aluminium at £125 is equivalent to copper at £60 per ton.

Against the use of aluminium for power lines is the greater

TABLE OF TENSILE STRENGTH, ETC.

Material.	Conductivity. Pure soft copper=100.	Specific gravity.	Weight of equivalent conductors. Pure soft copper = 100.	Conductivity for equal weights.	Ultimate strength. ⁴		Conductivity for equal weights × tenacity. Hard Cu. =100.
					Lbs. per sq. inch.	Tons per sq. inch.	
Copper, hard drawn.....	97	8.9	103	97	60,000 ⁵	27	100
Aluminium, commercial hard drawn.....	59 ³	2.715	51	194	28,000	12.5	93
Steel wire (for ropes).....	10.5	7.85	833	12	140,000	62.5	29
Galvanized iron wire.....	14	7.7	625	16	56,000	25	15
Silicon Bronze.....	(?) 97	8.9	103	97	63,000	28	105
" (different	80	8.9	125	80	76,000	34	105
" composition)	45	abt. 8.9	222	45	110,000	49	85
Aluminium Bronze.....	abt. 50	abt. 3.5	79	127	63,000	28 to 32	137
Light ² } Al 97.8, Cu 1.58.....	50.7	2.75	60.7	165	39,000	17.5	110
alum- } " 96.8, " 1.08,							
inium } Ni 1.29.....	49.7	2.75	61.8	162	45,900	20.5	127
alloys } Al 97.1, Cu .09,							
} Mn 1.78.....	48.6	2.75	63.4	158	35,300	16	102

¹ See *The Development and Transmission of Power*, Unwin, 1893, and Wilson, *Jour. Inst. E. E.*, 1902.² Wilson, *loc. cit.* Tests on wires $\frac{1}{8}$ " diam. Each of the three samples contains in addition 4% of Fe and of Ni.³ Soft aluminium can be obtained exceeding 60 per cent. conductivity (*vide ante*, p. 389).⁴ The limit of elasticity of hard drawn copper and aluminium wires can be taken at about 12 tons and 8½ tons per square inch respectively.⁵ The tensile strength of hard drawn copper varies from 50,000 lbs. per square inch for large sizes (as No. 0000), to 65,000 lbs. per square inch for smaller wires (as No. 10).

size of wire (28 % above diameter of equivalent copper) and consequent increased effect of wind pressure and snow accumulation, especially since tensile strength is less (p. 391). But, on the other hand, the larger diameter admits of working at a higher voltage without **discharge loss** (p. 398).

Mechanical joints in aluminium feeders are generally used; though soldered, electrically welded, and cast joints have been tried. The McIntyre joint* is effected by placing the ends in a flat tube of thin aluminium, 9 inches long and $\frac{1}{16}$ thick, giving the tube two or three complete twists, and securing the projecting ends. A good **solder for aluminium** is said† to consist of 70 parts of the metal to 20 of tin and 10 of copper. It gives no galvanic action; and the most suitable flux is potassium fluoride. But as the alloys of aluminium formed near soldered joints are often unstable and liable to become crystalline, a really successful solder is extremely difficult to find. Aluminium is also so *electro-positive* that the presence of almost any other metal will give rise to corrosion.

At its present price, aluminium cannot compete with copper for *insulated* cables, owing to the cost of the increased insulation required to cover the larger wire. The larger wire, on the other hand, has this advantage, that it involves a correspondingly smaller gradient of dielectric stress‡ in the insulating material next the wire.

In column 8 will be found relative values of the product of conductivity for equal weights, with the ultimate strength; this quantity is a measure of the value of the material for electrical conductors having long overhead spans.

Tension and Sag.

The sag depends on the weight of the wire per foot run, on the square of the span, and varies inversely with the tension. For practical purposes,§ the curve is assumed a parabola for which

$$\text{sag} = \frac{\left(\frac{\text{wt. per unit length}}{\text{of wire}} \right) \times \left(\text{span} \right)^2}{8 \times \text{tension}},$$

consistent units of length and weight being employed. Whence, for copper (p. 389),

$$\text{sag in inches} = \frac{5.8 \times \text{sect. area (}\square\text{'')} \times [\text{span (ft.)}]^2}{\text{tension in lbs.}}$$

* *Electrician*, October 27, 1899, p. 9.

† *Electrician*, 1898, p. 389.

‡ See O'Gorman, *Jour. Inst. E. E.*, 1901.

§ Otto, *E. T. Z.*, 1903, p. 37; giving tables and diagrams for 40 m. span.

Owing however to the elastic extension of the wire the sag for a given tension is more than this. Accurate methods of calculation including the effects of temperature have been given by Hopkinson * and Sengel.†

In calculating the **tension and sag** ‡ of overhead conductors, and the change of sag with temperature, account must be taken of the alteration of elastic constant with temperature when choosing a value of the temperature coefficient. (See Perrine and Baum, *Am. I. E. E.*, 1900, and Wilson, *loc. cit.*, in the case of aluminium and light aluminium alloys.)

The German I. E. E. state that **wind pressure** to the extent of 125 kg. per sq. m. (25·6 lbs. per sq. ft.) should be allowed for on ·8 of projected area of wire surface. This amounts to 20½ lbs. per sq. foot on the projected area. Ample margin should be left for snow deposits.

The relative weights of electrical equivalent conductors will be found in the fourth column of foregoing table. Aluminium has in this respect the advantage.

As a conductor of direct currents iron is considerably the cheapest, but its weight, its volume and its liability to corrosion preclude its use for overhead transmission. The use of iron as a "third rail" in electric railways accounts largely for the economy of transmission effected by such systems (see p. 410 and p. 101).

The presence of less than 1 % of copper and other metals in aluminium may increase the ultimate strength in a much greater ratio than it diminishes its conducting power (see table).

For Board of Trade regulations for overhead wires, see pp. 442 and 544.

Capacity.

The **electrostatic capacity of overhead lines** is given approximately by the following formulæ—

(a) *Two wires side by side* (at least 10 diameters apart and relatively far from the ground)—

$$\begin{array}{l}
 D = \text{distance between wires} \\
 d = \text{diameter of wires}
 \end{array}
 \left. \vphantom{\begin{array}{l} D \\ d \end{array}} \right\} \text{in same units, ins. or cms.}$$

$$\text{Capacity} = \frac{0.0194}{\log_{10} \frac{2D}{d}} \text{ per mile} = \frac{0.0121}{\log_{10} \frac{2D}{d}} \text{ per km.}$$

in mf.

* B. Hopkinson, *Electrician*, Jan. 25, 1901; with tables for 120 ft. span.

† Sengel, *E. T. Z.*, 1903, p. 802.

‡ See also p. 441.

(b) *One wire alone*—

h = height above ground
 d = diameter of wire } in same units, ins. or cms.

$$\text{Capacity in mf.} = \frac{.0388}{\log_{10} \frac{4h}{d}} \text{ per mile} = \frac{.0241}{\log_{10} \frac{4h}{d}} \text{ per km.}$$

The charging current in each wire of a three-phase line amounts to $2/\sqrt{3} \times$ the charging current of a single-phase line spaced with wires the same distance apart.*

Neighbouring trees or buildings may increase the capacity of a single wire by 20 % above the value calculated by above formulæ.

The apparent capacity of long H.T. lines is often greater in wet weather than would correspond to the formulæ.†

Electrostatic capacity of insulated cables ‡ is as follows—

d_1 = diameter of inner conductor.

d_2 = inner diameter of outer conductor, if concentric cable ;
of sheathing, if a single conductor.

These being expressed in terms of any one unit, ins. or cms.—

k = dielectric constant of insulating material (from 2 to 4)
(see p. 84).

$$\text{Capacity} = \frac{.0388k}{\log_{10} \frac{d_2}{d_1}} \text{ microfarads per mile.}$$

The capacity current into the outer conductor of a concentric cable = that into inner (as by formula below) + that due to outer sheath (also calculated by formula, inserting half-voltage if the sheath have the neutral voltage).

The “capacity” (defined to answer to the following formula for capacity current) of a **three-phase cable** (not concentric) is

$$\frac{.0448k}{\log_{10} \frac{3a^2}{r^2} \left(\frac{R^2 - a^2}{R^6 - a^6} \right)} \text{ mf. per mile.}$$

where r is the radius and a the distance between centres of the conductors ; and R , the inner radius of the sheath.

The **capacity current** (in each conductor if three-phase) in amperes is given by the formula—

$$2\pi \times \text{frequency} \times (\text{R.M.S. voltage}) \times \text{capacity (mf.)} \div 10^6.$$

* Perrine and Baum: *Am. I. E. E. Trans.*, 1900, and *Science Abstracts*, No. 2325, 1900.

† See *Electrician*, October 20, 1899, p. 911.

‡ For full discussion see Lichtenstein, *E. T. Z.*, 1904, pp. 106 and 124. Other references are:—Fleming: *Handbook to the Electrical Laboratory*, vol. ii., and *Proc. Phys. Soc.*, 1904; Leblanc: *Tr. Am. I. E. E.*, 1902, p. 525; Kath: *E. T. Z.*, 1903, p. 38.

In using this it must be noted that relatively large high frequency capacity currents may flow from low frequency generators owing to periodic irregularities in the wave form, such as occur by reason of the teeth of the slotted stator. And if, as may happen, the voltage at which such capacity current of higher frequency flows is raised greatly through effects of resonance; then, since the frequency is also high, this subsidiary capacity current may even be greater than the normal capacity current.

Inductance.

The **inductance** (coefficient of self-induction) of **two parallel wires** carrying same current in opposite direction —

Let D = distance between wires }
 d = diameter of wires } in same units, ins. or cms.

$$\begin{aligned}\text{Inductance} &= \cdot 00148 \log_{10} \frac{D}{d} \text{ henries per mile.} \\ &= \cdot 00092 \log_{10} \frac{D}{d} \text{ henries per kilometre.}\end{aligned}$$

Inductance of a **single wire**—

h = height above ground }
 d = diameter of wire } in same unit.

$$\begin{aligned}\text{Inductance} &= \cdot 00074 \log_{10} \frac{2h}{d} \text{ henries per mile.} \\ &= \cdot 00046 \log_{10} \frac{2h}{d} \text{ henries per kilometre.}\end{aligned}$$

Inductance of concentric cables is very small and for practical purposes negligible.

A table of the resistance and reactance (from which the impedance can be obtained) per mile of line calculated for different distances between wires, for different frequencies, and for every gauge No. (B. and S.) between 0000 and 10, is to be found in Franklin and Williamson's *The Elements of Alternating Currents*, New York, 1899, p. 200.

Louis Bell* gives the following table of inductance and capacity of ordinary overhead lines per mile. The figures apply to copper wires of the gauge stated and suspended parallel and 18 inches apart. Under these circumstances the height of the wires above ground does not affect the results—

* *Electric Power Transmission*, New York, 1897, p. 394.

Gauge No. B. and S. (see p. 442 for S. W. G.)	Inductance. per mile.	Capacity. per mile.	Impedance factor.	
	Millihenries.	Microfarads.	60 ~	120 ~
0000	1.48	.0102	2.3	4.3
000	1.52	.0099	2.0	3.7
00	1.56	.0097	1.79	3.08
0	1.59	.0095	1.53	2.54
1	1.63	.0093	1.39	2.16
2	1.67	.0090	1.28	1.85
3	1.71	.0088	1.19	1.61
4	1.74	.0086	1.12	1.42
5	1.78	.0084	1.08	1.30
6	1.82	.0083	1.05	1.21

In long overhead high-tension transmission lines, and in most underground electric cable transmissions the **capacity** of the line at light loads not only **neutralizes** the relatively small **lag** due to large polyphase motors, but converts it into a lead ; the C^2R losses in the line being much reduced, but the difficulties of regulation greatly increased.*

Insulation.

The insulation of H.T. apparatus should be subjected to testing pressure for at least an hour (6 hours according to many continental specifications). But the testing pressure, especially for cables, should not be excessive for fear of permanent injury to insulation.

The insulation of **cables** must, by the Board of Trade Regulations, be subjected before putting into service to a testing voltage double that for which they are intended, provided that the working voltage does not exceed 10,000 volts. If it does, the testing voltage must be 10,000 volts more than the service voltage. The voltage for testing cables both for insulation and capacity is generally obtained from a small independent motor-generator set and H.T. transformer ; the output of the generator and transformer being several times greater than that of the motor owing to low power factor of current taken by the cable.

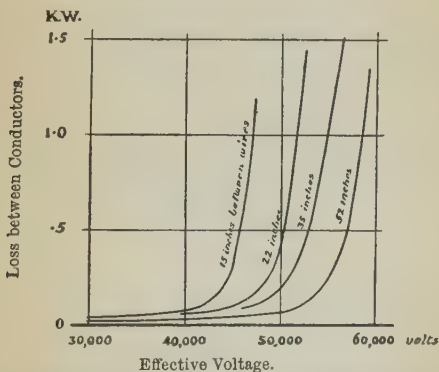
The **insulators** used on high pressure transmission lines

* See pp. 383-4 ; also W. M. Mordey, " Capacity in Alternate Current-working," *Jour. Inst. E. E.*, January 1901.

should be of porcelain, or better still of glass, doubly or trebly petticoated. For work above 10,000 volts, they must be selected by subjection to an electrical pressure double that for which they are intended, and frequently to a shower of water equivalent to $\frac{1}{4}$ " per min.

Insulators using oil must be so constructed as not to be liable to be short circuited by insects.

The figures in the table of striking distances (p. 96) are used in **testing** insulators (and also cables and transformers, etc.) for high-tension work. An adjustable spark-gap in series with a H.T. fuse (and occasionally a choking coil or high resistance) being generally used in place of a voltmeter.



Loss per mile of H.T. Transmission with conductors at various distances at 60 ~ (C. F. Scott).

FIG. 147.

A **limit of high pressure** is reached owing to silent discharge which takes place from bare wires and renders them slightly luminous when the pressure is sufficiently high. C. F. Scott* showed that the loss from this cause may become appreciable at pressures exceeding 40,000 volts, especially if wires at this P.D. are supported not very far apart.

In a three-phase 58,000 volt line, a loss of 10 kw. was observed for 124 km. length of line. The neutral point was earthed.† The loss on a $2\frac{1}{4}$ mile three-phase line between wires at different distances apart amounted to $\frac{1}{2}$ kw. and to 2 kw. respectively

* *Jour. Amer. Inst. E. E.*, vol. xv., 1898, pp. 673-718.

† *Elektr. Zeitschr.*, 1904, p. 598.

where the pressures reached the following values (see also Fig. 147)—

Distance between wires. Inches.	Effective Pressure.	
	Loss = $\frac{1}{2}$ kw. at	Loss = 2 kw. at
15	44,000 volts	46,000 volts
22	48,000 "	51,000 "
35	51,000 "	54,000 "
52	54,000 "	58,000 "

At still higher pressures the loss increased rapidly (nearly 1 kw. for every 1000 volts increase). Below 40,000 volts the loss did not exceed 200 watts on this length of line.

For two-metre spacing and for wires of 2 mm. or more in diameter (d mm.), the a.c. kilovolts (r. m. s.) at which discharge should commence are approximately $40 + 12d$.

The **loss through discharge** depends on the maximum dielectric stress at the surface of the wires and is thus larger with a peaked voltage wave. It increases with the amount of moisture in the air. It is slightly less when the barometer is high and it is less between conductors of large diameter, or if placed wider apart.

The **loss through "creeping"** is also serious at very high voltages, but in dry weather it is small compared with the discharge loss.

The losses due to both causes have been carefully studied by Prof. H. J. Ryan, Ralph D. Mershon, and E. A. Watson,* the latter giving a complete discussion of all available information, together with experimental data of H. T. direct current loss.

Lightning Arresters.

Lightning arresters† are required on all overhead power transmission lines (including feeders for traction work), and even on underground feeders when these form part of a circuit exposed at any point. They are especially wanted in exposed parts of overhead lines traversing hilly country. The rapidly

* "Conductivity of Atmosphere at High Voltages," Ryan, *Proc. Am. I. E. E.*, 1904, pp. 275-318. "High Voltage Measurements," Mershon, *Proc. Am. I. E. E.*, June 1908, pp. 1027-1083. "Losses off Transmission Lines," Watson, *Jour. I. E. E.*, 1910, vol. xlv, p. 5.

† "The Protection of Electric Power Transmissions from Lightning," by J. T. Morris, *Electrical Review*, January 5-26, 1899.

extending use of high pressure overhead power transmission lines is largely due to the recent introduction of efficient lightning arresters.

Their **object** is not to prevent the lightning from striking the line—this is a rare occurrence, and protection from direct stroke is not possible—but to afford protection from the results of a charging of the line due either to induced surgings caused by a lightning discharge in the vicinity, or to slow accumulation of charge in the line. If the line-wire becomes charged, a spark to earth will ensue at some point, and the object of the lightning arrester is to prevent the line current from following this spark as an arc to earth.

Excessive **static charging** of the line during thundery weather can be **prevented** by the following means—

(a) A wire suspended above transmission line and earthed at intervals. Thus at Niagara, a barbed wire is strung 15 inches above the transmission wire, and earthed every 200 or 300 yards.

(b) A permanent leak to earth, enabling the line to discharge itself. The **Tank lightning arrester** is an example of this. The line is earthed during sultry or stormy weather by connection with a well-earthed tank full of running water. This is used in America for the protection of overhead tramwires. A continual leakage amounting to 12 amps. in all from an extensive 50,000 volt circuit effectually prevents any but impulsive charges on the line, and these are discharged by ordinary lightning arresters.

On the Continent, the **Water-Jet Discharge** is much used. A jet of pure water flowing under high pressure impinges on a metal plate attached to each H.T. line, and then falls away in a discontinuous stream. The jet is sometimes replaced by a column of flowing water in a glass tube.

(c) Generally the protection is as follows:—Between line and machines, choking coils are placed to intercept impulsive discharges; while between line and earth, one or more spark gaps are used, generally in series with some form of non-inductive high resistance. The various forms of gap are described below. For the resistance, where flowing water cannot be used, or if electrolytic resistances are unsuitable, a rod of high-resistance carbon composition is best to employ.

The **Brazil Resistance** consists of fire-clay slabs, each moulded with a zig-zag groove which is filled with composition of high resistance leading out to suitable terminals. This resistance possesses the great advantage that it is practically indestructible, cannot burn, and does not clog up, boil away, or freeze.

In the "**Ball**" or "**short gap**" lightning arresters the spark occurs between two or a series of massive two-inch metal cylinders

which promote the extinction of the arc by simple cooling. These cylinders are placed about $\frac{1}{3}\frac{1}{2}$ inch apart, and are in series with a high carbon resistance. Fourteen gaps are claimed to break an 11,000 volt arc. (Fig. 148.)

Each cylinder may also be separately connected by a fuse to earth, this affording an additional protection. The fuses then go one by one.

The **Wurts** 1892 arrester (for continuous current only) depends for its action upon the fact that an alternate current electric arc is only maintained with great difficulty between points of zinc and antimony and certain alloys. This property is confined

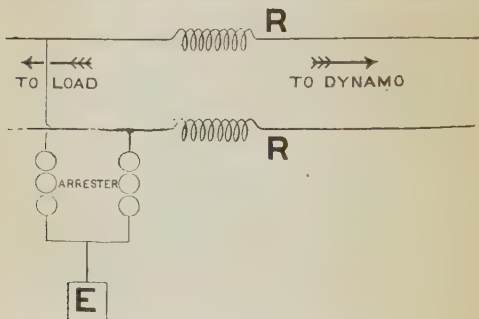


FIG. 148.

to short air-gaps only, in which the vapour fills the intervening space during the arc. Arresters depending entirely on the use of non-arcing metals are found not to work well in series with a resistance to earth.

The arrester of the type made by the **General Electric Co.**, of U.S.A., for 1000 volt circuits, consists of seven solid cylinders of an alloy of copper and zinc, each one inch diameter by three inches long, having a rough surface, and supported side by side in a porcelain frame with an air-gap of $\frac{1}{3}\frac{1}{2}$ inch between each. The cylinders can be revolved to expose fresh surface. The central cylinder is to earth and the others to the two line wires. This firm recommended one gap for every 300 effective volts to earth.

Siemens or "Horn" lightning arrester.—This is the simplest type and the most extensively employed on alternate current H.T. power transmission circuits on the Continent. The arrangement is seen in Fig. 149, one horn being connected to the line and the other to earth. In some cases for high-tension

three-phase lines, a central wire is connected to earth and three bent wire horns, about a foot in length, are arranged symmetrically around it, but three separate arresters are employed by preference.

The extinction of the arc in this arrester is rather slow, producing a heavy momentary overload; but it is found to be very sure in its action, besides being relatively cheap and having neither covered wire coils nor moving parts to get out of order.

It is in all cases essential that the **earth connection** of a lightning arrester, or of any other lightning protection device, shall be short, stout and straight, and leading to a good earth. Lightning arresters or conductors installed with a bad earth are a source of increased, not reduced danger from lightning.

In **damp soil** a coil of several turns (about two feet diameter) of the wire from the arrester may be buried in the ground. In **dryer** parts the arrester wire may be soldered to a 1-inch iron pipe 8 to 12 feet long, driven in the ground, the upper end of which projects into a large hole filled with coke well beaten down and covered with earth.

In **towns**, water or gas supply mains form an excellent earth, and may be used whenever no continuous current to earth is likely to flow for any lengthened period.

"**Choke-coils**" or "**Reactances**" are often employed (see Fig. 148, which shows a pair adapted to a Wirt arrester) to stop impulsive discharges and are to be found on most transmission lines where the line enters a station, just past the last arrester.

The General Electric Co., of America, recommend placing one choke-coil between each lightning arrester, and construct the coils out of 150 feet of line wire.

Distribution of lightning arresters along a power transmission line.—In long lines, or tramway feeders where no power is used except from the end of the line, arresters are needed only at the ends of the line; but if power is used at intermediate points along the line, as on tramway overhead wires or in industrial districts, from one arrester per mile in sheltered places, to twelve per mile in exposed parts, are required.

For tramcar lightning arresters, see p. 431.



Siemens Lightning Arrester on overhead line.

FIG. 149.

LIGHTNING CONDUCTORS.

The rules for erecting these are given differently by different authorities. The following are to some extent agreed upon—

Points from 2 to 4 feet high, and attached to all prominent parts of roof of building—not much more than 50 feet apart.

(According to Preece, a single conductor protects a conical space whose base has a radius equal to the height of the conductor.)

The points to be screwed or riveted to conductors running along all ridges of the roof, arching (with a point attached) over all tall chimneys, and connected to all external masses of metal, lead roofing, cisterns, etc. (which should preferably be separately earthed as well by straight wire).

The conductor (if for an extensive building) is then to be led down near each corner and well earthed. All **bends** should be as gradual as possible.

The rules for **earthing** lightning arresters (given on p. 400) hold good also for the “earth” and for the conductors leading to earth of lightning conductors.

The conductor must be all of one metal (galvanized iron or copper) and have as few **joints** as possible; and these, riveted rather than soldered. Allowance must also be made for expansion (about 1 inch in 60 feet).

(According to Lodge, the only advantage of copper over iron is that it does not rust. Iron besides being cheaper, is in every other way to be preferred.)

The **Cross section** may be either flat or round according to convenience. The area is usually not less than $\frac{1}{4}$ square inch, or No. 3 gauge. Iron conductors should be of same sectional area (Lodge); of same conductivity (Kelvin and Jamieson) as conductors of copper.

The conductor should *not be insulated* unless in the case of tall chimneys (Lodge); and it should be kept away from fusible gas-pipes inside the building.

The resistance from the tip of the conductor to an “earth” near to that of the conductor itself should not be greater than 1 or 2 ohms.

Electrical tests * to be of value in maintaining a conductor in good condition must be frequent and always recorded (Bucknill).

The War Office rules for ammunition, stores, etc., and other official instructions will be found in Lodge's *Lightning Conductors*, as also Admiralty rules for protecting wooden vessels.

* For this purpose a bridge set similar to that described on p. 165, only employing an alternating current, is the handiest.

DANGER FROM ELECTRIC SHOCK.

Messrs. Siemens and Halske of Berlin, in some experiments preliminary to drawing up rules for dealing with dangerous electric circuits, have determined the following values *—

Resistance of arms and breast (*i.e.* from hand to hand neglecting skin) 500 ohms.

Resistance of the skin (moist) 50,000 ohms per square cm.
or 7750 ohms per square inch.

A surface of contact of 10 square inches for each hand means therefore a resistance of over 2000 ohms between the hands. An even less resistance to earth has been found through moist hands and wet feet. In such extreme cases of good contact, even 100 volts may prove dangerous.

Messrs. Siemens and Halske regard a current of .03 amps. as dangerous and $\frac{1}{10}$ amp. as deadly; and they issue rules for special cases based on these limits.

Prof. H. F. Weber by direct experiment finds a current of .02 and .03 amp., alternating (or even less continuous), to be unbearable; and states that an electric shock can rarely be tolerated unless the body have a total resistance of at least 40 ohms per volt.† Reference should also be made to Trotter's paper (*Jour. I. E. E.*, 1902, p. 762).

Treatment of a Case of Apparent Death from Electric Shock.

—An electric shock, especially if from an alternating current circuit, rarely does more at first than arrest respiration.

In case of any one rendered insensible through electric shock, any live wire with which he may be in contact must first be disconnected, or, failing this, removed with proper precautions such as an insulating mat and rubber gloves, or, failing these, thick dry cloth; and, a doctor having been sent for, resuscitation must be attempted by the methods adopted in cases of apparent drowning.‡

* *Electrotechnische Zeitschrift*, Aug. 24, 1899, p. 602.

† See also Boyd, *E. T. Z.*, 1898, 728.

‡ Cf. *Elec. Review*, "Suggestions re Electric Shock."

ELECTRIC TRACTION.

By D. K. MORRIS, PH.D.

SYSTEMS EMPLOYED FOR STREET LOCOMOTION.

1. OVERHEAD WIRE SYSTEMS.

THE **single trolley** system is the most generally employed. It is the cheapest to erect, the easiest to repair, and will pay in a scattered district where no other system would.

The **overhead wire** (p. 440) is supported 16 feet to 20 feet above the track by—

(1) **Span wires** stretched across the street between poles on either side. This construction is often objected to as unsightly, and the passage of vehicles exceeding a certain height is prevented.

(2) Brackets from **side poles**.

(3) Brackets from a **centre pole**.

The brackets may be relatively short if the side trolley is employed.

The **overhead contact** (p. 439) is generally made either—

(1) By **trolley pole** or arm. A light steel tube trailing upwards over the car carries at its upper end a small flanged bronze pulley, pressing it firmly against the overhead wire by means of springs at the base of the arm. The trolley wheel is connected by a cable passing down the arm to the car controller. The arm can swivel at its base about a vertical axis, and thus follow the wire. The trolley wheel may also swivel about a vertical axis (**Dickinson side trolley**), and the overhead wire may then be supported over the side of the track. Where single track is laid with rail switches at intervals for passing cars, a double overhead wire is usually provided over the whole length of the line, to avoid overhead points or "**frogs**" at these places.

(2) By **Sliding bow** (Siemens). An aluminium rod, often grooved for carrying lubricant, is supported at right angles to the overhead wire by a light steel frame, and pressed upwards against the conductor by springs. The overhead wire must in this case conform roughly to the track.

Of these constructions, the most common in this country (where roof seats are generally provided) is the side trolley,

mounted on a hollow cast-iron standard inside which are the springs controlling the trolley arm and down which the main cable passes.

Another system is used in special cases in which the current returns by a **second trolley** arm and overhead wire instead of by the rails.

A similar construction with two side trolleys was introduced near Dresden (1901) for electrical **omnibuses** running on an ordinary road **without rails**. This system has since been considerably extended both in Germany and France. Three methods* of controlling the overhead contact are employed. These are—(1) the Schiemann system with two sliding swivel contacts on ordinary trolley poles; (2) the A. E. G. (Stoll) system with two rolling contacts (over the wires) which are joined by a frame attached to the car by flexible cables; and (3) the Lombard-Gérin system with a motor-driven contact-carriage (50 lb.) running on the wires ahead of the car to which it is joined as in the last case by flexible leads. The two wires are suspended about 15" apart over the side or the middle of the road. Cars can run 12 ft. aside from the wires. About 20 miles of systems (1) and (2) were installed (1904) near Dresden, Berlin, etc. More energy is needed on account of the absence of rails—200 watt-hours per ton-mile as against about 80 watt-hours with rails. But the capital cost is much less in the former case. (The energy for electric cabs in New York averages (1904) 1 B. T. U. per car-mile.†)

Such methods entirely avoid earth currents and consequent electrolytic corrosion of pipes, disturbances of telegraphs, magnetic instruments, etc. But the overhead work is difficult owing to the electrical pressure between the wires where lines intersect.

Where the lines are long, or where from any cause the electric power must be transmitted over long distances, **three-phase** high-tension alternating currents are employed. To use these on tramway circuits they are commonly transformed down to about 350 volts on each phase, and then converted by means of rotaries or by motor-generators to 500 volts direct current.

High-tension currents cannot well be used for tram wires over streets, but the following **three-phase system** avoids the conversion to direct currents. Two overhead wires are needed, the rails forming the third conductor. The cars are driven by three-phase induction motors. Such motors do not run economically except at their full speed. The possibility of using relatively high tension on the motors is of no advantage for

* Schiemann, *E. T. Z.*, 1903, pp. 705 and 1020; A. E. G., *E. T. Z.*, 1903, pp. 668 and 705.

† See also p. 439.

trams, as such pressures cannot be permitted on bare wires over roads. The overhead work is very complicated at crossings. On the other hand, alternating earth currents cause little corrosion, and magnetic disturbances are small and confined to telephone circuits.

2. CONDUIT SYSTEMS.

(a) **Open Conduit.**—Two conducting rails are carried on insulators in a slotted conduit built either between the rails or under one rail. Continuous current for the motors comes and returns by sliding, or sometimes by rolling, contacts attached to two conducting pieces, which pass insulated through the slot. The rail return is not employed unless as an auxiliary to one conductor. (Three-phase conduit systems with rails as third conductor have not yet been used.)

Advantages.—Absence of overhead construction; reduction or elimination of earth currents.

Drawbacks.—Large initial cost; serious disturbance of traffic during construction; slot in roadway; difficulty of maintaining good insulation in conduit; complication of inspection and repairs; unsatisfactory nature of rail switch at the points (with rail-slot construction); special shallow conduit needed on bridges.

Some *examples* of this system are: (1) Sliding contacts—L.C.C. Trams, Bournemouth, Buda-Pest, New York, Berlin, Brussels, and Paris. (2) Rolling contacts (Love conduit system)—Washington and New York.

(b) **Surface Contact** or Closed Conduit System.—The “live” conductor is supported on insulators in a closed conduit (not necessarily under the track). A long insulated sliding shoe under the car makes contact with at least one of a number of studs placed along the centre of the track and momentarily connected with 500-volt conductor. The rails are used as a return.

Advantages.—No overhead construction or slot in the street.

Drawbacks.—Initial expense; great multiplication of moving parts along the line, and consequent risk of breakdown; difficulty of preventing serious leakage in wet weather at the studs during the passage of the car.

Many surface-contact systems have been put forward, but only short sections have so far been tried. These systems can be classified according to the way in which the section switches are actuated as follows*—

* Kingsland, *Jour. Inst. E. E.*, 1902, p. 879.

		DOUBLE-STUD.	SINGLE-STUD.
Electro-magnetic Systems	Magnet in Switch	Westinghouse Johnson-Lundell Thomson-Houston McElroy	Claret-Vuilleumier (sections controlled in groups) Schücker Thompson-Walker Kingsland
	Magnet in Skate	---	Diatto Dolter Lorain Steel Co. G. B. System.
Mechanical Systems	No Magnet	Anderson	Kingsland.

The Paris surface contact trams are partly on the Diatto and partly on the Claret-Vuilleumier systems. The former system has also been installed at Tours, and the latter at Lyons. The Lorain system is in use in Wolverhampton (p. 438), and the G. B. system has been tried in Lincoln and on the Mile End Road in East London.

The danger from studs partially "live" through leakage is much reduced by surrounding each stud by an iron shield bonded to the track. The studs should be definitely earthed or connected to such shield except when the car is passing.

In the G. B. system the live conductor consists of a cable of galvanized iron supported on porcelain insulators. The magnetic parts are so arranged that when magnetized by the passage of the skate-magnet, a carbon contact is drawn *down* on to the live conductor; the road stud connected to this contact then becomes "live" and supplies current to the car. A very shallow conduit only is required.

"**Induction**" trams having no contact have been tried in France, the driving force being obtained between car and track as between rotor and stator of an induction motor.*

3. STORAGE BATTERY SYSTEMS.

(a) **Accumulator Cars.**—The car-motors obtain current from a battery of storage cells on the car which is charged periodically at the station.

Advantages.—Entire absence of electrical construction along

* *Light Ry. Jour.*, Oct. 3, 1903; *Electr.*, Jan. 29, 1904, p. 561.

the track: cars self-contained; no earth returns with stray currents.

Drawbacks.—High initial cost, and great weight of battery to be carried; excessive cost of maintenance; cars idle during charging of battery, causing serious increase in the number of cars necessary.

Examples (relatively short distances).—Paris, Ghent, and in Germany.

(b) **"Mixed" or "Hanover" System.**—Outlying parts only of line are equipped with overhead wire. The central sections are devoid of electrical equipment, the motive power being obtained from a battery carried either (1) on the car and charged from the trolley wire when running on the outer sections of the line, or (2) on a "dummy" car or trailer which is hooked on when the end of the overhead wire is reached.

Advantages.—Combines those of trolley and accumulator systems; economical working with overhead wire in all but the central portions of the line; the cars do not stand idle during charging.

Drawbacks on central sections of line.—Those of battery cars in a lesser degree; delay in attaching battery locomotive.

Examples.—Chiefly in Germany (see below).

Choice of System.

In outlying parts where the traffic is small and the lines extended, the overhead wire is the only system practicable.

On sections of the line where overhead wires are not permitted, and the traffic is large enough to warrant considerable expenditure, the conduit system is generally used. Over short and level sections, however, in which the gradient does not exceed 5 %, the "mixed" system has in some cases given good results. Surface-contact systems have proved commercially successful in one or two cases, but are still not widely used.

In England in 1909, few electric trams were run on any other system than the overhead trolley, though an extensive and well-equipped installation of conduit lines has been laid down for the L.C.C. trams. Short conduit lines exist in some other towns, while in Wolverhampton about 14 miles of the Lorain surface contact system has been working since 1903.

In Germany the "mixed" system has been much used. In 1902, the mileage in Berlin, Dresden, Hanover and other towns operated on this system exceeded 200. But most of these lines are now running on the conduit or the overhead system.

Except where high tension transmission is used, the **generating plant** is of the multipolar 500 to 600 volt d.c. type, compound wound with fixed carbon brushes, designed for large and rapid fluctuation of load.

A **battery** (240 to 270 cells) is frequently used to steady the load and improve the load-factor.* Traction batteries are more largely used in Germany than elsewhere. In October 1901 the batteries installed there for traction amounted to 38,700 kw. output as compared with 133,000 kw. of generating machinery, and this proportion of nearly 30 % has since increased. The minimum cost of such a battery is from £25 to £30 per kw. of output.

Electrical Efficiency.—The accumulator system is probably the least efficient. Cells may at high discharge rates return but 60 % of the energy spent in charging. On the other hand, overhead and conduit systems involve continual line losses, and (particularly in small installations) an unsteady load on the generators, which must therefore be heavier built.

The systems differ as to the **cost of electrical conductors** along the line. Iron is cheaper than copper as a bare conductor for direct currents in the ratio of about 7 to 10 (see p. 389). Conduit systems possess in this respect the advantages of "third-rail" systems (see below). In the ordinary overhead wire system at least one-half, and in three-phase two-thirds, of the length of the line conductors are of copper, iron being, from its bulk, out of the question either for overhead use or for covered feeders. Accumulator systems require, of course, no copper along the line, unless as feeders to an outlying charging station.

Electric traction without rails or line equipment, by means of battery cars, has not hitherto been successful, owing to excessive weight and deterioration of the cells requisite even to run a moderate distance (as 50 miles) without recharging. But with two overhead wires it is possible to dispense with rails, and run electrically-driven cars on a common road (p. 405).

SYSTEMS EMPLOYED FOR ELECTRIC RAILWAYS

(viz. for *Heavy Traffic on Fenced-in Railroads*).

1. "THIRD-RAIL" SYSTEM.

A rail of inverted channel section is carried on insulators beside or between the rails. Direct current at from 500 to 750 volts reaches the car-motors through a sliding cast-iron shoe running along on the insulated conductor. Current returns

* On the economical charging of traction batteries, see J. S. Highfield, "Storage Batteries controlled by Reversible Boosters," *Electrician*, June 14, 1901, p. 291 *et seq.*

through car-wheels and rails or by a fourth rail. (This system is not used for three-phase currents, the higher tension which would render such a system advisable being impracticable on conductors so easily accessible.)

Advantages.—All line conductors of iron, which is cheaper than copper (see p. 389); inexpensive and strong construction of insulated conductors; cheapest system of heavy electrical traction for short lines and frequent service.

Drawbacks.—Danger to persons crossing the line in wet weather; * a slight accident may cause a short circuit and fire risk.

Examples. †—Underground Railways of London, and underground and overhead railways in many of the largest cities.

A modified track construction suitable for high speeds and sharp curves is known as the **Mono-rail System**, an example of which is the Elberfeld Barmen line. ‡ The car hangs from a single rail by two pairs of wheels, each pair being actuated by a motor taking current from an insulated conductor. No attempt is made to keep the cars vertical, and in rounding curves they swing out at an angle.

2. SYSTEMS WITH OVERHEAD CONDUCTORS.

(a) **Direct current** with one overhead wire.—System the same as the overhead tramway system, but with heavier construction; and in place of one overhead contact, two or more on the one overhead wire, in order to carry the heavier currents wanted for greater loads.

The system involves the use of a *copper* overhead conductor, the track forming the return circuit. Iron conductors, though cheaper, are, owing to their weight, too costly to support overhead. (An exception is found in the Baltimore and Ohio line, where a heavy iron overhead conductor is installed.) Sliding bow contacts appear to be more suitable for heavy currents and high speeds than rolling or trolley contacts.

Examples.—Pennsylvania, Baltimore and Ohio, New York, Munich, and many lines for moderately-heavy traffic.

* See also Trotter, *Jour. Inst. E. E.*, 1902, p. 276.

† For details of equipment of English electric railways consult the following—

City and South London—*Proc. Inst. C. E.*, vol. cxii., p. 209, and vol. cxiii., p. 39.

Liverpool Overhead—*Proc. Inst. C. E.*, vol. cxvii., 1894, p. 51.

Waterloo and City—*Proc. Inst. C. E.*, vol. cxxxix., 1900, p. 57.

Central London—*Engineering*, Feb. 18 and 25, and March 4, 1898.

Metropolitan District Railway—*Electrician*, April 24 and May 1, 1903.

‡ See *Electrician*, Feb. 1901.

The long distances which necessitate high-tension transmission with polyphase currents for ordinary tramway schemes occur almost always with any but suburban electric railway projects. Both the above systems, therefore, are often combined with a power transmission by such currents, these being transformed to low pressures and converted to direct currents for use in the motors.

The following systems avoid this conversion, reduce very considerably the line losses, and in certain cases eliminate the transformation, the motors taking current at high-tension straight from the generator.

(b) **Single-phase**, with one overhead wire.—Single-phase motors both of the series and repulsion type and also combinations of these two types are being developed and introduced for traction work (see p. 343 and table on p. 380).*

(c) **Three-phase**, with two overhead wires.—Duplicate sliding bows or trolleys are required on each wire to deal with the heavier currents. The track constitutes the third conductor.

This system was introduced about 1900 by Messrs. Brown and Boveri on the Burgdorf-Thun † section (25 miles) of the Swiss Railways, and on some short mountain lines, the induction motors being wound for 750 volts. Messrs. Ganz installed in the following year a similar system on the Northern Italian Railways, in which 3000-volt motors are used. **High-tension motors** are also used on the experimental railways in Berlin ‡ and on the line through the Simplon Tunnel (1904).§

The polyphase motor is more durable, though less convenient, than the direct-current motor, or even the single-phase a.c. motor. It is better adapted to long runs at or near full speed than to frequent starting and stopping.

The three-phase, and probably also the single-phase, system, combined with the use of high-tension motors, appear to be the most economical electrical systems for long-distance traffic. The series H.T. direct current system of power transmission (p. 385) with substations and polyphase distribution for three-phase motors of two or three thousand volts should also secure great economy.

In addition to street and railway traffic, electric traction (overhead system) is extensively employed in coal and other

* A table with particulars of single-phase railways installed up to 1906, is given in Fowler's *Electrical Engineering Pocket-Book*, pp. 582-3.

† Burgdorf-Thun and other Swiss lines, *Jour. Inst. E. E.*, vol. xxix., p. 224, March 1900.

‡ "High Speed Electric Railways," Lasche, *Jour. Inst. E. E.*, 1901; "The Ganz Electric Traction," *The Engineer*, Jan.—March 1901. Consult also the evidence at the Board of Trade Arbitration on the case of the Metropolitan and District Railways; *Electrical Press*, Nov. and Dec. 1901.

§ See *Proc. Inst. C. E.*, Nov. 1909.

mines, and for the haulage of large canal boats.* In the latter case it is claimed that under favourable conditions (*e. g.* cheap coal) and at a slow speed (3 miles per hour) the cost is as low as '032*d.* per ton-mile.† The "tractor" is a small locomotive running on rails along the tow-path and supplied with either direct or polyphase current. References are given below.‡

Systems of Train-haulage.—Locomotives *v.* Motor Carriages (or multiple-unit system).—Where possible the multiple-unit system should be employed. The load on the track is more evenly distributed; a lighter construction may be employed for the track, short-span bridges, etc.; depreciation of adjacent property, and of line also, by vibration, is reduced; and higher rates of acceleration, which may be advantageous for short-section traffic, can be counted upon, since adhesion increases with the load.

On the other hand, shunting operations and goods traffic require locomotives; and train-couplings are much simplified when no motor connections have to be brought through. Storage battery shunting locomotives have been constructed.

POWER REQUIRED FOR CARS AND TRAINS.

Apart from that lost in the gearing and motor itself, the power required may be used—

(1) To overcome resistance of track, journal friction, and air resistance;

(2) To climb gradients;

(3) To increase speed of car.

On downward slopes and with decreasing speed the last two items represent negative power required. The first is always positive, but varies with the force and direction of the wind, as well as with the state of the track.

In the following formulæ—

W = weight of loaded car in tons.

V = speed in miles per hour ($= \cdot 682v$).

v = ,, ,, feet per second ($= 1\cdot467 V$).

E = combined efficiency *per cent.* of motors and gearing (if any), and taking account of any losses in regulating resistances. For tramcars E = about 75 % or less.

* "Electric Coal Mining Plant," Gresley, *Proc. Inst. C. E.*, vol. cxxxi., 1897-8, p. 100.

† Allen, *Electrician*, 1898, vol. 41, p. 851.

‡ Brussels-Charleroi, 50 miles, three-phase; *Sci. Abstr.*, No. 2114, 1900 and 509B, 1903; Erie Canal, *Elec. World and Engineer*, Nov. 14, 1903, "Cost of Traction on Canals"; also *Electr. Eng.*, Aug. 19, 1904, p. 277.

(1) Power for **level track**.

Let μ = traction coefficient expressed as a percentage of weight of loaded car, and

μ' = traction coefficient expressed in lbs. per ton * weight of loaded car ($= 22.4\mu$); then $22.4\mu W = \mu' W$ = tractive force in lb. required; and

$$\left\{ \begin{array}{l} \text{H.P. at car-wheel rim to drive a } W\text{-ton} \\ \text{car on level track at } V \text{ miles an hour...} \end{array} \right\} = .0507\mu VW$$

$$\text{or} = .06\mu VW$$

for practical purposes. This is also $= .00267\mu' VW$.

(2) Excess power for **running up-hill**.

Gradient = s per cent., or 1 in 100/ s . Then

$$\left\{ \begin{array}{l} \text{H.P. to lift a } W\text{-ton car up } s \text{ per} \\ \text{cent. gradient at } V \text{ miles an hour} \end{array} \right\} = .06s VW.$$

For climbing a hill, therefore, at a uniform speed,

$$\left\{ \begin{array}{l} \text{H.P. to be exerted upon} \\ \text{car-wheel rims ...} \end{array} \right\} = .06 VW(\mu + s),$$

and the

$$\left\{ \begin{array}{l} \text{Actual E.H.P. taken} \\ \text{from overhead wire} \end{array} \right\} = \frac{6 VW(\mu + s)}{E}.$$

For downward gradients the sign of s is reversed.

(3) Power absorbed in **accelerating cars**.

The energy in foot-pounds of any moving car or train is

$$\begin{aligned} &= 74.8 W \times (\text{speed in miles per hour})^\dagger \\ &= 34.8 W \times (\text{speed in feet per second}).^\dagger \end{aligned}$$

The *extra H.P. required to increase speed* at the rate of a feet per second each second is proportional to the actual speed, and is

$$\begin{aligned} &= .186 a W \times (\text{existing speed in miles per hour}) \\ \text{or} &= .127 a W \times (\text{existing speed in feet per second}), \end{aligned}$$

viz. the power which goes in accelerating (or which must be absorbed in retarding) a car at the rate of 1 foot per second in every second is .186 V horse-power per ton of loaded car, V being the actual speed in miles per hour.†

The motors will take current corresponding to $100/E$ times the above amount of power, E standing for gear and motor combined efficiency per cent.

A general expression for the **E.H.P. required by any car** increasing or decreasing speed up-hill or down-hill is therefore

* For values corresponding to different states of track, and various speeds, see p. 420 *et seq.*

† The energy required to start the motor armature and other rotating parts depends upon the construction, gear ratio, etc. It can be allowed for in most cases by adding about 10% to the car weight.

$$\left\{ \begin{array}{l} \text{E.H.P. taken from} \\ \text{overhead wire} \end{array} \right\} = \frac{6VW(\mu \pm s \pm 3.1a)}{E} \text{ horse-power}^* ;$$

or,

$$\text{Power} = \frac{4.45VW(\mu \pm s \pm 3.1a)}{E} \text{ kilowatts}^* ;$$

and,

$$\text{Current} = \frac{8.9VW(\mu \pm s \pm 3.1a)}{E} \text{ amps. at 500 volts.}^*$$

Or, since the traction coefficient is often expressed in lb. per ton (μ')—

$$\text{Current} = \frac{8.9VW(0.0447\mu' \pm s \pm 3.1a)^*}{E} \text{ amps. at 500 volts,}$$

and,

$$\text{E.H.P. required} = \frac{6VW(0.0447\mu' \pm s \pm 3.1a)^*}{E}$$

where V = speed in miles per hour.

W = weight in tons of loaded car.

E = combined efficiency per cent. of gear and motors.

μ = traction coefficient per cent. ; μ' = same in lb. per ton.

s = gradient per cent.

a = acceleration or retardation of car in feet per sec. per sec.

The positive signs of s and a in these formulæ apply to up-hill gradients and acceleration, and the negative signs to down-hill slopes and retardation.

If the power is negative, it stands for energy which is available for return to the line (see under Electric Brakes, p. 428), but most commonly represents power to be absorbed by friction of brakes or heating of resistances.

The traction effort which (apart from friction) will increase the speed of a car a ft. per second every second is the same as that which will overcome a gradient of $3.1a$ per cent. or 1 in $\frac{32}{a}$. *

The actual **tractive effort** on the car or locomotive wheels answering to the above three items of power are as follows—

- (1) Track resistance, μ' or 22.4μ lb. per ton.
- (2) Gradients, $22.4s$ lb. per ton.
- (3) Acceleration (including angular acceleration of rotating parts) about $76a$ lb. per ton.

Thus, exclusive of friction, tractive efforts of about 112 lb.

* Increased from $3.1a$ to $3.4a$ to allow for energy of rotating parts. See Field, *Jour. I. E. E.*, 1902, p. 1297.

per ton and 76 lb. per ton can increase speed at the rate of 1 mile an hour and 1 foot per second, respectively, in each second. These quantities must be multiplied by W to obtain the actual tractive effort on the wheel rims.

A logarithmic chart is given by A. G. Hansard * for finding rapidly the tractive effort and power required by cars for different speeds, gradients and efficiencies in which the track resistance has been taken at 30 lb. per ton.

The following constants are often wanted in calculating power for cars—

7·22 ft.-lb. = 1 kilogr.-metre.

1 H.P. = 550 ft.-lb. per sec. = 76·3 kilogr.-metres per sec.

1 ton = 1·017 metrical tons = 1017 kilogr.

1 mile = 1·609 km. ; 1 kg. = 2·202 lb. ; 1 m. = 3·28 ft.

1 mile per hour = 1·467 ft. per sec. = ·0447 metre per sec.

MOTORS.

1. Direct Current Series Motors.

Series Motors are almost always employed for continuous-current traction work. (For other motors see p. 418.) The armature reaction is much smaller than in a shunt-motor, the field strength increasing always with the armature current, thus—

(a) Fixed carbon brushes include the sparkless position at all loads when running either forward or backward ;

(b) A large starting effort for a given current is obtained, the effort per unit current increasing with the current, instead of slightly diminishing as with shunt motors.

The motor almost universally used is the 500-volt enclosed series motor ; four-pole type, two poles and one-half of the case being generally formed of a single casting of soft steel. Armatures, drum (p. 417) or ring, slotted or tunnel-wound (the former usually) ; fixed carbon brushes. Commutator, hard drawn copper segments with considerable depth for wear. Case watertight when closed ; lid formed on case for ready inspection of commutator.

Field windings are generally provided on all four poles, but sometimes upon two only, the remaining two being consequent poles.

Lubrication of motor bearings usually by pads pressed against spindle. Oil rings rarely used owing to relatively slow speed.

For average tramcars, two such motors are provided ; each of about 25 H.P. for single truck cars, or of about 35 H.P. for

* *Electrician*, October 11, 1901, p. 937.

bogie cars ; and running normally at 450 revolutions a minute. These act upon the car wheels, 30 to 33 inches diameter, through single-reduction spur-wheel gear (p. 424), reducing speed in ratio of 4 : 1 or 5 : 1. To avoid mechanical shocks, the motors are supported by springs from the truck, independently of the car body, in such a way that the distance between the gear wheel-centres cannot alter.

The **rating of car motors**, or their nominal output, is now usually stated as that H.P. which they can exert continuously for one hour without excessive rise of temperature. With the General Electric and the English Electric Companies, this limiting temperature rise after 1 hour is 135° F. (75° C.), the surrounding air not to be more than 77° F. (25° C.). Other firms specify 126° F. (70° C.), or less still as the limiting rise for 1 hour's run at the rating stated.

With cars for not more than 60 passengers, two motors each rated at 25 H.P. will suffice unless the track gradients are exceptionally heavy—over 1 in 12, when at least 35 H.P. for each motor would be necessary. A 25 H.P. motor, with its gear, will weigh on the average about 1800 lb., of which about four-fifths will be due to the motor alone.

The **efficiency** of motor and gear at the rated output is usually about 80 %. The average of published tests* on five different firms' tram-motors of about 25 H.P. size gives the following figures—

Percentage of normal current	}	30	50	75	100	130	%
Efficiency when warm (about 50° C. rise), taking account of gear losses		71	78	79	79	76	%

The current is given as a percentage of that current which corresponds to the rated horse-power. The greatest efficiency generally occurs at less than the full rated load. The efficiency of the motor alone is much higher than the above figures. That of railway motors having no gear may be 93 % or more.

The figures and particulars given in the table opposite† illustrate the **design** of traction motors.

Westinghouse standard traction motors. Those usually supplied now by this firm for tramway work are motor No. 49B, and for the narrower gauges, No. 46.

Weight.—No. 49B: total 1,900 lb. ; exclusive of gear and case 1,600 lb. ; armature, 438 lb. No. 46: total, 1,870 lb. ; exclusive of gear and case, 1,530 lb. ; armature, 383 lb.

* The figures in this and the following table are derived partly from data given in Arnold's *Gleichstrom maschine*, and Dawson's *Pocket-book*.

† *Loc. cit.*, and from information kindly supplied by Messrs. Dick, Kerr and Co., and the British Westinghouse Electric Co., Ltd.

Data of Standard 4-pole Traction Motors.
(About 25 B.H.P., 500 v.)

	English Electric Mfg. Co., Preston.	General Electric	General Electric.	Rieter Winter- thur.	Union Co., Berlin.
	No. 25A.	G.E. 800	G.E. 52		
Normal amps. and volts.	44 × 500	45 × 500	50 × 500	50 × 500	44 × 500
Rated H.P.	25	25	27	27	24
Normal revs. per min..	550	abt. 400	abt. 500	480	290
Gear ratio	5.07 : 1	4.78 : 1	4.78 : 1	4.45 : 1	3.6 : 1
Teeth ratio	71 : 14	67 : 14	67 : 14	67 : 15	54 : 15
Armature diameter.....	13½"	16"	11"	16.5"	16"
„ length of core	7½"	8"	9"	9¼"	12"
„ no. of slots...	41	105	29"	63	105
Conductors per slot.....	24	8	24	12	8
Commutator bars, no...	123	105	87	63?	105
Field coil turns.....	4 × 134	4 × 203	4 × 155	2 × 288	2 × 198
Axle diameter	2½"	3¼" to 3½"	3⅝" to 4"	3½"	3"
Bearings diameter	2¼" and 2⅝"	2½"	2½"	2¾"	2⅝"
„ length, pinion					
„ end	8"	6"	7¾"	5¾"	7½"
„ length, com- mutator end	6½"	4½"	6⅜"	5½"	5"

Armature.—Slotted drum two-circuit armature built up of soft steel laminations; ventilated core and winding; small diameter of armature (about 12 inches) for minimum flywheel effect; number of slots, 35.

Lubrication.—Grease cup above bearing and oil reservoir with wick feed below bearing; either method of lubrication can be used, or both.

The **English Electric** Co.'s car motors (Short's design).* The weight of their type No. 25A is 1,934 lb., complete with gear. The current density at the brushes does not exceed 25 amps. per sq. inch.

Armature winding.—The two-circuit single winding is generally used for car-motor armatures.

The coils are former-wound, while the slots are deep and have parallel sides. The use of open slots tends to minimize the *inductance per coil*; and this, in conjunction with the strong field in the air-gap, which is provided in modern motors (from 7,000 to 9,000 lines per sq. cm.) serves to facilitate sparkless commutation.

Each coil is made to occupy part of the bottom half of one

* For description see *El. Rev.*, March 29 and April 5, 1901 pp. 545 and 589.

slot and the top half of a slot just short of $\frac{1}{4}$ of the way round the armature. A connector then leads to a commutator segment, and on again to a similar coil occupying a similar position almost opposite to the first coil. A wire then leads from this second coil to a commutator segment almost opposite to the one previously mentioned, and thence another lead makes connection with the coil adjacent to the first coil. And so on round the armature. Only two brushes are required, 90° apart.

Three coils are often bound with micanite cloth into one compact section. There are then three times as many commutator segments as slots. The insulation should withstand about 2,000 volts alternating. Good insulation is needed between the top and bottom half of a slot, since full voltage occurs between these. To remove a burnt-out section, $\frac{1}{4}$ of the coils must be partially lifted.

The small number of armature teeth in modern car-motors involves the use of laminated pole-pieces to avoid eddy current loss from varying flux.

Speed Regulation is effected by resistances in series with the motor, but occasionally also by shunting the field.

In the ordinary tramcar with two motors, the **series-parallel control** is always used. On starting, the two motors are connected in series, through resistances which keep the current down to the largest safe starting current. These resistances are gradually cut out, and when all out the car will run at half the full speed. The motors are then placed in parallel with the help of resistances, and the car can then run at full speed (p. 426, Controllers).

2. Other Motors in Use.

The **shunt motor**, though less suitable than the series motor for variable speed work, offers many advantages including increased economy and reduction of wear, by the use of **regenerative control**; in which the braking is normally effected by returning current to the line to an extent determined by the field excitation. Comparative tests by Raworth* at Devonport and Gravesend show that about 30% of the energy can be conveniently returned to the line on an undulating track, bringing the energy-consumption below 50 watt-hours per ton-mile as against 70 to 80 watt-hours normally. But even on level tracks where saving of energy is small, there is still large economy through not using mechanical brakes. The motors for this system, being in use also during braking, should be more liberally rated, viz. slightly larger than when

* *Traction and Transmission*, No. 37, April 1904; *El. Rev.*, Feb. 26, 1904.

mechanical braking is adopted. The system is in use (1905) at Southport, Halifax, Glossop, Scarborough, and on one of the Birmingham routes, as well as in the two towns named above.

In conjunction with the **Ward-Leonard system** of control, direct current motors can be economically operated over wide ranges of speed; as in the recent experiments near Zurich.* The full starting effort can be obtained in this way with only 30 % of full current.

Alternating Current.—(a) **Single-phase series motor.**—The direct current series motor can be operated at low frequency as an alternating current motor, if constructed with laminated field cores. Special care is required to avoid sparking. Finzi,† whose motors have been used on the Milan tramways, finds a power factor of .7 during acceleration, and .8 and over during running. The frequency was 18 periods and the voltage, 160 to 500, obtained as required by means of an auto-transformer. The Westinghouse-Lamme series motor is applied to an electric railway, about 20 miles long, between Baltimore and Annapolis.‡

The speed-torque characteristic of such alternating current series motor is a little inferior to that of the direct current series motor; but the advantages, which include (1) economic obtaining of proper voltage for motor supply; (2) raised line pressure with light overhead construction; and, (3) suppression of rotaries and moving machinery in substations, render the motor well suited to certain kinds of traffic.

(b) **Single-phase repulsion motors** of various types are also employed for traction (p. 343).

(c) **Three-phase induction motors** are widely used on electric railways (see p. 411) and are well suited for constant speed running.

STARTING EFFORT.

A motor which, taking a given current, exerts P horse-power when running at R revolutions per minute will give, for the same current in armature and fields (and neglecting excess of static over running friction, see below), a **starting effort** or torque

$$= 5250 \frac{P}{R} \text{ lb. at 1 foot radius ;}$$

and if this is transmitted through reduction gear having a ratio = $n : 1$, and an efficiency = $E\%$, the **starting effort on car wheel rims** of d inches diameter is

$$= 1260 \frac{EPn}{Rd} \text{ lb.}$$

* Mordey, *Electr. Mag.*, vol. 1, pp. 265 and 489.

† *Sci. Abstr.*, 123 and 313B, 1904. ‡ *Electr. Mag.*, May 26, 1904, p. 494.

when the starting current is such that the motor would exert P horse-power at R revs. per minute. The value of P corresponding to any given current must be obtained from a knowledge of the motor characteristic. The starting current will usually be very much in excess of the current which the motor could continuously bear.

If there are two motors, the starting effort is twice the above. With the series-parallel control, this doubled effort is obtained by the current taken normally by one motor.

The starting effort of any motor is always reduced by the subsidence of the shaft in the journal while at rest. The weight of a direct driving electric locomotive (armatures excepted) is much greater than the armatures themselves, and the force producing subsidence is correspondingly greater than if the motors were detached from the locomotive. McMahon * has shown that in the case of a motor of a City and South London Railway locomotive (no gearing) the starting effort is 27 % below that of the same motor mounted on a foundation. After starting, this difference falls to only 6 % at 8 miles an hour.

Where locomotives are used, it is this reduced starting effort alone which is available to meet the increased starting resistance (due to same cause) of the rest of the train.

The resistance at starting can be much reduced by **roller bearings**, which have been tried on the trams in Hull, Zürich, † etc., and on the Waterloo and City Railway. These must, however, be very carefully and amply constructed to last under heavy railway loads or rough tramway usage. The Kynoch and Hyatt bearings, in which the rollers are flexible, being formed of stout steel spirals, appear to be less liable to derangement than solid rubber bearings.

TRACTION FORCE REQUIRED WHEN RUNNING.

Ordinary rails.—Experiments on the C. and S. L. Railway ‡ have shown the **running train resistance** on **ordinary rails** to vary as in the following table and curve—

* *Jour. Inst. E. E.*, 1899, p. 508.

† For careful tests see *Elektrotechnische Zeitschrift*, 1899, p. 72

‡ *Jour. Inst. E. E.*, 1899, vol. xxviii., fig. 24, p. 533.

TRAIN RESISTANCE AT DIFFERENT SPEEDS IN TUNNEL
(McMahon *).

SPEED.	RESISTANCE.	SPEED.	RESISTANCE.
Miles per hour.	lb. per ton.	Miles per hour.	lb. per ton.
0 (starting resistance)	40	12	9.5
1	29	16	12
2	20	20	15.5
4	13	24	19
6	10	28	23
8	9		

Train resistance in
tunnels. (McMahon)
(City and South London Railway.)

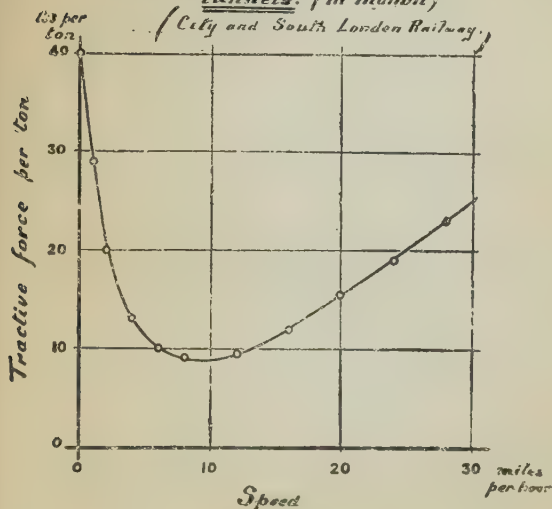


FIG. 150.

* Jour. Inst. E. E., 1899, vol. xxviii., p. 540.

These results, which are shown graphically in Fig. 150, refer to a 30-ton train—locomotive and two trailers—in a tunnel, the front surface occupying 56 % of the section of the tunnel. The minimum resistance here stated was found to be doubled on a **curve** of 540 feet radius.

Smooth rails in the open air offer a minimum resistance of 5 to 10 lb. per ton.

The **air resistance** is negligible below 5 miles an hour, and (below 25 miles an hour) is always a small fraction of the total resistance, even in a narrow tunnel. It is stated to be 1 to $1\frac{1}{2}$ lb. per square foot front surface at 14 miles an hour in above case.

Air resistance is of much greater importance at **higher speeds**. The recent experiments near Berlin on high-speed electric railways have included determinations of air resistance at high speeds* by means of special revolving apparatus. The results include the following—

Speed; miles an hour		60	70	80	90	100	110	120	130
		Pounds per square ft.							
Force on a square surface 1 sq. metre in area; lb. per sq. ft.	plane ...	13	19	26	36	—	—	—	—
	parabolic	—	4.9	6.3	8.0	9.7	12.2	15.5	21

[NOTE.—1 lb. per sq. ft. = 4.88 kgr. per sq. metre.]

The figures show the advantage of a properly shaped surface.

The effect of **head-wind** is very great since the effective train speed is the sum of the actual speed of the train and that of the wind.

The effect of **side-wind** has been found in cases to be even greater, since it forces all the flanges to grind on one rail and so increases largely the rail friction.

The **resistance** offered by cars on **grooved rails** depends very largely on the condition of the track, and may vary from 50 to 60 (or with snow, etc., even 100) lb. per ton at starting, down to 20 to 40 lb. when running at about 8 miles an hour; the larger figures referring in each case to a dirty track. With large cars, the resistance per ton is less than with small.

Experiments by Parrett on Nottingham horse trams (two-ton, $5\frac{1}{2}$ feet between axles) seem to show that the running resistance of grooved rails in moderate condition is increased

* See *L'Eclairage Électrique*, April 5, 1902, p. 9.

by 11 and 30 lb. per ton on **curves** of 45 and 22 feet radius respectively.

The resistance due to gradients and car acceleration which must be superposed on the above is given on p. 413.

The H.P. corresponding to given tractive efforts at different speeds will also be found on p. 414. In addition, it may be noted that a tractive force of F lb. exerted at a speed of V miles an hour

represents $\frac{FV}{375} = .00267 FV$ horse-power; and requires a

current of $\frac{FV}{2.52 E} = \frac{.397 FV}{E}$ amps. at 500 volts, if E is the

combined efficiency of the motor and gear per cent.

The **aggregate H.P.** of motors required for a given car is approximately

$$= \frac{\text{weight of car in tons} \times \text{maximum speed in miles an hour}}{4}$$

the motors being rated for a 75° C. rise in 1 hour (p. 416).

To give satisfactory **adhesion**, the weight on each driving wheel should be 5 or 6 times the greatest tractive (or retarding) effort which its rim has to exert on the rail at the wheel tread (the latter figure referring to street lines). Any slipping which may then occur on "greasy" rails will be easily prevented by sand. Such a condition does not occur in "tube" railways and a smaller margin of weight is possible. Field * gives the following values of

ADHESIVE FORCE.

	% of dead weight.
Greasy rail . . .	8-10
Wet rail . . .	15
Dry and clean . . .	20
Gritty . . .	25
Sanded . . .	30

As regards the **most economical acceleration**, it will be found that for short-section running, a high acceleration, allowing for a considerable amount of "coasting" before applying the brakes, will prove most economical; while for running longer sections a lower rate of starting will be best.

Very great starting acceleration (over 1½ feet per second per second, or 1 mile an hour every second), however, is not economical, as it involves greater weight of motor and of central station equipment.

The economy of short-section running is much increased by building the stations a little above the average level of the line, the gradients helping both starting and stopping of trains.

* *Jour. Inst. E. E.*, 1902.

CAR EQUIPMENT EXCLUSIVE OF MOTOR.

1. Gearing.

Single reduction tramcar gearing of the type most generally employed consists of a bronze, or better, a steel motor-pinion having about 14 teeth, gearing into a cast-iron or sometimes steel spur wheel with about 65 teeth (according to gear ratio).

The teeth have usually a width of 4 to 5 inches and a pitch of about 1 inch, giving pitch-circle diameters of $4\frac{1}{2}$ inches and about 21 inches for pinion and large spur wheel respectively. The teeth are cut by special milling tools, though steel pinions pressed through a die are occasionally used. The large spur wheel is made in two halves to bolt together on to a $3\frac{1}{2}$ to 4-inch rolled or drawn steel car-wheel axle.

The gear is enclosed and runs in grease usually mixed with graphite.

REVOLUTIONS PER MINUTE OF TRAMWAY MOTORS WITH DIFFERENT GEARS AND CAR WHEELS.

Speed of Car.	8 Miles an hour.				12 Miles an hour.			
Diam. of car wheel.	24 ins.	27 ins.	30 ins.	33 ins.	24 ins.	27 ins.	30 ins.	33 ins.
Gear ratio.	revs.	revs.	revs.	revs.	revs.	revs.	revs.	revs.
3.5 : 1	392	349	314	285	588	523	470	428
4 : 1	448	398	358	326	672	597	538	489
4.5 : 1	504	448	403	367	756	672	605	550
5 : 1	560	498	448	407	840	747	672	611
5.5 : 1	616	548	493	448	924	822	739	672

Formula connecting revolutions of motor with speed of car along track—

$$\left. \begin{array}{l} \text{Revs. of motor} \\ \text{per minute} \end{array} \right\} = 336 \frac{Vn}{d}$$

where V = car speed in miles an hour.

n = gear ratio.

d = diameter of car-wheel in inches.

Roller-chain gearing runs very silently, but is rarely used on tramways, and is not adapted to heavy work.

Worm gearing is no longer employed, except for light cars and electric automobiles.

2. Truck.

The **frame** of an electric **tramcar** must be very strongly built to resist the stresses due to the motors; it must be sufficiently rigid even with a very short wheel base and a greatly-overhanging car; but the car-floor must not be more than 28 to 30 inches above the track. (For construction of car-frames see Dawson, *Electric Railways and Tramways*.)

English electric tramcars with 4 wheels weigh on the average 8 to 9 tons when full, have a length of 25 feet, a wheel base of $5\frac{1}{2}$ to $6\frac{1}{2}$ feet, and carry 50 persons inside and out. Twelve-ton double bogie cars carrying 70 to 75 are now much used. A four-motor equipment (20 H.P. motor on each axle) has been occasionally used on bogie cars. The **car-wheels** are usually 30 inches in diameter, and are of cast-iron with cast-steel tyres shrunk on and sometimes pinned; the complete wheel being pressed on to the car axle. For easy replacement, the wheels are sometimes bolted to a flange shrunk on to the axle instead of being themselves shrunk on. Such wheels have an average "life" of 20,000 to 30,000 miles with ordinary braking and frequent stops, the wear being about $\frac{3}{4}$ in. (viz. diam. reduced by $1\frac{1}{2}$ ins.). A large part (nearly half) of the wear is caused by the brake-blocks.

The following table * shows the maximum rigid **wheel base** which is safe on curves. The wheels are supposed not greatly to exceed 30 inches diameter.

Minimum Radius of Curve.	Maximum Safe Wheel Base.	
ft.	ft.	in.
20	5	4
25	6	0
30	6	6
40	7	0
50	7	6
60	8	0
70	8	5

Curves of smaller radius are only possible with good track, very slow running at curves, and special wide-groove rails.

With double bogie cars, as on the larger tramway systems, the weight is more uniformly distributed, there is less "hammering" at the rail-joints, and less depreciation of the track, and especially of the bonding; and smoothness of running is much greater than with four-wheel tramcars on any but new tracks.

* Cottrell, *Electrical Review*, Apr 1 28, 1899, p. 669.

3. Controller.

The conversion of connections from series to parallel is generally carried out by the **controller*** as below, A_1 , F_1 , A_2 , F_2 , standing for armatures and fields of first and second motor respectively, and R representing a resistance section.

CONTROLLER FOR 2 MOTORS.

Controller Point.	Connections.
0	Circuit broken
1	$4 R + (A_1 + F_1) + (A_2 + F_2)$starting
2	$3 R + \quad \quad \quad + \quad \quad \quad$
3	$2 R + \quad \quad \quad + \quad \quad \quad$
4	$R + \quad \quad \quad + \quad \quad \quad$
5	$(A_1 + F_1) + (A_2 + F_2)$series
{ 6	Same as 2 }.....transition points
7	
8	
9	$\quad \quad + \left\{ \begin{array}{l} A_1 + F_1 \\ A_2 + F_2 \end{array} \right\}$
10	$2 R + \left\{ \begin{array}{l} \quad \quad \quad \\ \quad \quad \quad \end{array} \right\}$
11	$R + \left\{ \begin{array}{l} \quad \quad \quad \\ \quad \quad \quad \end{array} \right\}$
12	$\left\{ \begin{array}{l} A_1 + F_1 \\ A_2 + F_2 \end{array} \right\}$parallel

Points 5 and 12 are the only points of *efficient running*; they correspond to half and full speeds.

Extra points between 5 and 6, and after 12, giving efficient running at other speeds, are often provided by *shunting the fields* with suitable resistances. The number of resistance points—1 to 4 and 9 to 11—may be reduced.

The **sparking at the controller** contacts is generally extinguished electro-magnetically.

Controllers may be grouped according to the mode of extinguishing sparks as follows (Eborall)—

* See also under Electric Brakes, p. 428.

Action.	Name of Controller.
1. Magnetic blow-out with iron }	Westinghouse. Gen. Electr. (Thomson-Houston).
2. Magnetic blow-out without iron }	Walker. Steel-motor.
3. Multiple break . .	Short.
4. Quick break . .	Electr. Constr. Co.

In the Oerlikon controller the central cylinder of the controller is of iron, and the contacts work in "vulcabeston"-lined cavities backed with soft iron with suitably-shaped poles. A coil connected across the full pressure converts the whole controller frame into an electro-magnet, the contacts being broken in its air-gaps.

The extinction of the sparks is somewhat different in the widely-used General Electric Co.'s controller. Solenoids of stout copper ribbon well insulated with vulcabeston are fixed, one opposite where each spark will occur. Just before each controller finger breaks contact, the main current is diverted through the corresponding solenoid, which, having no iron core and very little resistance, becomes magnetized without delay and without interfering with the main circuit.

The **controller resistances** must be made of fireproof material. Messrs. Siemens and Halske make their resistance sections of stampings interleaved with asbestos or mica and connected by electric welding at alternate ends, the joints being allowed to project to promote radiation. A more common construction consists of coils of flat strip (iron or iron-resistance-alloy) insulated with vulcabeston or asbestos and arranged for rapid cooling. Resistance sections formed of light zig-zag iron castings with suitable end-pieces are also common. The frame holding the resistance sections must be fireproof and not placed near inflammable material.

The controller case must be metallically connected with the car frame.

A controller is required at each end of the car. The rear controller must be disconnected and the handle removed.

The controlling of the **four motors** on the Waterloo and City trains is managed as in the table on next page; M standing for a motor and R for a resistance sections.

Speed diagrams showing the effect of this controller on the

CONTROLLER FOR 4 MOTORS.

Controller Point.	Connections.
0	Circuit broken
1	$4 R + M_1 + M_2 + M_3 + M_4$starting
2	$2 R + \text{,,} + \text{,,} + \text{,,} + \text{,,}$
3	$M_1 + M_2 + M_3 + M_4$all in series
{ 4	$2 R + M_1 + M_2$; M_3 & M_4 short circuited } transition
{ 5	$\text{,,} + \text{,,} + \text{,,}$; ,, disconnected } points
6	$\text{,,} + \begin{Bmatrix} M_1 + M_2 \\ M_3 + M_4 \end{Bmatrix}$
7	$R + \begin{Bmatrix} \text{,,} \end{Bmatrix}$
8	$\begin{Bmatrix} M_1 + M_2 \\ M_3 + M_4 \end{Bmatrix}$semi-parallel
{ 9	$R + \begin{Bmatrix} M_1 \\ M_3 \end{Bmatrix}$; M_2 & M_4 short circuited } transition
10	$R + \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{Bmatrix}$
11	$\begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{Bmatrix}$all in parallel

acceleration will be found in the paper already mentioned.* Nos. 3, 8, and 11 are the main running positions.

4. Brakes.

The **Brakes** in use may be grouped as follows—

(1) Those applied by hand.

(a) *Common wheel brake.* Brake blocks worked by levers.

(b) *Friction "power" brake* (Price's). The brake blocks are applied by a friction clutch which connects a disc on the car axle with a drum over which the brake-lever chain passes.

(c) *Rail brake.* Sliding shoes are pressed down on to rail by screw-jack actuated by worm-gear, and capable of

* See Jenkins, 'Waterloo and City Railway,' *Proc. Inst. C. E.*, vol. cxxxix, 1900, p. 74.

supporting weight of car. (Usual emergency brake for cable cars.)

- (*a*) Block with *teeth pressed on to wood by side of track*. (Powerful emergency brake used on excessive gradients, at Lausanne and elsewhere.)
- (*c*) In special work such as mountain railways, brakes *gripping the rail tread* (specially shaped), and *band brakes* working on a drum turned by a toothed wheel engaging the rack in centre of track are used.

(2) Electric Brakes.

- (*a*) Motors generate and return current to line. (Shunt and induction motors.)
- (*b*) Motors generate current; the energy absorbed in resistances on the car.
- (*c*) Eddy currents induced in a brake disc attached to car axle, and revolving in the field of an electromagnet excited by the current generated by the car-motors.
- (*d*) Same as (*c*) except that the magnet is excited from the line.
- (*c*) and (*d*) are usually assisted in their action by actual mechanical friction of the brake disc against the poles.
- (*e*) Magnetic rail brake (Schiemann's).* Between each pair of wheels an electromagnet having seven alternate poles clearing the rail by $\frac{1}{4}$ inch is fixed. Electromagnet excited through current generated by motor.
- (*f*) Same as (*e*) except that the electromagnet is excited from the line.
- (*g*) Brake blocks of ordinary hand brake pressed on by plunger of electromagnet excited by current from motor as generator. This may act through mechanism similar to (*1 b*).

Comparison of electric brakes.—Brake (*a*) is not of any emergency value. The motor must generate more than the line pressure before the braking action exists. It is stated to be of no value beyond 6 % gradients.

Electric brakes (*b*) and (*c*) are the most used, and require merely a change of motor-connection from the line to the resistances or the brake-magnet coil, when the controller can be again used for regulating the braking action. An interlocking device prevents this change of connections being made in any other than the "off" position of the controller handle. Brakes (*b*), (*c*) and (*g*) are similar electrically.

Brakes (*a*), (*d*) and (*f*) are dependent upon the overhead contact. This is undesirable. The removal of the contact during braking has given rise to serious accidents.

* *Elektrotechnische Zeitschrift*, 1899, p. 537.

On the other hand, all but the rail brakes (*e*) and (*f*) are dependent upon the wheels not slipping, and can be thrown out of action by incautious use of the hand brake. With proper precautions, however, this property is advantageous and not otherwise, for the retarding force of wheels is always greatest when they are *just not slipping*. But it is essential that the hand brake should not be used so as to lock the wheels.

For emergency use, eddy-current brakes—(*c*) to (*f*)—have the drawback that the previously-unexcited electromagnet takes one or two seconds to excite fully.

Electric brakes produce very gentle stopping as they all cease to act when the car is about to stop; and will therefore not bring a car to absolute standstill on a down grade, the hand brake being necessary. Excessive electric braking, however, will produce an intermittent skidding of the car wheel. The braking current in the motor which is just not too large to cause skidding is, owing to frictional losses, considerably less than the running current which will be just short of skidding.

When electric braking is employed, the hand brake should only be used to obtain stand-still on downward grades.

Polyphase induction motors, if run slightly above the speed of synchronism, will act as generators like the continuous-current motor, and return current to the line. This method of braking is used on the three-phase Swiss lines. The rotors cannot run more than 4 % above synchronism.

(3) **Compressed-air Brakes** are always used for trains, and often for large cars. Either a small motor on the car drives a compressor or else a steam air reservoir, charged at the station to about 100 lb. per sq. inch, is carried to work the brakes.

The **rate of stopping** except in emergencies should not exceed 2 ft. per second per second, and should be reduced just before stopping.

The **retarding force** (p. 413) required to stop at the rate of *a* ft. per second each second is—

70*a* lb. per ton of loaded car, less the loss, say, 20 % in gear, etc. (provided that the wheels are not locked, in which case this deduction must not be made).

The **time taken** in stopping at the rate of *a* ft. per second per second from *V* miles per hour is—

$$\frac{1.47 V}{a} \text{ seconds.}$$

The **distance run** before stopping is—

$$1.08 \frac{(V)^2}{a} \text{ feet.}$$

5. Miscellaneous Equipment.

Lightning Arresters.—A spark-gap to earth (viz. a good connection by a straight conductor with the car truck) with means for extinguishing the arc resulting from a lightning discharge must be provided at the base of the trolley arm.

The "Ajax" lightning arrester is much used in this country. It consists of two silk-covered wires pressed together in a small glass tube, thus forming a short spark-gap to earth through the silk. This gap is in series with a fuse, and a carbon-ball replacing device which throws in another gap and fuse when one has gone. The B. T.-H. arrester consists of a spark-gap to earth through a high carbon resistance in series with an automatic cut out. The Oerlikon mica disc type is common on Swiss electric cars.

The "Wurts" non-arcing arrester (p. 400) in series with a choking coil is part of the British Westinghouse standard car equipment. It is attached on the rolling-stock at the points where the trolley currents enter the car. This arrester is used on the Burgdorf-Thun (alternating) line.

Electrical **car heaters** are now much used, a common size being to take one up to 1 kilowatt. They are usually formed of a long narrow spiral of high-resistance iron alloy wire (not less than No. 20 gauge), threaded by a stout asbestos cord, and stretched back and forward on porcelain insulators secured in an iron frame. Two such heaters suffice for a small car, and four for a large one. Connections are such that for all stages of heating, every heater is partially used.

TRACK.

The track for electric tramways must be of very **solid construction**. An electric motor tramcar will weigh 2 or 3 times as much as a horse tramcar loaded. The average speed also is much greater.

Grooved girder rails are used in this country, but this section offers about 20 % greater rolling resistance than the "step" rail.

The rails are held to gauge by tie-bars about every 6 feet, and are laid on a concrete foundation ;—usually directly, but spring liners under the rails are occasionally used. An asphalt foundation is more elastic than concrete.

For minimum radius of curves, see p. 425.

Very heavy rails are found to be best ; 80 or 90 lb. per yard is common. Still heavier rails are also being used. In Manchester, 98 lb. ; in Glasgow, 100 lb. rails in 45 feet lengths ; while the New York conduit lines are laid with 106 lb. rails in 60 feet lengths.

These great lengths are preferred in order to reduce the number of rail joints and bonds.

The **sectional area** of rails of various weights per yard will be found in column 4 of the table on p. 436. The figures correspond to a

Specific Gravity of 7·85

for the steel rails. The following formulæ also hold—

Weight per cubic inch = ·283 lb.

Volume per lb. = 3·53 c. inches.

Sectional area of rail = $\frac{\text{lb. per yard}}{10\cdot2}$ sq. inches.

To reduce lb. per yard to kilogrammes per metre, multiply by ·495 ; reciprocal, 2·02.

Sectional area of rail = $\frac{\text{kilos. per metre}}{7\cdot85}$ sq. cms.

The following figures are given by Parshall,* and each represent the averages of a number of tests on **rail resistance** (eliminating joints).

ELECTRICAL RESISTANCE OF STEEL RAILS.† (Parshall.)

Description of Rails.	Resistance Relative to Copper.	Resistance of 1 Mile 1 sq. in. Sectional Area at 70° F.
76-lb. rails after 2½ years' use	10·4	·45 ohm.
65-lb. rails, old track	12	·52 „
90-lb. rails, newly laid down	10·5	·455 „
Average values	11·0	·475 ohm.

The Engineering Standards Committee (1903) specification for the composition of **steel** tramway rails is—carbon, ·4 to ·58 % ; manganese, ·7 to 1·0 % ; silicon, less than ·1 % ; and phosphorus and sulphur each less than ·08 %. This corresponds‡ to from 19,000 to 23,000 C.G.S. units specific resistance, or ·475 to ·575 ohm per mile of 1 sq. in. cross section.

Columns 5 to 8 of the table on p. 436 are based upon the lower figure ; and the resistance of rails may be taken as follows—

* H. F. Parshall, "Earth Returns for Electric Tramways," *Electrician*, May 6, 1898, p. 57.

† For resistance (apparent) with Alternating currents see pp. 101 and 116.

‡ *Cupp. Am. I. E. E. Journal*. See *The Engineer*, Dec. 25, 1903, p. 627.

$$\left. \begin{array}{l} \text{Resistance of one} \\ \text{rail at } 70^{\circ} \text{ F.} \end{array} \right\} = \frac{.473}{\text{section in sq. in.}} \text{ ohm per mile.}$$

$$\left. \begin{array}{l} \text{Resistance of one} \\ \text{rail at } 70^{\circ} \text{ F.} \end{array} \right\} = \frac{4.83}{\text{wt. in lb. per yd.}} \text{ ohms per mile.}$$

Allowance for **wear** of rails to the extent of 1 or 2 per cent. every year should be made.

Rolled **iron** (*e. g.*, for third rail) varies in specific resistance from 12,000 to 13,000 C.G.S. units, or .3 to .325 ohm per mile of 1 sq. in. cross sectional area, according to quality.

A **current density** of 90 amps. per sq. in. in such rails is equivalent to 1000 amps. per sq. in. in copper. At 90 amps. per sq. in., therefore, the figures for volts lost per mile, etc., given on p. 390 are correct.

RAIL-JOINTS AND BONDING.

1. Effects of Imperfect Earth Return.

The conducting power of ordinary **rail-joints** with fish-plates and bolts falls off rapidly after completion of the track ; and the difference of potential due to the passage of the return tramcar currents increases proportionately. When the rails are in contact with damp soil, a perceptible part of the return current leaves the rails, and, passing through the soil, **corrodes water** and other **pipes** in the vicinity, and disturbs telegraphs and all other sensitive apparatus using earth plates within a very much larger radius.

Where such **disturbances** must be absolutely **avoided**, either battery traction must be used or else one of the following three systems adopted—

- (1) A second overhead wire provided to take return current.
- (2) Conduit system with both conductors insulated in conduit.
- (3) Alternate current three-phase system (alternate currents though effecting no electrolysis produce a serious disturbance in local telephones which is worse than that which arises from the fluctuations of direct-current motor currents. Such disturbances are avoided by crossing the telephone wires at intervals).

A difference of potential between different parts of the track not exceeding 7 volts is allowed by the Board of Trade in this country ; and with a properly-bonded track, and, if necessary, the Return or Negative Booster system, this and other Board of Trade conditions (see p. 442) can be satisfied while using the earth return.

The "**Return Booster**" system (due to Major Cardew) consists in lowering the potential of some intermediate point or points on the track (in addition to the point where the earth return reaches the station) to earth potential by means of a booster and special return cables.

Special difficulty arises in connection with the **return of alternating current** along rails on account of their great inductive resistance arising from the high permeability and consequent large flux inside the section.* To overcome this, the secondary of a small series transformer (carrying the line current in the primary) may be inserted at intervals (Kapp) in the return circuit. Or else,† an auxiliary return line is run on poles besides the track, to which it is joined at intervals; and, by means of series transformers or boosters as before, the current is largely diverted from the rails except in the section on which the train or car happens to be.

2. Bonding.

When the rails are used as a return, they must be "**bonded**" to insure electrical connection.

Copper Bonds consist generally of one or two stout copper wires (about No. 0000 gauge) bent to allow for expansion of rails, and riveted to the web outside the fish-plates or else to the flange. About 30 inches long in first case, and less than half as long in the second. Such bonds are very widely used, but the riveted joint is liable to shake loose under certain conditions.

Electric welding of copper bonds to the rail has proved satisfactory, but is not generally employed.

Plastic Bonds, in which the contact is made between the fish-plate and the rails by means of some mercury amalgam or soft metal cast into grooves in rail or fish-plate, are claimed to have a lower resistance and to be more reliable than riveted bonds. Their resistance may be less than that of 1 foot of rail even after a year's working. They have been used on a number of important installations.

Rail switches and crossings are now made commonly of **manganese steel** to minimize wear. The very high electrical resistance of this material necessitates a special conductor or long copper bond to complete the return.

3. Solid Rail-Joints (welded or cast).

The stress due to extreme changes of temperature does not exceed one-fifth of the ultimate strength of the rail, or one-half of

* p. 101.

† Behn-Eschenburg, Oerlikon, *Elektr. Zeitschr.*, 1904, p. 311; *Sci. Abstr.*, B. 1904, No. 1647.

elastic limit, and is less in mild climates ; but bolted expansion joints are sometimes inserted every few hundred yards.

Electric Welds.—The rail ends are first cleaned by sand-blast. A rotary converter takes current from the trolley wire, and converts the 500-volt direct to about 300-volt alternating current. This is transformed down to about 6 volts in a transformer secondary having but one turn and furnishing current up to 25,000 amperes. This enormous current passes by stout clamps through the rail-joint heating it to the welding temperature.

A butt-weld may destroy the quality of the rail end. To avoid this, the fish-plates may be welded to the web of the rail instead of welding the rail-ends direct.

Cast Welds (Falk).—Cast-iron from a furnace on a truck is run into a mould around and below the rail joint. The joints are good mechanically, but occasionally increase in resistance. In the *Goldschmidt* process, a mixture of iron oxide and powdered aluminium (known as "thermit") is ignited by means of a suitable fuse, and furnishes the required molten iron at a very high temperature indeed.

4. Electrical Resistance of Joints.

The extra resistance per joint varies very considerably with different bonds, and with the tightness of the fish-plates. In a well-bonded track maintained in good condition it should not exceed that of 4 feet of rail.

Column 7 of the table on p. 436 is calculated on this basis (8 % added for 50 feet rails). This increase of "drop" will be inversely proportional to the length of rail. Thirty, 40, 50, and 60 feet rails have 176, 132, 106, and 88 joints per mile respectively.

The last column gives the "drop" per mile at 100 amps. for a single track cross-bonded to a single 0000 bare copper wire (diam. = .4" ; section = .1257 sq. inch ; .344 ohm per mile) laid between the rails.

For calculating the resistance of copper of any section used as an auxiliary to the rails for the return, the following is useful—

$$\left. \begin{array}{l} \text{Resistance of soft copper} \\ \text{along track at } 70^{\circ} \text{ C.} \end{array} \right\} = \frac{.0132}{\text{section in sq. inch}} \text{ ohm per mile.}$$

$$\left. \begin{array}{l} \text{Resistance of soft copper} \\ \text{along track at } 70^{\circ} \text{ C.} \end{array} \right\} = \frac{.0550}{(\text{diameter in inches})^2} \text{ ohm per mile,}$$

if a wire.

In supplementing the conducting power of the return, however, it must be remembered that for equal conducting power,

iron is cheaper than copper (see p. 389).^{*} Thus, worn-out rails previously existing along the track have been frequently bonded and used to supplement the return. Again, if the conducting power of the track rails has not been completely utilized, it is cheaper to put in double bonding than to lay copper down the track.

The figures given below for resistance and "drop" per mile must be halved for double track (4 rails). The saving arising from this in addition to the dispensing with overhead and track switches, has often determined the laying of double instead of single track.

5. Data for Rails of Different Weights.

Weight and Sectional Area of a single rail.				Figures relating to 1 mile of single track (2 rails).			
Weight.			Sec- tional Area.	Electrical Resistance without joints.†	“ Drop ” in Volts for 100 amps. ‡		
					No. joints.	Joints equal to 4 ft. of rail occurring every 50 ft.‡	
						No wire along track.	Cross- bonded to 0000 cop- per wire.
lb. per yard.	tons per mile.	kilogs. per metre.	sq. ins.	ohm.	volts.	volts.	volts.
60	47·1	29·7	5·88	·0402	4·02	4·34	3·86
65	51·0	32·2	6·37	·0371	3·71	4·01	3·59
70	55·0	34·7	6·86	·0345	3·45	3·73	3·37
75	58·9	37·2	7·35	·0322	3·22	3·48	3·16
80	62·8	39·6	7·84	·0301	3·01	3·25	2·97
85	66·7	42·1	8·33	·0283	2·83	3·06	2·81
90	70·7	44·6	8·82	·0268	2·68	2·89	2·67
95	74·6	47·0	9·31	·0254	2·54	2·74	2·54
100	78·6	49·5	9·80	·0241	2·41	2·60	2·42
105	82·5	52·1	10·30	·0230	2·30	2·48	2·32
110	86·5	54·6	10·79	·0220	2·20	2·38	2·23

^{*} See also Chatelier, "Resistance of Steels," *Sci. Abstr.*, No. 1352, 1898.

† Direct current (p. 432); for alternating currents see pp. 101 and 116.

‡ For lengths other than 50 ft. a correction as explained above must be made.

CONDUIT SYSTEMS.

All conduit systems* require a clear depth below the rail tread of 16 to 20 inches; a condition difficult to satisfy on ordinary *bridges*. If a specially shallow conduit is adopted at such places, the plough must admit of raising and lowering.

This raising and lowering, as well as that required when the same cars run on both conduit and overhead sections, is often done automatically by inclined planes placed in the conduit.

The **central-slot** in which the conduit is laid along the middle of the track is to be preferred, but is not always possible on account of interference with wood and other paving.

In comparison with the central-slot the **side-slot** construction presents many difficulties, among which are (1) the extreme difficulty of constructing the rail-points immediately over a conduit; (2) continual splashing into conduit from car wheel; and (3) the great strength of conduit construction required to carry the whole weight of the cars.

The **width of slot** should if possible not be more than $\frac{5}{8}$ to $\frac{7}{8}$ of an inch. The side-slot construction, however, necessitates a slot over an inch wide to accommodate the car wheel; and slots averaging 1·2 inches are found in Paris, Berlin, Brussels and Buda Pest. In the L.C.C. conduit trams in S. London, the slot is $\frac{3}{4}$ " and 1".

The **underground contact** may be either sliding or rolling. In the Love conduit system it is the latter, a spring frame carrying two trolley wheels rolling against the two conductors. The sliding contact seems, however, to be most satisfactory.

The plough attached to the car (see reference below: Paris †) and entering the slot contains two flat copper conductors about $\frac{3}{32}$ inch thick, and of sufficient breadth to carry the motor currents. These conductors are placed preferably one in advance of the other, are thoroughly insulated with fibre and mica, and are flanked by stout steel plates. Two cast-iron shoes are hinged to the lower part of the plough, and are pressed sideways by springs against the vertical faces of two insulated $\perp$ -iron conductors placed about 6 to 8 inches apart. The conductor-rails for the S. London trams weigh 21 lb. per yard.

The conduit must be efficiently **drained**. The rails forming the slot must have a ridge on the under side, so that water from above will drip clear down the centre of the conduit and not run down its sides.

It is estimated that, where heavy traffic has to be met, the **cost of installing the conduit system** is about £10,000 per mile

* See also p. 408.

† For descriptions and drawings—L.C.C. trams: *Engineering*, Oct. 10 Dec. 1902; *Tram Railway World*, Sept. 1902; Paris: *Electrician*, Jan. 12, 1900, p. 293; *Street Railway Journal*, Dec. 1899, p. 845.

above that for the overhead system; but even where large cars are used this does not represent more than 1*d.* per car mile out of, say, 5*d.* per car mile actual working expenses. Local conditions (bridges, etc., as in Paris) may increase this excess cost. The conduit system is more practicable in a dry climate than in a wet one.

The following comparative figures for **operating expenses** have been published* for electric conduit, cable and horse traction on the lines of the Metropolitan Street Railway Co. of New York.

	Rate of Operating Expenses to Receipts.	
	1898.	1899.
	per cent.	per cent.
Electric (conduit)	37·9	38·3
Cable	47·8	50·8
Horse.....	65·3	69·8

Since capital charges do not figure in the above, and maintenance expenses are in each case small, these figures should also give roughly the relative cost of working expenses for electric overhead-wire traction on a large scale.

Shawfield† gives the following figures for the **cost (for maintenance) of the surface contact system** (Lorain) at Wolverhampton as compared with the average cost of the overhead system in about 15 towns—

	Lorain System.	Overhead System.
	Pence per car mile.	
Overhead work	—	·316 ‡
Special track equipment...	·569	—
Car equipment	·230	·080 ‡
Electr. energy.....	2·310	1·900
Total	3·109	2·296

* *Street Railway Journal*, Sept. 1899, p. 579; *Science Abstract*, 288, 1900.

† *Electrician*, April 24, 1903, p. 44; *Science Abstract*, 1049 B, 1903.

‡ See *Electrical Review*, April 17, 1903, p. 549, where smaller values are forward.

The overhead equipment requires 1.15 units per car mile. The Lorain system requires extra energy as follows—

	Units per car mile
Increased weight of car	.131
Excitation of magnets	.111
Leakage	.008

THE OVERHEAD CONTACT.

(a) **Ordinary trolley** with wheel pin rigidly attached to the trolley arm. In this case the overhead wire must conform closely to the centre of the track, a large number of “pull-off” wires are required to keep the contact wire in position, and the trolley wheel must have a very deep and open groove.

The contact is made by a wheel cast in bronze with four ribs, weighing about $2\frac{1}{2}$ lb., and running between two phosphor bronze contact springs on a pin supported by a steel fork, which is insulated from the arm.

The trolley arm is formed of a thin tapered steel tube enclosing the cable which makes connection with the trolley wheel. This tube slopes upwards at an angle of about 30° to the trolley wire, and its lower end is attached to a swivel with ball bearing to enable the trolley to adjust itself to the position of the overhead wire.

Such a contact will carry 40 or 50 amperes (and more still if alternating) without undue sparking (at ears, etc.), if the car speed does not much exceed 15 miles an hour at the time that this current is taken.

Where roof-seats are fitted, the swivel is on a pillar which is electrically connected to the car truck to avoid possibility of leakage shocks.

The contact is pressed upwards by springs. (A coil and plunger device actuated by the main current has the advantage over springs that, should the trolley “jump” the wire, the arm will fall, instead of rebounding against the wire at the risk of fusing it if the motorman has not switched off at once. This device, however, is little used.)

This system gives the neatest trolley-arm construction.

(b) **Dickinson Side Trolley.**—In this, the trolley-wheel fork is carried on ball bearings to swivel about a vertical axis. The trolley wheel need not therefore be so broadly or deeply grooved.

This construction in combination with the Blackwell standard, in which the springs for pressing up the arm are enclosed in an enlargement at its lower end, is the *most suitable for roof-seated cars*, and is preferred in this country. It permits of a less

unsightly overhead construction ; the contact wire may be run quite at the side of the track. On the other hand, the trolley arm itself is more unsightly than the much lighter central trolley so widely used on the Continent, where roof-seats are not so common as here.

An average trolley wheel will last about 5000 miles of run.

(c) **Sliding Bow** (Siemens and Halske).—A sliding bow 4 or 5 feet wide at the top is pressed up sloping from the car by springs. The contact with the overhead wire is made with a bar of pure or nearly pure aluminium, having longitudinal grooves filled with lubricant. The aluminium, which is held by a steel frame, must be replaced after two months.

In this construction the contact wire need not conform very closely to the track, and the arm need not swivel. The overhead construction at corners, and especially at crossings, is much simplified. The running is almost perfectly noiseless (which cannot be said of the trolley wire). The lubricant, however, if used at all freely may drop from the wire in hot weather. The system also is not adopted to roof-seated cars.

(d) **Sliding Shoe** carried on a swivel head (Thury).—The contact bedded with light friction alloy is carried on an ordinary trolley arm. This has proved satisfactory in Geneva, Lausanne and elsewhere.

(e) A **Sliding Finger** contact offering many advantages for high speed traction has been introduced by the Oerlikon Co.* Contact occurs either above, below, or at the side of the line wire according to its position. Two contacts can pass on the same wire.

Overhead Wire Supports (see also p. 404).—These are usually spaced 120 feet apart and the wire is anchored at both ends of every straight section to resist longitudinal stress.

Mechanical clips are not secure on round wire. Grooved wire is now widely used, and is supported by such clips.

The smoothest running is obtained either with grooved wire or with soldered ears. In the latter case the wire is laid in a long groove in the under side of a brass casting hanging from an insulator by a bolt.

Soldering, especially if by blow-lamp instead of an iron, is liable to soften the hard-drawn copper used, and the wire is apt to break at the insulators.

The insulators should if possible not be supported directly from the side or centre poles ; and if suspension wires across the street are not desired, a short span or "fiddle-bow" suspension between two insulators attached to the pole arm is adopted to minimize the shock to the wire as the trolley passes the supports.

* *Elektr. Zeitschr.*, 1904, p. 298.

For heavy railway work with H.T. overhead wire, a special form of roller contact is used. It is supported by a system of levers, controlled by air pressure, by which the contact is pressed upwards vertically against the wire. The H.T. conductor itself is supported at frequent intervals from a span wire stretched immediately above it. These suspensions from the span wire are so chosen in length that the "live" wire hangs without perceptible sag practically at a uniform distance above the track (*catenary support*).

The **wire** most commonly used is No. 0 to 000 gauge hard-drawn copper wire. Aluminium, even if cheaper, must have a diameter $1\frac{1}{4}$ times as great for the same conducting power (see p. 391). It has been used for trolley wire in America.

The resistances in the following table are based upon the standard of hard-drawn high-conductivity wire given on p. 389, namely, two per cent. higher resistance than annealed copper.

Hard-Drawn High-Conductivity Copper Wire.

Gauge.	Diameter.		Sectional Area.	Data for 100 yards.			Break- ing Load.* approx.
				Resist- ance.	"Drop" with 100 amps.	Weight.	
S.W.G.	ins.	mm.	sq. in.	ohms.	volts.	lb.	lb.
4/0	·400	10·16	·1257	·0195	1·95	146·	6280
3/0	·372	9·45	·1087	·0226	2·26	126·	5240
2/0	·348	8·84	·0951	·0258	2·58	110·	4750
1/0	·324	8·23	·0824	·0297	2·97	96	4120
1	·300	7·62	·0707	·0347	3·47	82	3530
2	·276	7·01	·0598	·0410	4·10	70	2980

The wear of overhead wires is estimated at 1% per annum with sliding contact; and is not less with rolling contact, owing to rubbing at the sides of the trolley.

The **Sag** of overhead trolley wire is usually made about 10 inches on a span of 120 feet at about 70° Fahr. This means a **pull** of some 700 lb., which will increase to about 2000 lb. at the freezing point (32° F.).

The tension divided by half the weight of overhead structure supported is frequently given as equal to the ratio of the half-span to the sag. See, however, p. 392.

* Assumed to be 50,000 lb. per sq. inch.

COMPARISON OF S.W.G. AND B. AND S. GAUGE.

Gauge No.	2	1	0	00	000	0000
S. W. G.	·276	·300	·324	·348	·372	·400 inch diam.
B. and S.	·257	·289	·325	·364	·409	·460 „ „

Span wires are of stranded galvanized steel wire rope about $\frac{3}{8}$ inch in diameter.

The **Board of Trade Regulations** relating to Electric Traction include the following*—

1. **Street Tramways.**—**Dynamos**, if direct-current, shall furnish current with no appreciable pulsation.

Return.—Uninsulated return wire to be connected to rails at least every 100 feet by conductor equivalent to $\frac{1}{16}$ sq. inch of copper.

Negative terminal of generator to be connected to rails and also to an earth (of not greater than 1 ohm resistance (see p. 401); in each case through an ammeter. The current to earth at this connection shall not exceed either 5 % of the total current or 2 amps. per mile of single track in the system.

The **potential of the track** shall not be as much as that of 3 Leclanché cells above (rails +), or 1 Leclanché below (rails -) that of any pipe in the vicinity.

The potential difference between the station “earth” and the most remote point of the earthed return shall be continuously recorded (by pilot wire), and shall not exceed 7 volts. (Half volt between any points 300 yards apart is the rule in Chicago.)

The **overhead wire** (or insulated conductor in conduit system) to be constructed in half-mile sections connected by switches.

Leakage current from insulated conductors (which must be recorded daily) shall not exceed $\frac{1}{16}$ amp. per mile of track. Leakage above $\frac{1}{2}$ amp. per mile of track to be stopped within 24 hours.

Feeders to have insulation resistance (to be recorded monthly) of 10 megohms divided by length of track in miles.

The owners of telegraph or telephone wires, erected previous to tram line, may insert device to be approved of by tram company or Board of Trade to neutralize induction.

† See also p. 545.

The distance between line and return must be 3 feet if both are overhead, and 18 inches if both underground.

Conduits for the conduit system must be well drained and such as to be readily inspected and cleaned; and, if of metal and used as part of the return, must be electrically connected to the rails every 100 feet; otherwise, the conduit must be earthed at the station end through a detector.

2. **Electric Railways** in metal-lined tunnels.*

Dynamos.—Same as above.

Return.—Uninsulated return wire connected to rails as above.

Rails and tunnel lining to be connected to negative pole of generator through ammeter in each case.

Water and gas-pipes entering tunnel not to be in metallic contact with the tunnel.

If metallic contact between rails and tunnel lining occur at all, it must be every 100 feet by conductors able to carry 100 amps. without sensible heating. If such contact do not occur, except (as must be the case) through the negative terminal of the dynamo, then the extreme P.D. between rails and tunnel, which must be recorded once a month, shall not exceed 10 volts.

The **P.D.** between the extremities of the earthed return shall not exceed 7 volts. A daily record to be kept.

Insulation resistance of **feeders** outside tunnel same as for tram lines.

Leakage current from insulated conductors to be recorded daily.

See also p. 546.

SWITCHES AND SWITCH-GEAR.*

By H. W. W. Dix, A.M.I.E.E.

A WELL-DESIGNED switch should have a **temperature rise** not exceeding 35° F. after six hours' run at full load. The brush should make a tight but elastic **contact**, all cutting and undue wear of the sliding surfaces being avoided.

The **pivots** on which the switch turns should not be relied on for heavy currents, but if the pivot surfaces are so arranged as to be forced together by means of a spring, sufficiently good contact can be made to carry currents up to 50 amperes.

Switch **handles** should be strong and well insulated. If made of metal the handles must be exceptionally well insulated, and if possible should be earthed, otherwise static shocks may be obtained from them on breaking circuit. To give room for handling, a minimum distance of 2 inches between the outside of the handle and the nearest fixed object should be allowed.

Strong **stops**, at a leverage calculated to throw a minimum stress on any part of the switch, should be fitted.

All feathers and **keys** used in securing switch parts should be of ample strength, as the hurried operation of a switch subjects it to heavy stresses. In large switches it is always desirable to check their proportions by the ordinary rules of machine design.

Switches employed to interrupt currents should have a **quick break** movement of an amount suited to the voltage and current of the circuit they control; any unavoidable flashing being made to take place on distinct metal or carbon sparking pieces in order to avoid damage to the main contacts.

It is best for the **arc** to take place in a horizontal plane, as it then rises upwards, lengthens, and goes out. Whereas if it is vertical there is a greater chance of its being permanently kept up. Any object above the flash or near enough to be damaged by it should be protected by a ledge of fireproof insulating material.

The **type** of switch, and other switch-gear, adopted by different makers is extremely varied, but as any maker of experience and repute will supply switch-gear to satisfactorily meet any requirement, it will only be necessary to indicate here the salient points of the fittings usually in demand for continuous and alternating currents.

Fuses are used to protect circuits against exceptionally heavy

* See also p. 518.

overloads, and, in many cases also, as a means for disconnecting unloaded circuits. Switch-fuses are widely used on account of the ease with which they can be removed for examination or re-fusing. For circuits subject to severe and frequent overloads automatic cut-outs are to be preferred. Fuses form a useful protection for subsidiary circuits of moderate capacity, but heavy current main circuit fuses are somewhat unreliable.

For generators working in parallel reverse-current circuit breakers, viz. those which open the circuit when the generator receives a motoring current, should be used (sometimes in conjunction with a fuse, or excess current attachment on the circuit breaker, designed to operate when an exceptionally severe short circuit tending to shut down the plant has occurred).

Fuses for **high tension** circuits have to be specially designed with a view to extinguishing the arc liable to be formed when they melt. A great many excellent designs have been produced, one of the most successful and effective being to thread the fuse wire through an insulating tube open at both ends. The insulating tube is often of porcelain, lined with plaster of Paris, the hole through the plaster being slightly larger than the fuse wire. When the fuse melts, it does so with an explosive effect which very effectually extinguishes the arc. The plaster of Paris appears to act as a buffer to the shock caused by the fuse melting, and prevents, to a considerable extent, the breakage of the more brittle porcelain outer tube.

Dust-extinction * fuses are very effective for voltage up to 1000. Oil-extinction fuses should be used for high † and extra high tension alternating circuits as they have the property of extinguishing the arc when the current passes approximately through zero, and this practically prevents resonance effects breaking down the insulation of long feeders. Oil-break switches are used in alternating current circuits for the same reason.

There are four types of **automatic cut-outs** in general use.

(1) **Excess current** cut-outs take the place of a fuse and open the circuit when the current exceeds a certain fixed amount which is adjustable over a moderate range.

(2) **Excess non-return** current cut-outs take the place of fuses, but they operate only with a return current; they can also be designed to work as minimum non-return current cut-outs.

(3) **Minimum current** cut-outs open the circuit when the current fails to say 10 or 15% of the maximum. They are often used in the circuits of machines running in parallel,

* For tests see *Inst. E. E.*, paper read March 1908.

† Low tension is here taken as 600 volts and under, high tension as 600 to 3500 volts, extra high tension as 3500 to 20,000 volts.

their function being to cut-out a machine which has become disabled or underloaded, and thus prevent its being damaged by receiving heavy currents from other machines or cells working in parallel with it.

Reverse current cut-outs which operate with a motoring current are also used for generators working in parallel. Their tendency to act should increase with the strength of the motor-ing current. They are generally to be preferred to minimum current cut-outs.

(4) **Combined excess and minimum** current cut-outs combine the functions of (1) and (3) in one instrument.

All cut-outs consist of (*a*) a switch ; (*b*) a magnet and trigger which controls (*c*) a spring of weight, whose release causes the opening of the switch.

The parts forming the switch of the cut-out should have mechanical rubbing contacts protected from sparking by auxiliary contacts of metal or carbon. Mercury contacts work well in cut-outs that have to break only small currents, but they have many disadvantages and should never be employed to break heavy currents.

Excess or reverse current, or combined excess and reverse current cut-outs of the oil break type are used in H.T. and E.H.T. alternating current work. Used in generator circuits they should act if the field circuit fails and they should have an adjustable time limit device to prevent their acting too sensitively.

Plug-boards are employed (chiefly in low tension continuous current work) to avoid the use of a large number of switches. They consist of a suitable number of vertical metal bars, secured to the front of the slate or marble slab forming the base of the switch-board, and a number (usually three) of horizontal bus bars fixed at the back of the switch-board base. The back bus-bars are provided with tapped holes, opposite plain holes in the front bars and switch-board base. Plugs screwed to fit the holes in the bus-bars, and flanged to make contact with the front bars are used for connecting any front with any back bus-bar.

Plug-boards are not used in high tension work—section bus-bars and section switches or disconnecting links being used instead.

Owing to the fall in pressure of a **secondary cell** during discharge, some means of adjusting the pressure given by a battery of secondary cells is necessary. The adjustment is made by having a number of extra regulating cells which can be switched in or out by a **battery regulating switch**, or by adding or subtracting the required number of volts to or from the battery circuit by means of a booster.

Battery switches should have pilot brushes and resistances to prevent the dead short-circuiting of the cells or the breaking of the circuit, the main brushes being substantial and the connections thoroughly secure in order that discharges above the maximum may be carried with safety.

Where overhead lines are used, **lightning arresters*** or dischargers are employed to protect the plant as well as the line itself from the destructive effects of lightning discharges. It is doubtful if any form of arrester can prevent or mitigate the damage done by lightning *actually striking* the line. Arresters are, however, of great service in dealing with violent surgings of pressure induced in the line by a lightning discharge and slow accumulations of pressure in the line due to atmospheric electricity. The functions of an arrester are to provide a path to earth for high potential charges and to prevent the line current following to earth when a discharge takes place.

Automatic release starters for the protection of motors are in use to prevent flickering of lamps and burning out of armatures. The ideal starter would—

1. Prevent sudden switching-on.
2. Switch-off if supply is interrupted.
3. Switch-off if over-loaded.
4. Prevent the resistance being left in circuit.
5. Prevent arcing on the main contacts.
6. Have means for switching-off instantaneously.
7. Have its resistance coils enclosed in a ventilated, water-dust, and fire-proof box.

The radial switch, which is quite satisfactory up to 50 H.P. 500 volts, has a contact arm working radially over a set of flat contacts fixed to the switch base. This type permits of the use of functions 2, 3, 4, 6 and 7, indicated above. This design sometimes gives trouble through arcing at the contacts owing to the difficulty of switching the radial arm sparklessly from one flat contact to the next unless the surfaces are true and in good condition. The liability to arc is also increased by the practical impossibility of forcing the arm tightly on to the fixed contacts, as the spring operated arm requires that the friction at this point shall be small.

The radial type of starting switch has a spring which tends to keep the contact arm in the "off" position, and a "shunt" electro-magnet fixed near the "on" position of the switch, and which holds the contact arm in the "on" position when the motor is working normally. The windings of this "shunt" magnet are connected in series with the shunt winding of the motor.

* For further information, see p. 398.

If the motor is stopped by an interruption in the supply, the "shunt" magnet becomes demagnetized and releases the contact arm, which flies to the "off" position. In this way the switching on, when the supply is resumed, of standing motors without first inserting the starting resistance, is prevented, and the burning out of armatures and the flickering of lamps obviated. In case the shunt circuit of the motor should break the switch will also open and protect the armature.

The motor can be protected from overloading by an additional "series" magnet having a winding connected in series with the motor. The armature of this magnet is attracted, and short-circuits two contacts connected to the ends of the "shunt" magnet winding, when the current through the motor exceeds a predetermined amount, and the switch therefore flies off, stopping the motor.

The switch described is suitable for shunt or compound motors, but automatic release switches with a "shunt" magnet winding, designed to take the full supply voltage, can be obtained for series motors. Automatic release switches for induction motors are made on the same general principle.

This switch is not suitable for regulating purposes, as the spring prevents the contact arm from remaining in any but the "off" or "full on" positions; but automatic switches designed to allow of regulating, reversing, etc., are obtainable.

The springs of automatic switches must be *strong* and arranged so that they can be easily repaired. The spring stop that "catches" the contact arm must be of good design and construction, to prevent the switch from hammering itself to pieces.

MATERIALS USED FOR SWITCH-GEAR.

Wrought copper should be used as far as possible in all the current-carrying parts of switch fittings, on account of its high conductivity; cast copper, though necessarily employed for parts of complicated shape, is very inferior in this respect. Cast copper varies in conductivity, and may be anything from 18% to 89% of the conductivity of pure copper.

Parts of switch fittings not carrying current may be of wrought iron, cast iron or malleable iron.

Switch **bases** are always of some non-inflammable material,* **slate** or **marble** being the best to employ. Slate slabs of good quality and free from metallic veins should be well dried, shellac-varnished at the back and oiled on the front surface. Slates are also black stove-enamelled and polished, the enamelling increasing the insulation of the slates and the polishing

* A so-called "non-inflammable wood" has been introduced during the last few years, and where wood must be employed is to be recommended.

greatly improving the appearance of the switch-board when new. Plain oiled slates are often to be preferred, as they wear better. Polished slates soon lose their high finish, and it cannot be readily restored.

Amongst the **marbles** White Sicilian, St. Anne's, Mexican onyx, and Tennessee grey are all good switch bases, but the soft marbles must be avoided, as they are poor insulators and easily spoiled by oil spots.

In order to facilitate handling and packing, the slabs of material forming the switch-board base should not exceed about twelve square feet in area, and are usually $1\frac{1}{2}$ inches or 2 inches thick.

For general **insulating** purposes **ebonite** is the best material to use, but it is damaged by oil or a temperature exceeding about 150° F. Vulcanized **fibre**, though tough and not brittle like ebonite, is liable to absorb moisture which impairs its insulating properties and causes it to expand and contract or warp very much. **Stoneware** and **porcelain**, though poor mechanically, have first-rate insulating properties. Cheap kinds of stoneware, which depend only on the glaze for their insulation, are to be avoided.

SWITCH-BOARDS.

The **arrangement** of fittings on the panels, and the grouping of the panels themselves, vary with the system of generation and distribution, and with the ideas of the designer. No hard-and-fast rules for the arrangement of switch-gear can be laid down, but it is a good plan to make the arrangement of fittings follow the diagram of connections as far as possible, so arranging matters that there shall be a minimum amount of crossing of the conductors at the back of the board. The fittings should be symmetrically and compactly arranged, but undue crowding must be avoided.

Substantial **frames** of wrought iron or wood * are employed to hold the switch-panels, the frame being fixed to stand about 4 feet from the switch-room wall, so that access to the back of the board can be obtained. Wrought iron or steel frames are mechanically the best and most compact, and are much to be preferred. With iron frames, all live metal should be kept at least 2 inches away from the framing (voltage not exceeding 600 volts to earth), and the switch panels should not be allowed to come into contact with it, the bolts fixing the panels being insulated with ebonite or other suitable material. Switch-board frames are sometimes built into a party wall, so that the face of the board is flush with the wall, the back connections

* See previous foot-note.

being cased in with a shallow box fixed to the other side of the wall. This arrangement is a very neat one, though it does not give so much freedom for extending the board as that first mentioned.

There are three principal **Systems** on which the panels and fittings can be grouped—

- I. The Three Section System (Fig. 151).
- II. The Strict Panel System (Fig. 153).
- III. The Mixed Panel System (Fig. 152).

In the first system (see Fig. 151), which applies only to three-wire methods of distribution with continuous current, the switch-board is divided into three sections, the one on the right hand carrying all the Positive fittings, the one on the left hand carrying all the Negative fittings, both for generation and distribution, and the one in the centre carrying all the Neutral, Booster, and Balancer fittings. This system allows of the neatest arrangement of back connections with a minimum expenditure for copper, its only possible drawback being that the room available for the switch-board may not be used to the best advantage, as the extensions to the feeders may not keep pace with the extensions to the generating plant.

In the second system, which applies to all methods of distribution, each panel carries fittings for one particular function only. Thus in a three-wire board, the right-hand panels would be devoted to the feeder fittings only, and two or three panels in the centre of the board would be reserved for the Booster, Balance Neutral and Battery fittings. The switch-gear on the centre panels not generally needing extension, this board is free to grow at either end, feeder or dynamo panels being added as the demand arises.

In the third system, which applies to all methods of distribution and covers small boards for isolated plants, the panels are arranged systematically to suit the fittings required, but without restricting the fittings on any panel to any particular class of plant.

Remote control,* or the operation of switch-gear at a distance, is often necessary to ensure safe and convenient working. It applies only to H.T. and E.H.T. work, and more particularly to large power schemes where the switch-gear occupies so much space (owing to the separate poles of the gear having to be enclosed in fire-proof chambers) that it cannot be efficiently operated directly. In such cases the various switches, etc., can be connected to a control desk or board, which may be

* See *Journal Inst. E. E.*, vol. 33, 1904 ; vol. 34, p. 438 ; and vol. 38, 1907. Also *Electrician*, May 13, 1904, p. 133 ; and Oct. 20, 1905, p. 5. Also Inst. E. E. paper by Dr. Garward on relays, read May 14, 1908.

of small size and placed in the most convenient position. As the control desk carries no high-pressure current and as a view of the running machinery may be obtained while operating it, the station plant can be handled with the maximum of convenience and security. The connection between the switch-gear proper and the control desk is either Mechanical (viz. by rods and lever), or Electrical (viz. by electro-magnet or motor-operated switches). Mechanical remote control is generally used for plants of moderate power, Electrical remote control with electro-magnet operated switches being used for plants of large power and high voltage, or in cases where mechanical control is inconvenient or impossible to arrange.

The Westinghouse Oil-break Circuit-breaker arranged for remote electrical control is closed by an iron-clad solenoid acting on a core suspended from the switch lever, and is kept "on" by means of a toggle-jointed support. It is opened by a second solenoid sucking up a core attached to a hammer, which knocks over the toggle support and allows the switch parts to fall to the "off" position by their own weight. These two solenoids are actuated by a small two-way control switch on the distant control desk, a low-voltage direct current being used in the control circuits. The automatic or circuit-breaker action is obtained by means of a relay, which is actuated from a series transformer, and which closes the tripping circuit. Two lamps on the control desk indicate respectively whether the circuit-breaker is open or closed.

The following types of **relays** are in use—Overload current, Reverse current, and Overload current with time element adjustable from one to several seconds. (For further details, see p. 445.)

In **preparing designs for new switch-boards**, it is advisable to adhere as far as possible to the generally accepted methods of making and arranging the fittings, as special designs are always costly. For the same reason it is advisable to avoid interlocking the fittings as far as possible.

All ornamentation is, as a rule, out of place on a switch-board and should be avoided; a plain moulding run along the top of the board and a clock fixed to a cast-iron bracket of simple design mounted on the moulding being all that is necessary.

TYPICAL DIAGRAMS OF CONNECTION FOR SWITCH-BOARDS,
ETC. REFERENCES TO FIGS.

- A = ammeter.
- L = link for disconnecting feeder, fitted with over-load alarm.
- PB = plug board.
- SRS = shunt regulating switch.
- ACO = automatic cut-out, non-return current.
- CZ = central zero, ammeter, or with current direction indicator.
- BS = battery switch.
- V = voltmeter.
- Z = zero or non-return current cut-out.
- G = generator.
- M = motor.
- MT = meter.
- B = booster.
- BAL = balancer.
- CCO = charging cut-out.
- FRS = field regulation switch.
- FB and FBS = field breaking switch.
- S = shunt winding.
- M = main (series) winding.
- R = starting resistance for motor.
- D = dynamo.
- F = feeder.

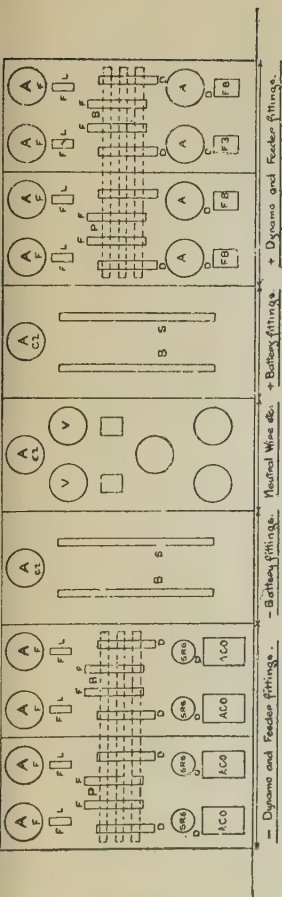


FIG. 151.

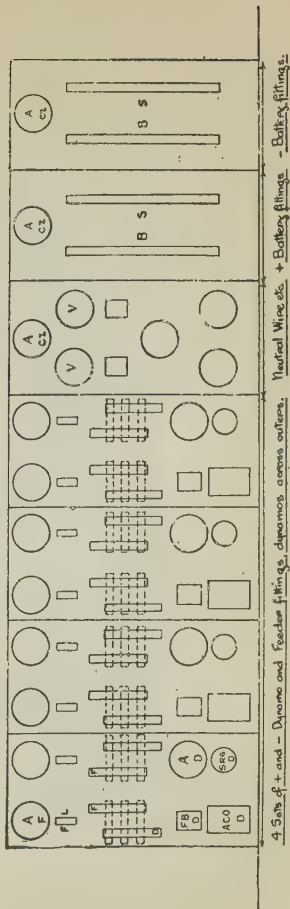


FIG. 152.

Three-section System Switch-board.

Mixed Panel System Switch-board.

Feeder fittings marked F. Dynamo fittings marked D.

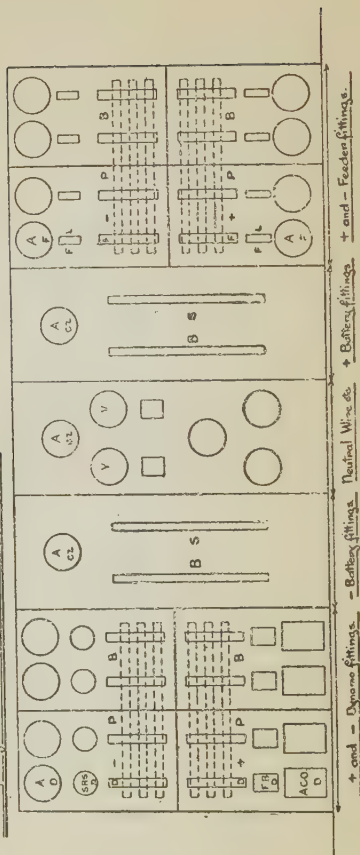


FIG. 153.

Strict Panel System Switch-board.

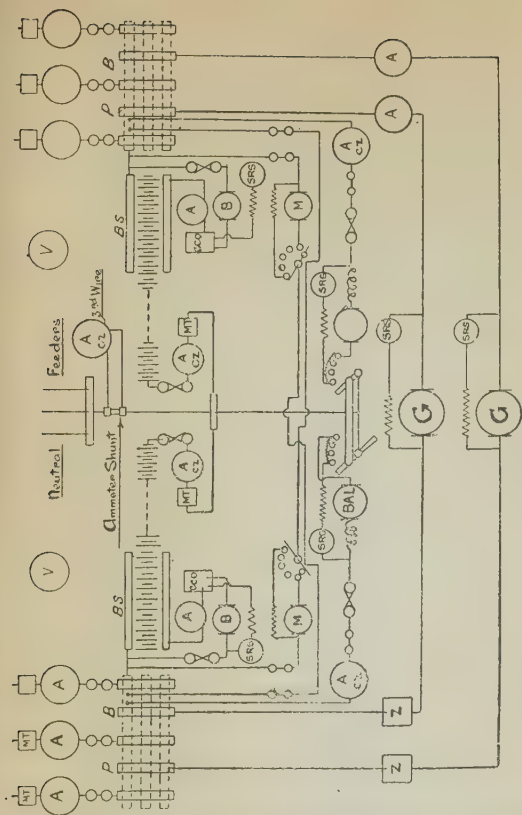


FIG. 154.

Three-wire Switch-gear Diagram.—This arrangement of generators, balancing motor-generator, battery, and boosters, is in common use in Electricity Works.

* To make it as self-regulating as possible, the balancing motor-generator is either compound-wound, or shunt-wound and "cross-excited." The diagram shows a compound-wound motor-generator, but if a shunt-wound machine is used, the shunt windings of the positive and negative halves of the machine must be connected to the neutral and negative, and the positive and neutral bars respectively by means of a double pole starting switch, precautions being taken to prevent the armature of either half of the machine being switched on with its field unexcited.

In this arrangement the battery switches are worked either by hand or automatically, generally the former. The boosters are used for *charging* the end cells only.

* See *Elec. Rev.*, May 10-17, 1907.

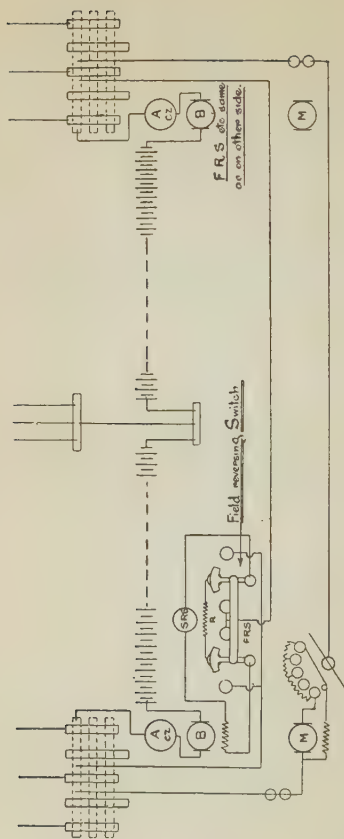


FIG. 155.

Battery Switch-gear for Three-Wire System.—In this arrangement the boosters charge, discharge, and regulate the battery, the battery-regulating switches with their attendant end cells being dispensed with. When end cells are used, it is found that they are very liable to rapid deterioration, owing to the irregular and improper charge and discharge which they receive. The advantage of booster-regulation is greatly discounted by the fact that the boosters have to run practically continuously. The method is not much used in purely lighting work, but an analogous system (see Fig. 158) is frequently employed for traction generating plants.

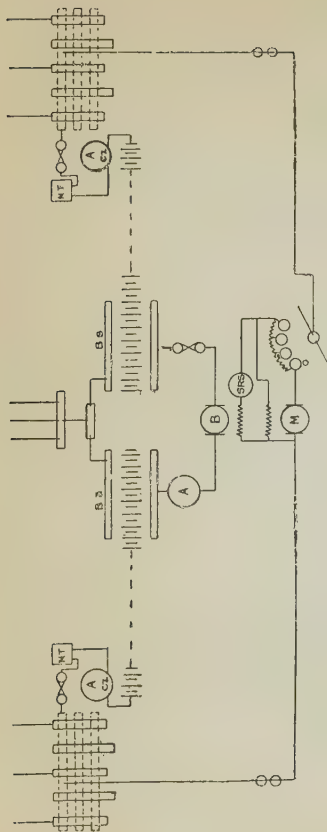


FIG. 156.

Battery Switch-gear for Three-Wire System.—In this arrangement one booster, with end cell regulating switches connected to the neutral wire, is used. Note that all battery-regulating switches should be provided with stops on the brushes to prevent the charge brush from being put *below* the discharge brush; otherwise it may frequently occur that the brushes of the switches are left in such position that the charging current is flowing in the wrong direction through some of the cells.

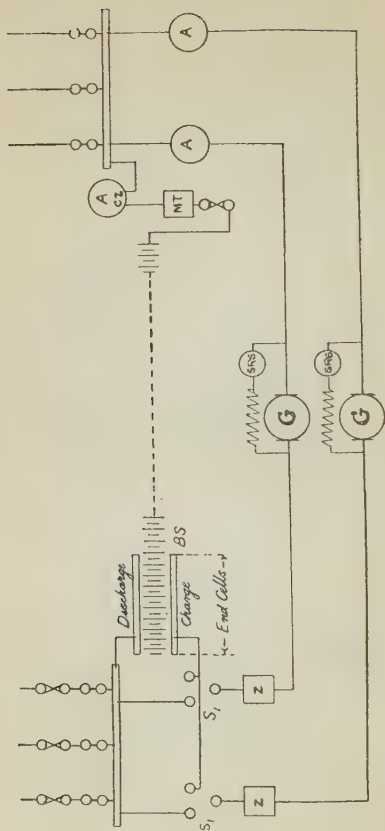


FIG. 157.

Two-Wire System with Battery.—In the majority of cases it is best to employ a booster for obtaining the extra pressure required to charge the end cells of a battery, though in some cases this extra pressure is obtained by raising the voltage of the generators. The diagram shows the connections for the latter method of working, the two-way switches S_1 being provided to enable either generator to be connected to the charge switch or directly to the bus-bar.

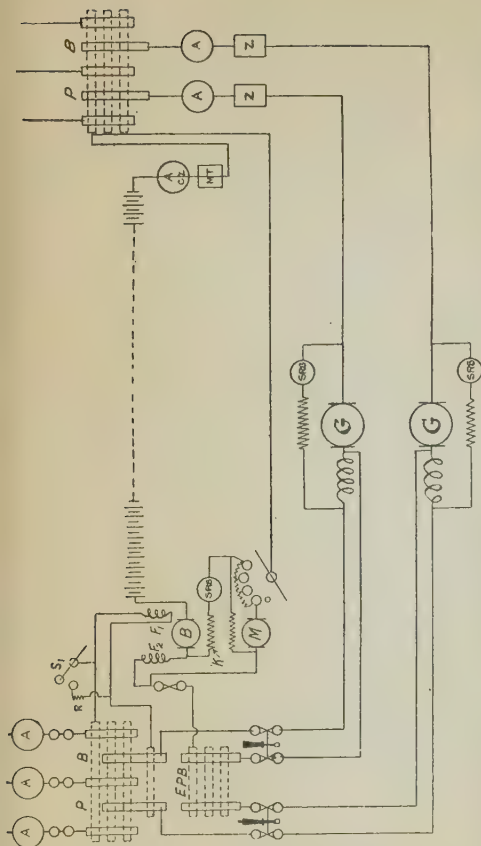
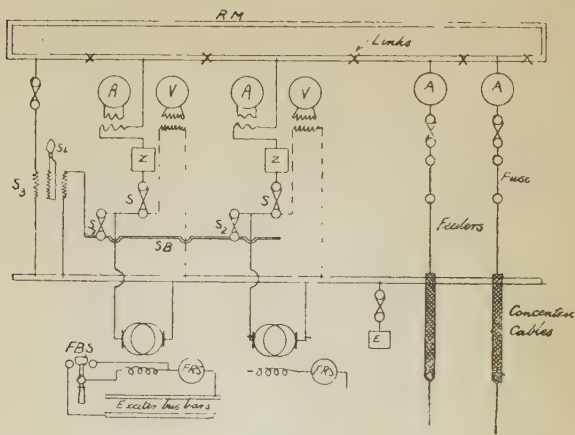


FIG. 158.

Two-Wire System with Automatically Regulated Battery.—In power plants where the fluctuations in load are rapid and considerable, a system comprising compound-wound generators running in parallel with an automatic booster-regulated battery is often employed, especially for traction work. The diagram indicates the connections of such a system. The equalizing plug-board, EPB, is provided to enable a third generator to be run on a separate circuit, should it be necessary to run the remaining generators and battery on a distinct circuit at a slightly different pressure. The booster B is differentially wound, and tends to keep the load on the generators constant by automatically discharging or charging the battery as the external load exceeds or falls below the normal. The booster series fields F_1 and F_2 oppose the booster shunt field K , the field F_1 being regulated to suit the number of generators running by the shunting switch S_1 and the resistance R . The switch S_1 can have as many contacts as there will be generators when the installation is complete, but to allow a margin for adjustment it is better to have three contacts per generator.



Single-phase H.T. System.—The diagram shows an alternating current system with one pole earthed. RM is the ring main with disconnecting links for isolating the circuits if necessary, and SB, S₁, S₂, and S₃ are the synchronizing bar, lamp (see p. 319), switches, and transformer respectively.

Automatic Cut-in and Cut-out.—This diagram gives the connections of an automatic switch used in battery circuits. It cuts the generator into circuit when its pressure reaches the right value, or cuts it out of circuit in the event of the generator pressure falling enough to allow the battery to discharge through the generator.

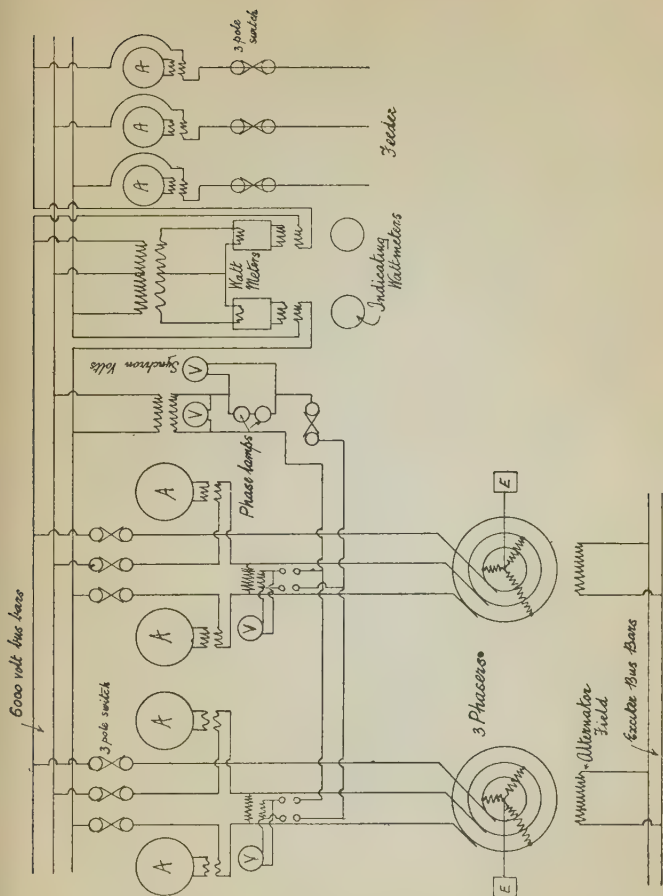


FIG. 161.

Three-phase H.T. System.—The ammeters and wattmeters are worked off the secondaries of Transformers so as to be absolutely safe.

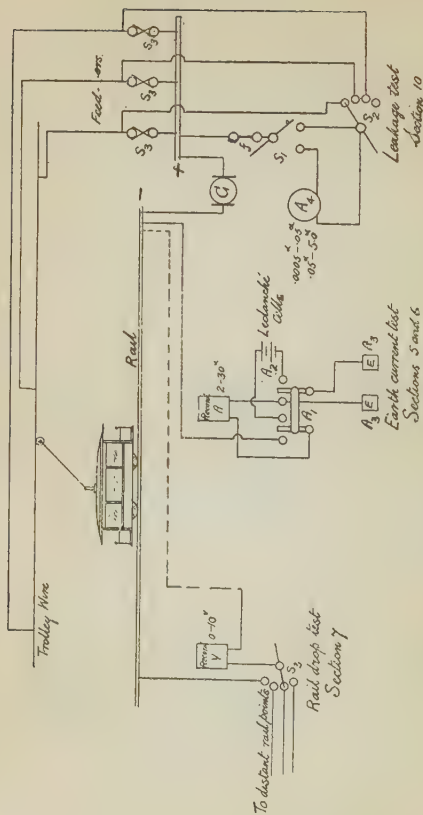


FIG. 162.

Diagram of B. o. T. Tramway Tests.—The instruments and connections for making the principal Board of Trade tests on systems employing uninsulated conductors are shown in this diagram. To prevent the ammeter A_4 from being frequently burnt out when making the leakage test, the switch S_1 is put on the first contact, which does not include the testing ammeter A_4 in the circuit. In the event of a strong current accidentally flowing, the 10 ampere fuse f will give warning by blowing that something is wrong, and the ammeter will escape damage.

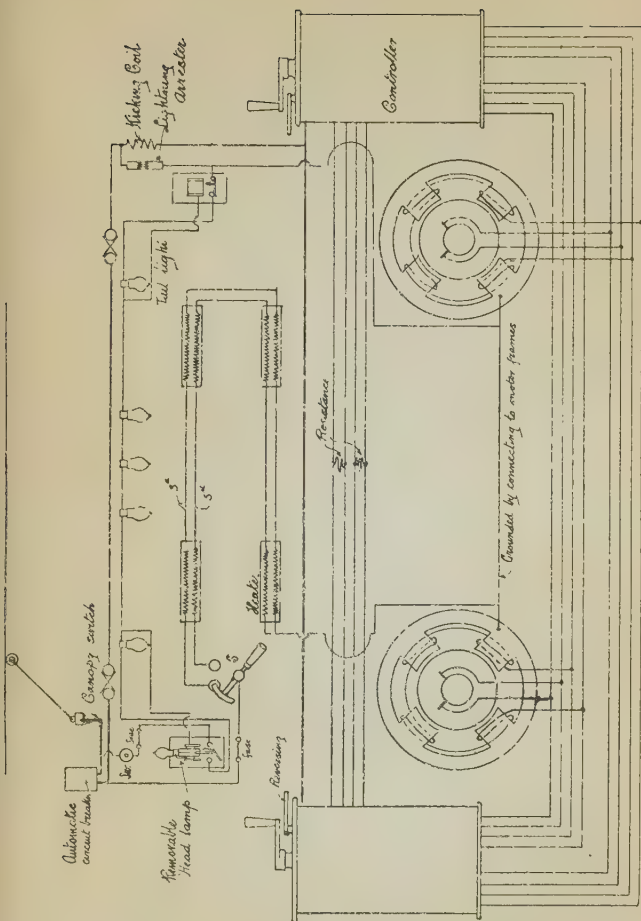


FIG. 163.

General Scheme of Tramcar Wiring.

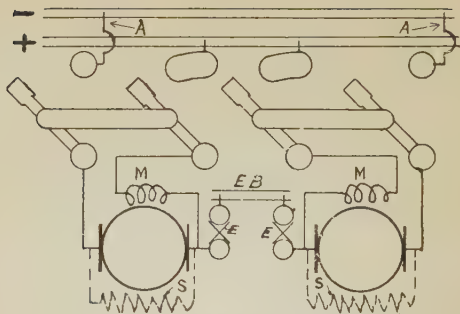


FIG. 164.

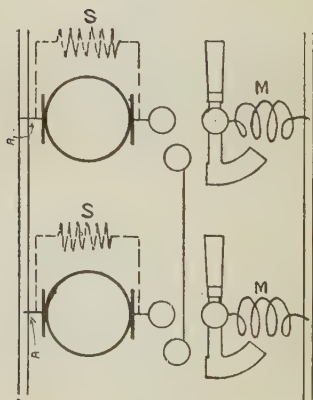


FIG. 165.

Switches for the parallel running of compound-wound dynamos.

In Fig. 164 the equalizing switch *E* must be put in first when switching on a dynamo, and taken out last when switching off.

In both diagrams the main switches must be put on "half-cock" and the generator pressure adjusted, before they are finally put right home. The dynamo ammeters must be inserted in the leads marked *A*; if they are put next to the main bus-bar on the pole containing the equalizing lead, they will not indicate the dynamo armature current correctly.

ELECTRO-METALLURGY AND ELECTRO-CHEMISTRY.

By E. G. LL. ROBERTS.

THE metals are often arranged in what is known as the **electro-chemical series**. This is convenient, as from the position of an element in this series many of its properties can be foretold with fair accuracy. The order is based upon the relative "heats of formation" of compounds of the various metals. The following are so arranged :—

1 Potassium.	7 Cadmium.	13 Copper.
2 Sodium.	8 Nickel.	14 Mercury.
3 Magnesium.	9 Cobalt.	15 Silver.
4 Manganese.	10 Lead.	16 Platinum.
5 Zinc.	11 Tin.	17 Gold.
6 Iron.	12 Bismuth.	18 Antimony.

Those occurring at the head of the list are said to be **electro-positive** to those below, which are said to be **electro-negative**. These terms are, however, only relative. Thus copper is electro- to iron and electro + to silver or gold. It will be noticed that where two metals are constituents of a voltaic cell, one (most generally zinc) occurs at the electro-positive end, and the other, as, for instance, platinum (Grove), silver (Snec), or copper (Daniell and simple cells), at the electro-negative end. This is absolutely essential with a view to obtaining a high E.M.F. from the "element," as the heat evolved during the "combustion" of the more positive metal in acid, is a measure of the E.M.F. to be expected when the cell is in good working order. Conversely, the arrangement is indicative of the E.M.F. which must be maintained at the electrodes when such solutions are electrolyzed. For those compounds which, during their formation, give out large quantities of heat, require relatively large amounts of energy in some form or other, to effect their decomposition.

Calculation of E.M.F.s from "heats of formation." As stated above, the E.M.F. developed in any combination is proportional to the heat of formation of the resulting compound. The heat of formation of H_2O from hydrogen and oxygen is 68,360 gramme-calories, and the E.M.F. is 1.478 volts. Both these figures have been experimentally determined (though the latter can be deduced from the former), and they imply that when one "gramme molecular weight" (*i.e.* $2 \times 1 + 16 = 18$ grammes) of water is formed, sufficient heat is liberated to raise 68,360 grammes of water through $1^\circ C.$, and that further, if this energy is employed to generate electricity, the corresponding E.M.F. will be 1.478 volts. Hence it follows that in any reaction the E.M.F. developed = $\frac{\text{heat of formation}}{68,360} \times 1.478 \text{ volts.}$

The following table (based on Naumann's "Thermochemie") gives the heats of formation of various compounds:—

Metal.	Combination with Oxygen.		Combination with Chlorine.		
	Reaction.	Grm. calories.	Reaction.	Grm. cals. dry.	Grm. cals. in soln.
Potassium ...	K_2, O, aq	164,560	K_2, Cl_2	211,220	202,340
Sodium ...	Na_2, O, aq	155,260	Na_2, Cl_2	195,380	193,020
Calcium ...	Ca, O, aq	149,460	Ca, Cl_2	169,820	187,230
Magnesium ...	Mg, O, H_2O	148,960	Mg, Cl_2	151,010	186,930
Aluminium ...	Al_2, O_3, aq	388,800	Al_2, Cl_6	321,870	475,560
Manganese {	Mn, O, H_2O	94,770	Mn, Cl_2	111,990	128,000
Zinc ...	Mn, O_2, H_2O	116,280	—	—	—
Iron ...	Zn, O, H_2O	82,680	Zn, Cl_2	97,210	112,840
Cobalt {	Fe, O, H_2O	68,280	Fe, Cl_2	82,050	99,950
Nickel {	Fe_2, O_3, aq	191,130	Fe_2, Cl_6	192,080	255,420
Copper {	Co, O, H_2O	63,400	Co, Cl_2	76,480	94,820
Lead ...	Co_2, O_3, aq	149,300	—	—	—
Tin ...	Ni, O, H_2O	60,840	$NiCl_2$	74,530	93,700
Mercury {	Ni_2, O_3, aq	120,380	—	—	—
Silver {	Cu, O, H_2O	7,520	—	—	—
Gold ...	Cu, O	37,160	Cu, Cl_2	51,630	62,710
	Cu_2, O	40,810	Cu_2, Cl_2	65,750	—
	Pb, O	50,300	Pb, Cl_2	82,770	75,970
	Sn, O, H_2O	68,090	Sn, Cl_2	80,790	81,140
	Sn, O_2, H_2O	133,490	Sn, Cl_4	127,250	157,170
	Hg, O	30,670	Hg, Cl_2	63,160	59,860
	Hg_2, O	42,200	Hg_2, Cl_2	82,550	—
	Ag_2, O	5,900	Ag_2, Cl_2	58,760	—
	Au_2, O_3, aq	13,190	Au, Cl_3	22,810	27,270
	—	—	Au_2, Cl_2	11,620	—

Metal.	Combination with Sulphuric Acid.		Combination with Nitric Acid.	
	Reaction.	Grm. cals.	Reaction.	Grm. cals.
Potassium ...	K_2, O, SO_3aq	195,850	K_2, O, N_2O_5aq	192,100
Sodium ...	Na_2, O, SO_3aq	186,640	Na_2, O, N_2O_5aq	182,620
Calcium ...	Ca, O, SO_3aq	180,600	Ca, O, N_2O_5aq	177,360
Magnesium ...	Mg, O, SO_3aq	180,180	Mg, O, N_2O_5aq	176,480
Aluminium ...	$Al_2, O_3, 3SO_3aq$	150,630	—	—
Manganese ...	Mn, O, SO_3aq	121,250	Mn, O, N_2O_5aq	117,720
Zinc ...	Zn, O, SO_3aq	106,090	Zn, O, N_2O_5aq	102,510
Iron ...	Fe, O, SO_3aq	74,990	Fe, O, N_2O_5aq	89,670
Cobalt ...	$Fe_2, O_3, 3SO_3aq$	224,880	—	—
Nickel ...	Co, O, SO_3aq	88,070	Co, O, N_2O_5aq	84,540
Copper ...	Ni, O, SO_3aq	86,950	Ni, O, N_2O_5aq	83,420
Lead ...	Cu, O, SO_3aq	55,960	Cu, O, N_2O_5aq	52,410
Silver ...	Pb, O, SO_3aq	73,800	Pb, O, N_2O_5aq	68,070
	—	—	Hg, O, N_2O_5aq	37,320
	—	—	Hg_2, O, N_2O_5aq	47,990
	Ag_2, O, SO_3aq	20,390	Ag_2O, N_2O_5aq	16,780

Miscellaneous Combinations.				Reaction.	Grm. cals.	Gr. cals. in water.
Oxygen and Hydrogen	...	...		$2\text{H}, \bar{\text{O}}$	58,200	78,020
Oxygen and Nitrogen	...		{	N_2O_5	—	72,970
				$\text{N}_2\text{O}, \text{O}_4$	—	48,140
				$\text{N}, \text{O}_3, \text{H}$	41,510	49,090
				$\text{NO}, \text{O}_2, \text{H}$	63,085	70,665
				NO_2, OH	43,515	51,095
				$\text{N}_2\text{O}_4, \text{O}, \text{H}_2\text{O}$	18,670	—
Nitrogen and Hydrogen	...	...		N, H_3	11,890	20,230
				$2\text{NH}_3, \frac{1}{2}\text{O}_4, \text{H}_2$	65,250	28,150 *
				NH_3, HCl	41,910	12,270 *
				$\text{N}, \text{H}_4, \text{Cl}$	75,800	—
Chromic acid...	...	...	...	$\text{Cr}_2\text{O}_3, \text{O}_3, \text{aq}$	—	18,900
Hydrochloric acid	...	...	...	HCl aq	22,000	39,315
Hydrogen sulphide	...	...	...	$\text{H}_2, \text{Sa q}$	9,300	
Sulphuric acid	...	...	{	$\text{SO}_2, \text{O}, \text{H}_2\text{O}$	53,480	
				$\text{SO}_3, \text{H}_2\text{O}$	21,320	
Carbonic oxide	...	...	...	C, O	29,000	
Carbon dioxide	...	...	{	C, O_2	96,960	
				CO, O	73,840	

In using the table, it should be noted that the heats generated in the formation of a compound in the dry state, and in solution, are not as a rule the same, owing to the "heat of solution" of the compound—Aq in the formula denotes that excess of water is present, while the H_2O means that only enough to satisfy the formula is employed. If two chemical reactions take place in the same cell, the algebraic sum of the E.M.F.s produced must be taken.

Example—Daniell's Cell.

- (a) Formation of zinc sulphate + 106,090 cals. = + 2.29 volts.
 (b) Decomposition of copper sulphate - 55,960 cals. = - 1.21 ,,

Hence E.M.F. of cell = 1.08 volts.

ELECTROLYSIS.

Electrolysis is the splitting up of a chemical compound into its constituents by means of an electric current. The compound to be separated is called the "electrolyte," the plates or poles of the electrolyzing bath or tank the electrodes, and the plate where the current enters the anode, while the other plate is spoken of as the cathode.

From a solution of silver nitrate (containing preferably 20 % AgNO_3) 0.001118 gram of metallic silver will be deposited per

* Components in solution.

second by the passage of one ampere. On this fact is based the silver voltameter for measuring current. The amount of hydrogen evolved by an equal current is 0.00001038 gm. per second. Multiplying the figures by the chemical equivalent of any metal, the amount of metal deposited by a given current in a certain time can be calculated.

Thus, the current taken to liberate 1 gram of hydrogen will yield 8 of oxygen, or 108 of silver, these numbers being the chemical equivalents of those elements.*

To calculate the amount of metal deposited in a certain time by a known current, find the weight of hydrogen evolved, and multiply by the chemical equivalent of the metal.

ELECTRO-CHEMICAL EQUIVALENTS.

Elements.	Valency.	Atomic Weight.	Chemical Equivalent.	Electro-chemical Equivalent (milligrammes per coulomb).	Coulombs per gramme.	Grammes per ampere hour.
ELECTRO-POSITIVE.						
Hydrogen	H ₁	1.00	1.00	.010384	96293.00	0.03738
Potassium	K ₁	39.04	39.04	.40539	2467.50	1.45950
Sodium	Na ₁	22.99	22.99	.23873	4188.90	0.85942
Aluminium	Al ₃	27.3	9.1	.09449	1058.30	0.34018
Magnesium	Mg ₂	23.94	11.97	.12430	804.03	0.44747
Gold	Au ₃	196.2	65.4	.67911	1473.50	2.44480
Silver	Ag ₁	107.66	107.66	1.11800	894.41	4.02500
Copper (cupric)	Cu ₂	63.00	31.5	.32709	3058.60	1.17700
„ (cuprous)	Cu ₁	63.00	63.00	.65419	1525.30	2.35500
Mercury (mercuric) ..	Hg ₂	199.8	99.9	1.03740	963.99	3.73450
„ (mercurous)	Hg ₁	199.8	199.8	2.07470	481.99	7.46900
Tin (stannic)	Sn ₄	117.8	29.45	.30581	3270.00	1.10090
„ (stannous)	Sn ₂	117.8	58.9	.61162	1635.00	2.20180
Iron (ferric)	Fe ₃	55.9	18.64	.19356	5166.4	0.69681
„ (ferrous)	Fe ₂	55.9	27.95	.29035	3445.50	1.04480
Nickel	Ni ₂	58.6	29.3	.30425	3286.80	1.09530
Zinc	Zn ₂	64.9	32.45	.33696	2967.10	1.21330
Lead	Pb ₂	206.4	103.2	1.07160	933.26	3.85790
ELECTRO-NEGATIVE.						
Oxygen	O ₂	15.96	7.98	.08286		
Chlorine	Cl ₁	35.37	35.37	.36728		
Iodine	I ₁	126.53	126.53	1.31390		
Bromine	Br ₁	79.75	79.75	.82312		
Nitrogen	N ₃	14.01	4.67	.04849		

* Its law, namely, that the electro-chemical equivalent (*i. e.* the grams liberated by an ampere in one second) is proportional to the chemical equivalent, is due to Becquerel. The chemical equivalent is equal to the

ELECTROLYTIC COPPER REFINING.

The **crude copper** is cast into plates measuring 3 feet by 2 feet, which form the anodes in the refining bath, containing a solution of 1·5 to 2 lb. of copper sulphate and 5 to 10 oz. of sulphuric acid per gallon of water.

These plates are separated from the cathodes by a distance of $1\frac{1}{2}$ to 2 inches. The area of the cathode surface may total 200 sq. feet in each tank.

The **cathodes** now universally employed are thin electro-copper plates prepared by depositing on a blacklead plate and stripping the thin deposit of about $\frac{1}{32}$ inch. The tanks, measuring 8 feet $\times$ 4 feet wide $\times$ 3 feet in depth, are constructed of wood and lead lined, soldered joints being avoided. Tanks should be grouped in terrace fashion in order to take advantage of gravity for the circulation of the electrolyte.

Recording voltmeters are now used for the potential difference in each tank and give warning of short circuits.

The many **systems** in use may be divided into two groups, known respectively as the multiple and the series systems. For the comparison of these systems reference may be made to articles by Kellar (*Mineral Industry*, New York, 1899, vol. vii., p. 229). The current density commonly ranges from 7·5 to 15 (or even more) amperes per square foot. The E.M.F. necessary may be taken as ·3 volt for every pair of electrodes, when the resistance of electrolyte, impurities present, and distance between electrodes are normal.

The **yield of copper** of, say, one ounce per ampere per day, which Faraday's law implies, is never obtained, 95 % of this being a very average figure.

The **power required** with a current efficiency of 95 % and an E.M.F. of ·3 volt for refining 1 lb. of copper, is about ·16 electrical H.P. hours. In practice this figure may rise as high as ·25 electrical H.P. hours per lb. of copper.

Most of the **impurities** present in the copper anodes fall into the mud at the bottom of the bath, which is subsequently treated. This mud contains the gold, platinum and silver; bismuth, zinc, iron, nickel and cobalt dissolve in the electrolyte, and are never deposited upon the cathode to any serious extent.

The following analyses show the **composition of modern electro-copper** :—

combining weight, and not to the "atomic weight divided by valency." Ferric salts as also sesqui salts generally are not in accordance with the last-named rule, as defined by Thompson and others. This is shown in the table of Electro-Chemical Equivalents under Iron (ferric), where the chemical equivalent, instead of being $\frac{55\cdot9}{4}$ is actually 18·64.

	1	2
Cu	99.98	99.92
As	nil	nil
Sb	nil	trace
Pb	.01	.01
Bi	trace	trace
Fe	nil	.01
Ni	nil	trace
O	.01	.03
	100.0	99.97

Revolving cathodes are now used for deposition, and also for the manufacture of seamless tubes by the Cowper-Coles process,* where a peripheral velocity of at least 1000 feet per minute is used. By employing revolving cathodes, a very much higher current density may be employed.

ELECTROLYTIC REFINING OF SILVER.

Impure silver is refined in an acid solution of silver nitrate in a similar manner to copper. The deposit of silver is very incoherent and as deposited is automatically scraped from the cathode plate and caught in suspended cloth bags beneath. On account of the very electronegative character of silver no other metal will be deposited at moderate current densities. Therefore with a pure solution a **current density** of 30 amperes per square foot may be used, the density must be lowered as the copper in solution increases and also extra silver must necessarily be added.

With an electrode distance of 2 inches, an **E.M.F.** of 1.5 volts is sufficient.

Gold is recovered in the anode slime in bags of cloth.

The necessary **power** to deposit 100 ounces (troy) of refined silver may be taken as $1\frac{1}{4}$ electrical H.P. hours.

Other arrangements are in less general use for the refining of silver, and reference should be made to Dietzel's method.†

* *Jour. Inst. E. E.*, vol. xxix., 1900, p. 258.

† *Zeitschrift für Electrochemie*, 1899, vol. vi. p. 81.

REFINING OF GOLD.

In the refining of gold the electro-plating cyanide solution is unsuitable on account of the solution of other metals, as silver, copper, etc., and their consequent co-deposition.

A chloride solution is used in a glazed earthenware or china tank, the solution being kept at a temperature of about 70° C. (158° F.). The strength of solution should be 2.5 to 3 parts of pure gold per 100 parts of solution, with 2 to 8 % concentrated hydrochloric acid for the solution of the anodes. The bath is worked with an **E.M.F.** of one volt or higher, and a current density of 100 amperes per square foot, the electrodes being 1.25 inches apart.

All the anode metals will be dissolved in the bath with the exception of silver and iridium, lead and bismuth coming down in the slimes when the solution is saturated with their respective chlorides.

With pure electrolytes the **deposit** will be powdery, impure solutions forming a detachable plate. The purity (or fineness) of the gold may be as high as 999.8 (per 1,000).

Platinum and palladium are recovered from the solution by chemical precipitation; on no account should more than 5 % of palladium be present in the anode, as otherwise the metal will be co-deposited with the gold.

PRODUCTION OF ALUMINIUM.

The processes in present day use, namely, the Héroult, Hall, and Minet, are in principle the same and have entirely displaced the purely chemical methods.

The **furnaces** in use consist of steel tanks lined with crushed carbon dust, this lining forming the cathode. A single carbon rod or a bundle bound together forms the **anode**, which is capable of being lowered to any depth in the furnace or entirely withdrawn, as is necessary when re-lining and repairing the furnace. The furnace, which is generally circular in form, is provided with a cover, in which holes are left for the introduction of the anode carbon and the charge. The reduced aluminium is withdrawn from the interior of the furnace, by means of a tap hole situated at the side and on a level with the bottom, either by a ladle, or a siphon.

The **bath** consists of a mixture of aluminium, sodium and calcium fluorides, or of "cryolite," which is the natural double fluoride of aluminium and sodium; other fluorides can also be added.

The charge should be so arranged as to allow of its ready melting at a good red heat, and may be melted in a separate furnace or added solid, to the reducing furnace.

The mixture of fluorides is capable of dissolving 20% of their own weight of alumina, and it is this alumina that is electrolyzed through the fluoride media.

The theoretical **E.M.F.** for the electrolysis of alumina is 2.8 volts, but in order that sufficient heat shall be maintained to ensure a molten bath, four volts is required.

In the electrolysis of alumina the **metal is deposited** on the bottom and sides of the furnace, and the evolved oxygen combines with the red-hot anode carbon forming carbon monoxide. The anode is thus gradually exhausted. The amount used is however in excess of that required to satisfy the chemical reaction. For every pound of metal produced .7 to 1 lb. of anode carbon is required. The temperature of the molten bath is about 800° C. (1500° F.), thus ensuring a very fluid condition for the metal, which can consequently be easily withdrawn.

This process is absolutely continuous, the fluorides being capable of very long use.

For the manufacture of very pure metal, all materials charged into or used in the furnace must be as pure as possible, for on account of the high position which aluminium occupies in the order of electro-positive metals, certain metalloids would be co-reduced.

This fact, however, is utilized for the production of certain **alloys of aluminium**, used for casting and machining purposes, and, on that account, the production of such alloys is no more expensive than pure aluminium. These harder alloys contain lead, iron, chromium and copper.

The **current** employed in these furnaces may be 8000 amperes,* and current density of 35 amperes per square inch of anode surface, and five amperes per square inch of cathode surface.

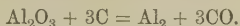
THE PREPARATION OF ALUMINIUM ALLOYS.

Ferro-aluminium and **aluminium bronze** are alloys which are made by a purely electro-thermal process. The use of a second metal, and the formation of an alloy of aluminium, in the furnace with the reduced metal is solely for the purpose of withdrawing the aluminium in a workable and usable condition. Ferro-aluminium is the more satisfactory alloy, as a certain percentage of carbon is sometimes necessary and hardly ever troublesome.

The **furnace** itself is a small brick chamber, in which the

* *Jour. Society of Chem. Indust.*, 1898, vol. xvii. p. 308.

carbon poles are introduced at opposite ends. The process is solely based upon the chemical reaction—



The reduced aluminium unites with the molten metal previously introduced, the resulting alloy being tapped out from the furnace at convenient intervals.

In the same way the oxides of **other metals, used in modern steel manufacture** are reduced. Iron forms the basis of all the alloys with which these reduced metals are alloyed. At the present day this is the only commercial method of preparation of such alloys as ferro-molybdenum, ferro-tungsten, ferro-uranium and high-grade ferro-silicon ; metals which require a temperature for reduction higher than that obtained by any iron-smelting furnace. It is necessary to allow an excess of the metallic oxides in the furnace, otherwise an alloy containing much carbon will be produced.

THE PRODUCTION OF CALCIUM CARBIDE.

The method of manufacture in use at the present day is dependent upon the reducing action of carbon on lime at a high temperature. Part of the reduced calcium combines with a further portion of carbon to form carbide, which sinks to the bottom of the furnace. For the production of calcium carbide, which is very widely used for illuminating and other purposes, a large industry is very rapidly rising in America, Germany and other countries where electrical power is cheaply obtained. In general practice now, the **furnace** employed consists of rectangular brickwork, the bottom of which is lined with carbon, circuit is made through a number of carbon rods introduced through the top and sides of the furnace. The lime employed should be pure, and the carbon, fed in the shape of good hard coke dust, should be mixed in the proportion of 10 parts of lime to 7 parts of coke. Arrangements are made for the carbide as produced, to be tapped out of the furnace in the continuous process, and to be withdrawn and allowed to cool in the intermittent. The current employed is alternating, and should be heavy enough to produce a **temperature** of 3400°C . to 4000°C ., between which temperatures calcium carbide flows freely, this being the essential feature in the continuous process. With the intermittent process the temperature required is not so great as is necessary in the continuous method. Sufficient current, however, should be supplied to enable the small globules of carbide to slightly cohere into an irregular mass. When sufficient is formed, the masses are withdrawn by means of a false bottom fitted to the furnace, and allowed to cool.

In some instances, means are provided by which the bottom can receive a slight jolting motion, which facilitates the cohesion of globules, and the consolidation of the charge. The materials are charged into the furnace by means of rotating blades working through slots in the walls of the furnace.

The **E.M.F.** in general use varies greatly, 50 and 110 volts being probably extreme figures. The necessary amperes are entirely dependent upon the form and size of the furnace in use.

The amount of **energy required** is, in good practice, about 1 electrical H.P. hour per 1 lb. of pure calcium carbide.

FIXATION OF ATMOSPHERIC NITROGEN.— CYANAMIDE.

Works are now (1911) erected for the production of calcium cyanamide, whereby 100,000 tons of nitrogen can be fixed annually.*

It is also called "nitrolim," its main use being as a fertilizer, the nitrogen being liberated by steam and the NH_3 caught as sulphate of ammonia by sulphuric acid.

The production of cyanamide is of much importance, it being made by heating calcium carbide, electrically or by gas, in an atmosphere of nitrogen, to a temperature of $800\text{--}1000^\circ\text{C}$.

Crude "nitrolim" contains—

57–63 %	pure cyanamide,
20 %	of lime,
14 %	of carbon,
8 %	of silica, iron oxide and alumina.

CARBORUNDUM.

During the investigations carried on in connection with the production of carbides, it was shown that under the influence of the electric arc a mixture of clay and charcoal gave a hard blue crystalline substance. To this the name carborundum was given, it being supposed to consist of alumina (corundum) and charcoal. On account of its extreme hardness it has found a wide application as an abrasive, and its economic production has received very considerable attention. It is now made by passing a current of 2000 to 6000 amperes at a pressure of from 100 to 125 volts through a mixture of coke and sand containing about 10 % sawdust and 1 or 2 % common salt. The charge is built round a core of granular carbon to allow the current to start. During the process, which lasts 36 hours, a large quantity of carbon monoxide is evolved. On breaking the circuit, the

* *Chemical Trade Journal*, Jan. 28, 1911, p. 87.

mass is allowed to cool for 24 hours, and the top of the charge is then raked off and used in another operation. Below this is a layer of amorphous carbide of silicon amounting to 40 % of the whole which is quite valueless. Round the carbon core is found a layer some 12 in. thick of brilliant crystals of silicon carbide (carborundum). The mass is broken up and washed, and treated with strong sulphuric acid for some days. After this treatment it is washed and sorted, different grades of the material having each their uses. Then the coarse material is mixed with clay or felspar and water and made up into wheels often as large as 36 in. in diameter. Finer grades are used for polishing pottery, boots, plate-glass, etc. Carborundum, according to analysis, has a composition in agreement with the chemical formula SiC . It is unattacked by acids, and no action can be detected when heated to redness in oxygen. Its hardness is 9 to 10, and it may therefore be used as an abrasive for diamonds.

ALUNDUM AND CORUNDUM.

Alundum is prepared from bauxite, which contains a high percentage of alumina, by preliminary heating, to drive off the water of combination, and by subsequent melting in an electric resistance furnace similar to that used for the production of carborundum. The melted bauxite is allowed to cool, and crystallizes out into solid masses of alundum of great purity. Its chief use is for the manufacture of artificial emery grinding wheels.

Corundum is native alumina containing a little silica, ferric oxide and water, being thus very similar to alundum.

MANUFACTURE OF GRAPHITE.

When compressed carbon-dust is subjected to the intense action of an electric arc, the carbon is changed into the graphitic form, the electrical conductivity being increased and the specific gravity raised. The first experiments made upon the phenomenon was due to Girard and Street, while their practical application was started by Acheson at Niagara.

Powdered coke or any other form of carbon, is mixed with some oxide or silicate; and this mass, moistened with water containing a small quantity of sugar, is then moulded into the required shape.

The moulded articles are next embedded in powdered carbon and surrounded by a very bad conductor of heat. They are then subjected to the action of a strong electric current.

The metals having been reduced, form carbides and probably

subsequently volatilize. The whole of the carbon may be turned into graphite if so desired; in practice, however, the whole change is seldom carried completely out, a small percentage of the carbon being left in the initial condition, this affording greater strength to the article.

In this manner most of the carbon used for dynamo brushes, electric furnaces, arc-lamps, battery-plates, etc., is manufactured.

THE ELECTRO-THERMAL PRODUCTION OF PHOSPHORUS.

Of late years a large amount of the commercial phosphorus sold has been manufactured electrically from the waste slag of basic steel furnaces and other sources.

Furnaces smaller than those used for the production of carborundum are employed, capable of making $1\frac{1}{2}$ cwt. of phosphorus per day. At the temperature obtained in the furnace, silica, fed in in the form of sand, attacks the calcium phosphate, forming calcium silicate and giving off phosphorous pentoxide, which is decomposed by carbon into carbon monoxide and phosphorus in the form of vapour. These reactions occur in the furnace itself, and an outlet is provided for the escape of the phosphorous vapour, this being led into condensers, where it is condensed and subsequently cast into commercial sticks under water. An alternating current is employed. The calcium silicate is run off from the furnace continuously, and sometimes kaolin is used in conjunction with sand, thus forming a double silicate which is more fusible than the single silicate of lime. Reference should be made to the *Journal of the Society of Chemical Industry*, 1891, vol. x., p. 445, where the Readman-Parker process is very fully described.

THE REDUCTION OF MAGNESIUM.

For many years the whole of the production of magnesium has been by electrical means.

The process is similar to that of aluminium reduction, the temperature employed consequent on the very low melting point of magnesium being very much lower. Carnallite, which is a double chloride of magnesium and sodium, has been latterly considerably employed. The process is worked at a dull red heat (say 700° C.). The reduced metal is caught by perforated ladles, similar to, but considerably smaller than, those employed in the Pattinson and Parke's processes for the desilverization of lead. The liquid chlorides drain through the holes, while the metal is left behind, on account of the high surface tension, and is subsequently remelted and cast into sticks or ingots.

The **current density** employed is about 0.7 ampere per square foot of cathode surface, with an **E.M.F.** of 7 volts.

MANUFACTURE OF POTASSIUM CHLORATE.

The manufacture of this important compound is largely carried on by passing chlorine gas into hot milk-of-lime and decomposing the resulting compound by means of salts of the alkali. A mixture of calcium chlorate and calcium chloride results. On treatment with carbonate of potash, potassium chlorate is formed and is obtained by concentration and crystallization. The electro-chemical preparation follows similar lines. A solution of potassium chloride is electrolyzed by a current of 50 amperes per square decm. at 5 volts pressure. The electrodes are so arranged that the liberated chlorine comes into intimate contact with the potash, and at a temperature of 50° C. a mixture of chlorate and chloride results, no hypochlorite being formed. The chlorate is obtained by evaporation, and the solution is maintained at a strength of 20% to 25% of chloride.

PRODUCTION OF SODIUM.

The demand for pure sodium, for the production of chemically pure sodium hydroxide, the reduction of organic substances for aniline dyes, and for the preparation of the double cyanides of sodium and potassium from potassium ferrocyanide (according to Erlenmeyer's reaction), gave rise to several processes which have now displaced the distillation methods.

The discovery of the alkaline metals, as is well known, was due to **Davy's electrolytic experiments**. Since then many processes have been evolved, the foremost being that of **Castner**, whose process is now universally adopted.

The apparatus consists of a melting pot of iron. The cathode is inserted in the bottom and the initial joint made through the melting pot with a packing of caustic alkali. The anode takes the form of a cylinder loosely fitting over the end of the cathode and may be made of metal in the case of caustic alkali electrolysis. The alkali is kept melted by a Bunsen gas flame placed underneath the melting pot. Immediately above the cathode and within the anode is placed the collecting tube, fitted with a loose lid, which allows of the escape of hydrogen. The metal is withdrawn by means of a perforated ladle, which allows of the removal of the metal by reason of its high surface tension, while the hydroxide drains away through the perforations.

Castner's apparatus has been largely modified by **Becker**,* in

* German patent 104,955, Jan. 21, 1899.

whose apparatus the cathode joint with the melting pot is made more safe by placing a water jacket round the bottom of the cathode, so that the fused salt becomes pasty. The end of the cathode is so shaped that the globules of liberated metal may easily escape to the collecting tube above.

The anode, which is ring-shaped as in Castner's apparatus, is held in position by suspended wires, and may be made in one or more pieces. The collecting tube is connected by an external wire to the cathode, and the metal is withdrawn by an inclined spout fixed to the top of the collecting tube, which is also arranged for the escape of hydrogen. A process devised by **Darling** for the production of sodium from sodium nitrate, has been in use for some time on a large scale in America.

The apparatus embodies the use of a diaphragm, which was mainly due to the many initial fruitless experiments. The material in use in the diaphragm is electrically-fused magnesia, which is crushed and pressed into the hollow space between two tubes, forming a double-walled vessel. The inner tube is perforated and made of steel, the thickness of the walls being about 4 inches. The vessel is 30 inches high and 15 inches in external diameter.

This double tube is set within a simple cast-iron melting pot from which it is insulated by some simple material. The intervening space between the diaphragm and the melting pot is filled with sodium nitrate, and the diaphragm is charged with sodium hydroxide into which the cathode is fixed. The electrolyte, which is slightly heated, yields sodium at the cathode and nitrogen dioxide and oxygen at the anode, these being subsequently converted into nitric acid, while the sodium is ladled from the vessel. A small amount of the total current is allowed to pass through the walls of the diaphragm and thus prevent its action as an intermediate electrode. A substitute for the magnesia packing now used consists of a mixture of finely-burnt clay, magnesite and cement.

The furnace takes a current of 400 amperes at 15 volts, and firing is only required after charging the diaphragm, which lasts for about 450 hours.

PRODUCTION OF CALCIUM.

The economic production of metallic calcium has recently been made possible by the use of movable electrodes in the electrolyzing bath.

Fused calcium chloride is generally used, and as the metal is deposited on the cathode an exact distance (dependent on the E.M.F. employed) between anode and cathode is automatically maintained.

At present the calcium is made exclusively in Germany, and its much reduced cost has given rise to many new applications for the metal.

OTHER METALS ELECTROLYTICALLY PRODUCED.

Amongst the metals produced on a large scale by the electrolysis of their fused salts, large amounts of **lead** (Betts' process) and **zinc** are now annually produced.

ELECTROLYTIC DETINNING.

The scrap, generally in the form of tinplate shearings or old tinned cans, is suspended in a basket forming the anode in a bath containing a solution of caustic soda. On electrolysis taking place we have the following reactions: The water decomposes into oxygen and hydrogen, the oxygen attacking the tin, yielding tin oxide, which is soluble in caustic soda, giving sodium stannate. This is, in its turn, reduced by the previously evolved hydrogen, the metallic tin being deposited on the iron cathode in the form of a spongy mass. This deposited metal on subsequent melting contains 96 % Sn.

Free caustic soda must always be present and has to be continually added, owing to its exhaustion as sodium carbonate by exposure to the atmosphere. It is also necessary to keep the tinned scrap loosely packed, in order to allow of the detinning action over the whole surface. Much care has, moreover, to be exercised to produce scrap containing little or no tin, since imperfectly detinned scrap is unsaleable.

To obtain good results it is necessary to work in small units, and to avoid loss of liquor which contains a large amount of tin. The final detinned scrap still contains from .04–.5 % Sn.

By this and other less known processes nearly 4 % of the total annual consumption of tin is recovered.

ELECTROPLATING.

The value of electroplated articles is dependent on the durability of the deposited metal; it is therefore necessary to ensure complete adhesion of the metal deposited, by keeping everything in use—and more particularly the article to be plated—**perfectly clean**.

All grease must be removed by washing the objects in benzine; in some cases, where a great quantity of grease has to be removed,

an alkali bath or even ignition in a muffle furnace may have to be resorted to. Tarnish is best removed by acid or potassium cyanide, using a large supply of water for washing after each operation.

Metals and alloys such as copper, brass, and German silver, may be brightened after the removal of dirt in the alkali bath, by immersion in fuming nitric acid. Iron and steel objects after thorough polishing are placed in boiling caustic potash solution before being placed in the pickling bath, made up of 10 % sulphuric acid or nitric acid or 25 % hydrochloric acid.

Should much iron oxide be present on the article, this may be greatly removed by long soaking in a 2 % sulphuric acid bath, a second bath or cleansing by rubbing in fine sand being resorted to for final polishing. Many objects are subjected to a process known as "**quicking**" before receiving their final coating of metal, in order to give a better and firmer deposit. This process, which consists in slightly amalgamating the object with mercury, also prevents the initial precipitation of the metal in solution by the object in the plating bath.

The **vats** for deposition may be made of earthenware, lead-lined wood, glass or enamelled ironware, etc.

Where a certain **electromotive force** is given for a particular deposition, it must be borne in mind that it depends greatly on the conditions, such as the temperature of the bath, which by influencing the conductivity alters the **current density**. Care should be exercised not to subject articles to too high a current density, this gives a "burnt" deposit which is dark-coloured, sometimes pulverulent and useless. Resistances should be provided for every tank to ensure the proper voltage, as also should voltmeters and ammeters.

Decorative plated work may be produced by the aid of a stopping out, non-conductive varnish such as copal. A design is formulated and certain portions stopped out with this varnish and metal deposited on the unvarnished surface, which in turn is stopped out and another metal deposited on a different place.

Copper.—Iron is practically the only metal which can be coated with metallic copper by **simple immersion**. The coating of metal thus obtained is very often deposited in a non-adhesive form which has to be mechanically worked on to the object on which it is precipitated by either rolling or burnishing. For this process, the process by which steel pens are coated with copper, an alkali bath should be used for cleansing purposes.

Most of the copper plating is now done with the electric current, generally produced from low voltage dynamos.

The following are some **solutions used for the deposition of copper** on different metals.

Metal to be deposited upon	Temperature	Copper acetate	Copper sulphate	Potassium cyanide	Ammonia ('880)	Sodium bisulphite	Sodium carbonate	Sulphuric acid	Water
Electrotypes	Cold	—	150	—	—	—	—	50	1000
Zinc		14	—	20	8	—	—	—	1000
Iron and Steel	Hot or cold	19	—	16	14	20	40	—	1000

For an **acid bath** the amperes per square inch of cathode surface should be about $\cdot 065 - \cdot 1$, and the voltage may vary between $0\cdot 5$ and $1\cdot 5$. For an **alkaline bath** from $\cdot 02 - \cdot 03$ ampere per square inch of cathode may be used with from $3\cdot 0 - 5\cdot 0$ volts.

With the same current density the deposit of copper varies with the percentage of copper sulphate in solution; thus 5% copper sulphate and $\cdot 193$ ampere per square inch of cathode gave a resulting deposit quite worthless, but altering the percentage of copper sulphate to 20 an excellent metal of very fine raying was obtained (Von Hübl).

Silver.—Silver may be thinly deposited by **immersion** or by means of **pastes**. By immersion a very thin film is obtained consequent on the coat of silver preventing further precipitation from the solution by the article to be plated. The solutions are generally weak mixtures of a silver salt and potassium cyanide. The pastes, which may be used for lamp reflectors and small brass and copper articles, are made of the same ingredients with sufficient water to form a paste.

Typical solutions for **depositing silver electrically** would contain silver cyanide 3–35 parts, potassium cyanide (95%) 15–100 parts, water 1,000 parts.

Amperes per sq. in. of cathode should vary from $\cdot 015 - \cdot 030$, and from $\cdot 75 - 1\cdot 0$ volts are required.

The solutions are generally used quite cold, although occasionally, for depositing silver upon iron, tin, lead or zinc previously copper plated, the solutions are heated.

Deposits exhibiting a **frosted appearance** may be obtained by using a solution of silver iodide in potassium iodide with a very low voltage (Zinin).

For **bright plating**, where it is not easy to burnish the deposit, a little carbon bisulphide added to the bath produces this effect. Only a mere trace is necessary, an excess giving stained deposits, a large excess a dead white deposit.

A higher voltage is necessary for bright plating, and a harder deposit also results.

The silver to be used for the **anodes** must be the purest obtainable, and should be annealed before use in order to render it more easily soluble.

The area of the anode should equal that of the cathode, the amount of free cyanide being normal, an excess requiring a smaller anode.

The suspension wires for the anodes, unless of platinum, should not be allowed to come into contact with the solution.

The deposited metal should have a pure white colour, a pink shade indicates the presence of copper in solution, and the yellowish tarnish is due to a subcyanide of silver which is soluble in the plating bath, and may thus be removed after the current is stopped.

Gold.—The **simple immersion** process is seldom used for gold deposition, though at one time it was extensively in use.

For deposition by **electrolysis** a current of .006 ampere per square inch of cathode and .5–4.0 volts should be used.

As with silver the double cyanide solution is the most suitable and gives the best results. A **solution** for general use giving good results may be prepared by dissolving $\frac{3}{4}$ oz. (troy) of gold, converting it into cyanide and adding 7 oz. of good potassium cyanide to every gallon of solution. The solution should have been previously boiled and is to be used at a temperature of 130° Fahr. The bath should be altered in composition as required, for no hard-and-fast rule can be laid down, the bath varying so according to requirements and conditions.

The purest gold should only be used, the presence of a trace of either copper or silver giving a coloured deposit. If pure gold cannot be obtained for the **anodes**, a platinum anode may be substituted and the bath replenished with gold as required.

A black deposit is produced by too great a current, and motion imparted to the articles gives a light deposit. The required **colour of deposit** can only be obtained by experience, and the amount of scratch-brushing and burnishing also alters the colour considerably. Dead gilding is obtained by depositing on a slightly roughened surface, produced by a miniature sand blast or by dipping the article to be plated in an acid bath.

Nickel.—Nickel plating is generally required on one of three surfaces, iron (or steel), brass and zinc. Nickel deposits require the most careful guidance, and the extra caution extended to the preliminary operations of **cleansing and polishing** are generally repaid by a perfect deposit. Nickel deposited under the best conditions is extremely hard and a little brittle. It cannot be burnished. A thin film of nickel well deposited is equal to a much thicker deposit of another metal. Metal deposited on

articles not chemically clean, generally peels off in time, as does a very thick deposit unless deposited under perfect conditions. Metal deposited with a high **E.M.F.** is grey in colour and pulverulent, while a deposit at too low an **E.M.F.** will separate from the article during polishing.

The **current density** should vary from $\cdot 09$ – $\cdot 10$ ampere per square inch of cathode on initial deposition at from $4\cdot 5$ – $5\cdot 0$ volts; on finishing, however, the current strength must be lowered to $\cdot 015$ – $\cdot 02$ ampere per square inch of cathode at from $1\cdot 5$ – $2\cdot 0$ volts. For deposition on zinc $\cdot 025$ ampere per square inch of cathode at from $4\cdot 0$ – $5\cdot 9$ volts should be used.

The **solutions** used invariably contain nickel-ammonio sulphate as the basis. For depositing on Britannia metal Watt recommends the following solution, nickel-ammonio sulphate $33\cdot 3$ parts, ammonium sulphate $6\cdot 6$ parts, water 1,000 parts.

For deposition on iron, Watt uses 40 parts nickel-ammonio sulphate, 6 parts of ammonium sulphate, to 1,000 of water.

For hard deposits, Weiss uses 40 parts of nickel-ammonio sulphate, 10 parts of cobalt-ammonio sulphate, 17 parts of ammonium sulphate, to 1,000 of water.

The **anodes** to be used should be of cast nickel of the highest purity.

The area of the anodes should be greater than that of the cathodes in order to keep the solution saturated with nickel.

Better results are obtained by using rolled anodes at a voltage from 3–4, than by using cast anodes at from 2– $2\cdot 5$ volts. The **plating vat** should exceed the required size by about 25 per cent.

Britannia metal and zinc should be copper plated previous to nickelling.

Cobalt has lately been used as a substitute for nickel. The deposit is quite as brilliant as nickel and is somewhat harder. For cooking utensils cobalt plating is preferable to nickelling, on account of the resistance offered to organic acids by cobalt.

Satisfactory deposition has been obtained by Professor S. P. Thompson by using a solution of 1 part of cobalt sulphate to 20 parts of magnesium sulphate.

Brass (Copper and Zinc). **Brass, bronze, and German silver** are practically the only alloys deposited electrically.

In the deposition of brass a solution of the cyanides of copper and zinc is used, as the zinc does not precipitate the copper from solution, and there is thus a better chance for the deposition of the two metals occurring at the same time. One of the best solutions is due to Roseleur and contains 10 parts of copper carbonate, 10 parts of zinc carbonate, 20 parts potassium cyanide, 20 parts sodium carbonate, 20 parts sodium bisulphite, $\cdot 2$ part arsenious acid, to every thousand parts of water. In this solu-

tion the copper and zinc carbonates are precipitated from copper and zinc sulphates by sodium carbonate. The anodes to be used may have the same composition brass as the brass to be deposited, or they may consist of alternate layers of copper and zinc. The latter arrangement has the advantage of being capable of adjustment, as more or less of layers of one metal can be employed.

Several conditions have to be fulfilled for satisfactory depositions, consequent on the presence of two metals. A weak current tends to deposit more copper and a strong current more zinc. Further, an increased temperature increases the deposit of zinc, while a motion imparted to the cathodes increases the copper.

The current density should vary between .03-.05 ampere per square inch of cathode, with an **E.M.F.** of 3-4 volts.

MANUFACTURE OF PARABOLIC MIRRORS.*

Many difficulties met with in the production of parabolic mirrors have recently been overcome by the introduction of the method of electrotyping. Glass moulds having a convex surface similar to the mirror required are first made, no difficulty being experienced in this process. They are then carefully cleaned and finally rubbed over with precipitated ferrie oxide. This surface is then rendered conductive by immersing in a mixture of equal parts of (1) a $\frac{1}{2}\%$ solution of silver nitrate ; (2) a $\frac{1}{2}\%$ solution of caustic potash ; and (3) a $\frac{1}{4}\%$ solution of glucose.† Under these conditions the surface receives a brilliant coating of silver upon which a deposit of copper may be placed in a sulphate solution. When the deposit is sufficiently thick, separation from the mould is effected by immersing in warm water at a temperature of 120° F., the copper leaving the glass and carrying the silver with it. The surface is finally coated with palladium from a solution of ammonium palladium chloride. A weak current is employed, and the solution held in the inverted mirror. The palladium deposit while still retaining a brilliant polish tarnishes much less readily than silver. The only trouble found with these mirrors is that, should they become tarnished by exposure, they are very difficult to clean owing to the extreme thinness of the palladium coating. They are extremely light, and far cheaper than glass mirrors.

ELECTRIC WELDING.

The apparatus most generally used is the transformer welding machine. The primary coil is fed by an alternating current

* See *Electro-chemist and Metallurgist*, vol. i. p. 61.

† This same solution may be employed to silver lamp-bulbs, etc.

usually at 100–400 volts, and the secondary coil consists of a large solid copper casting, to which the welding clamps are fixed. The E.M.F. in the secondary is 2–3 volts. The **material to be welded** has a great influence on the ease with which welding is effected, and the low conductivity of iron is beneficial in the welding of this metal.* **Iron** requires about 40 k.w. to weld one square inch of metal, **brass** about 25 kw., and **copper** about 100 kw.

“The cost of fuel for power used to effect a weld electrically is equal to the cost of fuel required to weld by hand, the metal being heated in a forge.” Welding machines are now made automatic in their action, namely, the pieces of metal to be welded having been placed in position in the clamps and the current turned on; after a degree of heat is reached sufficient to render the metal plastic, the clamps close up, being under control of a spring acting at a predetermined pressure. The amount which the clamps can travel determines the weld, for, after travelling a certain distance, they engage with a trip which throws out the switch controlling the welding current.

By these means great speed and similarity in welding can be obtained, and an examination of such electric welds in iron or steel under the microscope fails to reveal any want of homogeneity.

ELECTRIC FURNACES.

“An electric furnace is, strictly speaking, an apparatus whereby a physical or chemical change is effected by the transformation of electrical energy into heat.” This definition does not, however, cover those classes of furnaces in which an electrolytic as well as an electro-thermal action takes place. There are three main types of furnace:—

1. **Resistance**, as the **Heroult**, **Keller**, **Harmet**, **Gin**, **Borchers**, **Girod**, **Galbraith**, etc.
2. **Induction**, as the **Ferranti**, **Kjellin** and modifications.
3. **Arc**, as the **Siemens**, **Moissan** and **Stassano**.

Resistance furnaces are based on the production of heat from the passage of an electric current through a conductor of high resistance. In the case of electrometallurgical and electrolytic processes, this conductor is the actual material used for the production of the metal or alloy. For use in annealing muffles or furnaces this conductor is very often thin strip platinum, or even iron or nickel wire, suitably insulated.

In the Kryptol furnace and its various applications, the conductor is a compound of silicon and carbon, called “silixicon,”

* *Proc. Inst. C. E.*, Pt. IV. vol. cii. p. 1; *A. I. M. E.*, vol. xix. p. 877.

and having a high resistance. In the Heroult and Keller types of furnace the conductor is the bath of metal and slag, the carbon electrodes being the means of establishing contact with the molten bath.

In the Girod furnace the graphitic crucible itself is the resistance. The limits of such a principle being short-circuiting of the current from wall to wall at a high temperature, using moderate voltages.

In resistance types of furnace generally, the heat developed by the conductor is dependent on **resistance**, **current** and **time**, and these being capable of variation, the furnace is easily regulated. The limit of **temperature** is governed by the melting, and in some cases, the dissociation of the conductor.

This type of furnace is in more general use than any other, a great advantage being its efficiency; for neglecting radiation, all the energy is usefully employed. A low voltage is favoured, usually 60 volts, and now hardly ever exceeds 120 volts.

No rules can be laid down for even the approximate consumption of power in resistance furnaces, as in every case the size of furnace, material treated, melting point or tapping temperature required, together with any chemical reactions taking place, determine the current necessary. Therefore, only under similar conditions of use can the efficiency of various designs of furnaces be compared. (See Induction type.)

The resistance type of furnace in various forms is in use in France, Italy, Germany, England and America, for the production of iron from complex ores, the conversion of iron and steel scrap into steel, as well as for the general production of ferro-alloys, such as ferro-silicon, ferro-tungsten, ferro-uranium, silico-manganese, metallic manganese and the manufacture of calcium carbide, aluminium, magnesium, carborundum, graphite, phosphorus, etc. Except when power can be acquired at a very low cost, the use of electric furnaces for the competitive production of iron and steel is of no advantage, unless, as is the case with **induction** furnaces, a better class of material is produced. (See Induction type.)

In other directions such as making ferro-alloys, carborundum and calcium carbide, much new ground has been opened out by the ability to produce high temperatures, on which so many of the reactions now taken advantage of depend. It may be stated that resistance furnaces have so increased in size, that currents of 20,000–30,000 amperes at 50–60 volts are being employed,—such furnaces being used for steel manufacture in districts favoured with cheap power from waterfalls.

Induction furnaces.—These are solely based on the principle of a step-down static transformer, of which the primary coil is excited by a high voltage, generally 3000–5000 volts, while the

secondary consists of a single turn, formed by a circular bath of metal. The original form was invented by **Ferranti** in 1885, but only put into commercial use within the last ten years by **Kjellin** in Sweden.

Since the advent of Kjellin's furnace, numerous modifications have been evolved, all being based on the same principle of induction, but aiming at higher efficiency and ease of manipulation.

The **induction furnace** is naturally handicapped in its efficiency by reason of **self-induction**, but its great advantage is the ability to produce **high temperatures** in the absence of **gases**. This advantage is so great that it is now displacing the crucible and other processes for the production of high-grade steel; but it is necessarily limited in its use for treating non-metallic materials. In the original furnace at Gysinge built by Kjellin, and having one annular ring for molten metal on the secondary side of the furnace, the primary coil took 90 amperes at 3000 volts giving 7 volts in the secondary, the efficiency being about 40 %. **Hiorth** by elaborating the Kjellin furnace and using two or more annular rings for metal served by one coil, claimed an efficiency of 78 %, using 3000 volts in the primary coil with 90 amperes. In the same furnace a temperature of 2000° C. was attained. The current generally used is single phase. The efficiency of these furnaces being practically dependent on the efficiency of the transformer principle involved, and by reason of the fact that such furnaces are working under similar conditions, namely, refining molten iron, the direct comparison of the efficiency of various modifications of the induction furnace is made possible.

Such furnaces are now (1911) being made to hold up to thirty tons of steel in the annular ring or trough, and the general use of a tilting arrangement operating the whole of the furnace, to facilitate pouring, is establishing itself. By reason of the temperature obtainable in the absence of gases, many reactions are now possible, so much so that by taking an average grade of iron, the impurities can be eliminated, the finished steel having quite as good, and in some cases better, properties than tool steel made in crucibles in gas or coke-fired furnaces.

Arc furnaces.—These are a type more or less restricted to small experimental furnaces. In some instances furnaces have been designed having carbon electrodes, and the arc has been depended upon as the source of heat, but these cases have very often resolved themselves into a resistance type, and, except in the case of the **Stassano** furnaces in operation in Italy, no large commercial arc furnaces are in use.

The first arc furnace was brought out by **Siemens**, and the

heat from the craters was conveyed to the object to be heated, by reflection or radiation. The efficiency of such an arrangement is manifestly very low, and we therefore find the type restricted to the experimental forms of **Siemens** and **Moissan**. The voltage used is very variable, ranging from 40 to 200. In the case of high voltage arc furnaces, an arrangement of electro-magnets having a stationary or revolving field is sometimes used, in order to deflect or produce a whirling effect on the flame. This arrangement is very useful when work is confined to small operations.

ELECTRIC POWER STATIONS AND MAINS.*

By E. KILBURN SCOTT, M.I.E.E., A.M.Inst.C.E.

The Board of Trade Regulations regarding pressure of supply are as follow :—

- (a) Where the conditions of the supply are such that the pressure at the consumer's terminals does not exceed 250 volts, the supply shall be deemed a **low pressure** supply.
- (b) Where the conditions of the supply are such that the pressure between any two conductors, or between one conductor and earth, may at any time exceed 250 volts but does not exceed 650 volts, the supply shall be deemed a **medium pressure** supply.
- (c) Where the conditions of the supply are such that the pressure may exceed 650 volts but does not exceed 3,000 volts, the supply shall be deemed to be a **high pressure** supply ; and
- (d) Where the conditions of the supply are such that the pressure may exceed 3,000 volts, the supply shall be deemed an **extra high pressure** supply.

Standard Voltages.—It is clearly of great importance that certain voltages should be in general use so that generating machinery, transformers, motors, arc and incandescent lamps, etc., may be standarized and produced cheaply.

Such voltages have been arrived at by experience, and it may be well to note them down.

110 and 220 volts for lighting.

250 for mining installations.

500 for tramways.

650 for direct current railways.

2,200 for single-phase alternating current supply.

6,600, 11,000 and 22,000 for three-phase alternating current bulk-supply ; and

55,000 and 110,000 for extra high pressure transmission lines.

The standard periodicities for alternating currents are 25 and 50 periods per second.

A few cities, Glasgow for example, give a supply at 250 volts, and some employ 240 volts ; Sydney (N.S.W.) being one of them. The majority of places, however, supply at 220 volts.

One advantage of the 250 volts is that it is just half 500, which happens to be the pressure used on tramway trolley wires. Therefore, where a power station gives a supply for both lighting and tramways, the dynamos are slightly more

* In previous editions this section was contributed by Mr. Arthur T. Cooper, M.I.E.E., to whom the Editor wishes to acknowledge his indebtedness for allowing some of the matter to be incorporated in the present edition,

efficient than if wound for $2 \times 220 = 440$ volts. To get the extra voltage from, say, 500 to the 600 required for traction purposes is easier than from 440 to 600 volts.

Carbon Filament Lamp.—For many years the standard voltage for incandescent electric lighting was 110 volts, because makers did not care to supply other lamps. At this pressure power station engineers found it difficult to keep up the voltage of supply on the fringe of the lighting area, because of the drop of voltage in the feeder and distributor cables.

It was seen that by merely doubling the lamp voltage the area of economical supply for a given drop in the feeder cables could be made four times as great, the area varying as the square of the voltage. Lamp makers were therefore encouraged to meet the difficulty, and they produced 200 volts, then 220, and finally 250 volt lamps.

Metal Filament Lamp.—Although the increase of lamp voltage was at that time an advantage, it is of special interest to note that the introduction of the metal filament lamp has been handicapped to some extent by high voltage. Being made of metal instead of carbon, the filament has to be almost as many feet long as the carbon filament was inches. Consequently, it is difficult to find room for, and to support the filament inside the lamp bulb. The metals used are also brittle, especially after they have been in use for a few hundred hours.

At the present time the makers of metal filament lamps prefer to supply them for 110 volts or lower. On direct current 200 to 250 volt circuits the lamps are usually connected two in series, in which case care has to be taken to pair the lamps for the same current. On alternating current circuits auto-transformers are used to give just the right pressure for economical working and cheap lamps.

There is, of course, every probability that, as before, lamp makers will rise to the occasion, and eventually produce a lamp to work as well, and be as cheap, for 220, or even 250 volts, as the carbon filament is now.

At the time of writing (1910) a metal filament lamp costs about three times as much as a carbon filament lamp, and it takes just about one-third of the watts for the same amount of light.

The three-wire system is the usual plan of distributing current for town lighting. Current is delivered at pressures between 400 and 500 volts across two "outer" distributing conductors, an earthed "middle" or "neutral" conductor of small section forming the third wire. Consumers' installations are then, as a rule, coupled between one of the "outers" and the "middle," the connections being balanced as accurately as possible between the two sides. The consumer's voltage, being thus half the pressure across the "outers," comes within the limits of low pressure as defined in (*a*).

This arrangement, invented by the late Dr. Hopkinson, enables the distributing mains to deal with four times the load which could be put on a two-wire system, at the same pressure to consumers and the same percentage loss of voltage. This is done at the small cost of the extra "neutral" conductor.

One great advantage of direct current supply on this system is that it allows of the use of storage at the generating station. In a young undertaking this is most useful, as it enables a continuous night and day supply to be maintained from the outset, and provides an efficient load for the generating plant whenever it is running. From the consumer's standpoint, the fact that direct current arc lamps and motors are in many ways more convenient than those adapted for alternating currents, goes a long way in its favour. Many authorities go so far as to assert that no matter in what form the electrical energy is generated in the first instance, it should be supplied to the consumer as direct current.

Motors across Outers.—In order to improve the balance and prevent unsteadiness of lighting, all motors larger than, say, 3 horse power are wound for the higher voltages and connected direct across the outers. This would be the case with lift motors, for example, which take a heavy current at starting up.

As all such motors are wound for *medium pressure* they must be installed according to the following B.O.T. Regulations.

(1) The frame of every electric motor shall be efficiently connected with earth.

(2) The consumers' wires forming the connection to motors, or otherwise in connection with the supply, shall be, as far as practicable, completely enclosed in strong metal casing efficiently connected with earth, or they shall be fixed at such a distance apart, or in such a manner, that there shall be no danger of any shock.

(3) The supply to every motor shall be controlled by means of an efficient cut-off switch, placed in such a position as to be easily handled by the person in charge of the motor, and connected so that by its means all pressure can be cut off from the motor itself, and from any regulating switch resistance or other device in connection therewith.

(4) Switches, efficient fuses or other automatic cut-outs shall also be provided, so as to protect the circuits from excess of current, and all switches and cut-outs shall be so enclosed and protected that there shall be no danger of any shock being obtained in the ordinary handling thereof, or of any fire being caused by their normal or abnormal action.

(5) A notice shall be fixed in a conspicuous position at every motor and switch-board in connection with the supply, forbidding unauthorized persons to touch the motors or apparatus.

TYPES OF POWER STATIONS.

Some power stations differ from others by reason of the voltage being low tension or high tension or the supply being direct or alternating current. Some, again, differ by reason of the type of prime mover, for example, slow-speed or quick-speed steam engines, gas or oil engines, steam or water-turbines, etc.

Most of the power stations in Great Britain are steam driven, but in countries such as Norway and Switzerland, hydro-electric stations naturally predominate.

It is therefore a little difficult to summarize the various types, and more especially so because one station may contain several kinds of plant. However, the following will serve to draw attention to the leading features of power stations generally met with.

(1) **Direct current stations** giving low tension supply direct to consumers of light and power, and in some cases to tramways. Such stations are frequently owned by municipalities, the area over which they give a supply being bounded by the municipal area. In order to keep up the pressure of supply the stations have to be near the centre of the supply area. Storage batteries are usually prominent adjuncts in such stations. The voltage of supply is 400 to 500, and the three-wire system is employed.

(2) **Single-phase alternating current stations** are useful for giving a supply over a scattered area. Each consumer may have a small transformer, but this is not efficient because the iron losses of the transformer go on, whether a supply is being given or not. It is usual, therefore, to group the consumers together and give a low tension supply from small sub-stations. In this case the distributing cables are about the same as for low tension direct current, but of course there is a saving of copper in the feeders, which generally work at 2,200 volts.

(3) **Two-phase alternating current stations** are frequently single-phase stations that have been changed over, but there are cases where two-phase plant has been laid down from the outset. The original power station at Niagara is an example. When changing over from single phase to two phase the alternators only require an extra set of armature coils and readjustment of field coils. The mains have also to be duplicated. With such a two-phase supply, induction motors can be used, and they are more satisfactory in working and cheaper than single-phase motors.

(4) **Three-phase stations** may supply three-phase alternating current only, to consumers, but more often they give both an alternating and a direct current supply, the latter being generally by means of sub-stations equipped with rotary converters or motor generators.

This is the system employed by large municipalities such as Manchester, Glasgow, Sydney (N.S.W.), etc. The London County Council station at Greenwich, which supplies current to the tramways, is laid out on these lines, and so also are the power stations supplying electric railways, such as the Central London Tube, the London Underground, and the Liverpool to Southport branch of the Lancashire and Yorkshire Railway.

(5) **Bulk supply power stations** transmitting extra high tension three-phase current over whole or parts of counties. For example, the Yorkshire Power Co., the Lancashire Power Co., the Cleveland and Durham Power Co. These are always owned by companies and frequently sell current in bulk to municipalities for the latter to transform in sub-stations and retail to light and power consumers.

This division might also include the several large power stations near London, which exist for the special purpose of supplying power in bulk to companies like the St. James and Pall Mall, the Westminster Electric Supply, etc. There is also the case of the Durham Collieries Power Co.'s station, started to supply collieries with cheap power. This is now merged in the Newcastle Electric Supply Co.

In such power stations everything is on a very large scale, and the generating units are usually steam turbo alternators of several thousands of horse power each. The station is built on the unit system, each turbo alternator having its own bank of boilers. The alternating current goes to the mains through oil switches, these being in brick cells and actuated by a remote control gear. The pressure may be 6,600, 11,000, or 22,000 volts.

Stations using Steam raised from Refuse as well as Coal.—Combined refuse destructors and electric power stations are always controlled by municipalities, the principal idea in view being to make use of the waste heat from destructors. The destructor house and the electric station may be combined in one building, but more often they are separate, steam being conveyed from the boilers in the destructor house, by a long, well-lagged pipe. There are excellent examples of combined destructor and electric light stations at Shoreditch, Fulham, Maidstone, Wrexham, Greenock, etc. For many years there was doubt as to the advantages of the combination, but the bulk of experience seems to be favourable, if the combination is skilfully designed and managed.

Stations equipped with Internal Combustion Engines.—The engines may utilize gas from blast furnaces or coke ovens, and they are frequently found in iron and steel works, chemical works, etc. There is, however, an increasing number which use gas producers and supply light and power to towns.

This heading may also include the stations equipped with large oil engines, such, for example, as the Diesel engine. There is likely to be a considerable development of oil engine plants in the future.

Hydro-electric Power Stations.—There are not many large water powers in the British Isles, but in other parts of the Empire, particularly in Canada, India, South Africa and New Zealand, there are very large powers. Of those using high head-water powers running to hundreds of feet, the stations of the North Wales Power Co. at Snowdon, and those of the British Aluminium Co. in Scotland, are in some respects typical. The North Wales Power Co. uses 1,000 feet head.

Water-power plants are usually owned by companies, but there are a few municipally-owned water-power stations, *e. g.* Dunedin (New Zealand), and Launceston (Tasmania). Low head-water powers having heads of, say, 10 to 50 feet, require a very large volume of water, and are costly in hydraulic works. The Worcester power station has turbines working on a low head.

High Tension Direct Current Stations.—The generation of direct current at high tension, and its distribution by means of sub-stations, converting it to low pressure, is sometimes called the Oxford system, because it was first tried in that city. With modern methods of design and commutation, the manufacture of high tension dynamos is comparatively easy. The advantage of being able to use a storage battery without change of current from alternating to direct current is favourable to the direct current system.

It is well to bear in mind that the difficulties of commutators, for high speed steam turbine sets, may at any time be solved by the introduction of a really efficient homopolar dynamo.

M. Thury has equipped power stations in France and Switzerland which generate direct current at extra high pressure, in some cases at 55,000 volts. This is done by connecting several dynamos in series.

SITE, BUILDINGS AND CHIMNEY.

The correct choice of site for a power station is a very important matter, for many undertakings have been seriously handicapped by being placed away from a good water supply, or where fuel could not be delivered conveniently.

A waterside site is of course the best situation for a coal-fired station, and especially so if steam turbines are used as the prime movers. To be efficient, steam turbines require a high vacuum, and this in turn necessitates plenty of cooling water.

Stations having internal combustion engines are not so

dependent on water supply, but, other things being about equal, it is well to be near to it, as a fair amount of make-up water is required. A much cheaper site, with water and other convenience, usually obtainable some distance away from the centre of supply, is often preferred.

As a rule, however, low tension stations for light and power, and stations supplying direct current at 600 volts for tramways, should be near the centre of supply.

Refuse destructor stations are of course placed where the refuse can be promptly dealt with without nuisance. This being the deciding factor, it often happens that the electrical plant is not so favourably situated as it otherwise might be.

The site of a station using blast-furnace or coke-oven gas is naturally near the gas scrubbers, so as to shorten the piping as much as possible. Gas producers are liable to give off poisonous gases, so it is necessary to install them in the open. Sometimes they have a sulphate of ammonia recovery plant.

Hydro-electric stations are of course situated at or near the bottom of the fall, and conveniently placed for the pipe line and tail race. In difficult mountainous country it frequently happens that a place less favourable for efficiency is chosen, in order that the machinery can be more easily moved into position.

Foundations.—As low tension stations have to be in populous districts it is necessary to be careful about the foundations, so as not to cause a nuisance from vibration. As a rule, it only means going low enough with the concrete blocks, and arranging for each block to be quite independent of the walls of the building.

Banks of rivers and canals are frequently “made ground,” and consequently more excavation is required near the bank than at some distance back. Other things being equal, it may therefore be best to place the boilers near to the water, as they require lighter foundations. The most solid ground on the site should be reserved for the chimney.

In laying the foundations the usual plan is to excavate the full area of the building and lay concrete all over it a foot thick. On this the walls of the building and the concrete blocks for the engines are built.

Steam turbines require much smaller foundations than reciprocating engines, whilst the internal combustion engine requires heavier foundations than the steam engine.

The question of cost of foundations is often lost sight of when comparing tenders for plants of different types and by different makers.

Lay-out of Buildings.—In a steam station the building consists of two shells, one for the boiler house and the other for

the engines and dynamos and machinery. The roof of the boiler house is often occupied by a large coal store fitted with bins, into which the coal is fed by a bucket conveyor.

The roof of the generating room has usually a roof light and louvres, and care should be taken to make them weather tight so as to prevent water falling on to the machinery below. The height of the roof must permit an overhead traveller to pass over the largest generating unit likely to be installed.

The storage battery is usually in a separate room on the ground floor and near to the dynamos and switch-board, so as to economize connecting cables.

Stations equipped with internal combustion engines do not require a boiler house, but when producer gas is used, a roof has to be provided over the producers. Corrugated iron on steel girders is usually considered sufficient.

In hydro-electric stations there is only one very simple building to cover the turbines, generator, and switch-board. Such stations are often in very damp situations, where in stormy weather the building may be drenched in spray from the fall. Provision must be accordingly made to keep the water away from the electric machinery.

Chimney.—Chimneys are only required for steam stations and combined refuse and electricity works. Formerly, the usual plan was to build the chimney very solidly and large enough for all the extensions foreseen at the time. With the advance of the internal combustion engine, however, there is a feeling that long before a well-built brick chimney is worn to any extent it may become antiquated.

Self-supporting steel chimneys are therefore becoming common, and when made taper and kept well painted they have a good appearance. Their advantages may be summarized as follows:—

- (a) The cost is 25 to 50 % lower than a brick chimney.
- (b) The time for erection is only about as many weeks as a brick chimney takes months.
- (c) The efficiency is higher because there is no infiltration of air as there is through bricks.
- (d) A steel ladder forms part of the chimney, so that it is easy at any time to get to any part of it.

Of course, all chimneys require a good firebrick lining inside, because ordinary red bricks will only stand about 600° F. for short periods. In the case of a steel chimney it is necessary to keep the hot gases from the metal.

If the lining is built so that the whole weight comes on the bottom layers, then the great pressure is liable to destroy the bricks, and in case any renewals are required care has to be taken not to disturb the over-structure.

For this reason it is now usual to build the firebrick lining in sections, which are supported by corbels projecting inwards.

*BOILERS—ECONOMIZER—PUMPS—PIPING—
DESTRUCTORS.*

Boilers.—Three general types of boiler are used in this country, viz. Lancashire, Water-tube and Marine. Opinions as to their relative merits vary greatly, but the following tabular statement is a fair comparison:—

Type of Boiler.	Advantages.	Disadvantages.
Lancashire.	Low first cost. Large steam reserve. Accessibility. Low maintenance. Long life. Good evaporative efficiency when not pressed.	Large floor space. Slow steam raising. Want of circulation.
Water-tube.	Small floor space. Quick steam-raising.	Small steam reserve. High maintenance. Good water necessary. Possibility of wet steam when pressed. Tendency to cause smoke.
Marine.	Small floor space. Neatness. Good evaporative efficiency under all loads. Fairly quick steam-raising.	Inaccessibility. Fairly high maintenance. High first cost.

The Marine forms a medium type between the Lancashire and the Water-tube as regards rapidity of steaming and floor space required, and for this reason has been adopted in a good many instances. A combination of Water-tubes and Lancashires has also come into favour of late, the Lancashire boilers taking the steady long-hour load, and the Water-tubes being pressed up for the peaks.

Water-tube Boilers.—The Babcock & Wilcox and the Stirling types are best known, but the Climax and a boiler made like the Stirling, but with straight tubes, by Clarke, Chapman & Co., are also used. The ideal arrangement is to so arrange the tubes that they can be seen through from end to end, for of course their regular cleaning out, is a necessity.

The Lancashire.—One disadvantage of this type is that it requires very careful setting in brickwork. It is also long as

compared with its height, and a battery of such boilers therefore covers more area than Water-tubes.

Where carriage is a consideration, as when sending abroad, the Lancashire and Marine types are distinctly at a disadvantage compared with a Water-tube boiler, which easily takes to pieces.

It is now becoming common practice to arrange the flues and the chimney along the **tops of the boilers**, or at any rate at some height from the floor. They are in such cases made of steel plates lined with firebrick, and this has the advantage of giving a better draught than brickwork alone, because air can leak through the latter.

For economy of fuel and preventing smoke, it is now being more generally recognized that a large combustion chamber is a desirable feature of a boiler.

The **class of coal** used varies in almost every station, and is generally arrived at after careful tests and mixing of two or more qualities. The advantage of two kinds when hand stoking is that when starting up or during rush hours the better quality can be temporarily fed in, in larger quantity.

Mechanical Stokers.—As a broad principle it may be taken for granted that the dearer the fuel the greater the advantage of hand firing, and the cheaper the fuel the more advisable the use of automatic stokers. When combined with conveyors, a mechanical system of firing is most efficient for dealing with unscreened nuts or smudge. Usually these cannot be hand fired to advantage without much smoke and loss of economy. With screened steam coal of good quality, however, a good human stoker cannot be excelled, in spite of the extra cost of the labour required.

Mechanical stokers divide themselves into two types—coking and sprinkling. The former deposits all the coal at the front end of the grate, and gradually pushes it on towards the bridge, while the latter deposits it evenly over the whole of the grate area. The coking stoker is the better of the two where smokelessness is a necessity, while the sprinkling type gives the higher economy and rate of evaporation.

Draught.—Besides the height of the chimney there are two ways of increasing the draught through a boiler firegrate.

One is to *force* the air into a closed ashpit by means of steam jets, the other is to *induce* the air by means of a suction fan fixed in the main flue, preferably at the base of the chimney.

The increase in the quantity of fuel burnt per unit of grate area is most marked, and in spite of a slightly lessened economy, the ability to force the boilers over the peaks of the load is worth a great deal. It has been urged that the use of forced or induced draught lessens the creation of smoke, but this seems somewhat doubtful unless provision be made for heating the air

going to the furnaces. Otherwise the extra amount of cold air passing through is apt to cool the furnace gases down below flaming point.

The difference between forced and induced draught has been very aptly compared with the difference between the effects of *pushing* and *pulling* a rope through a crooked pipe. The smoother and more efficient system is at once evident from this analogy.

Economizer.—There are several fuel economizers on the market, but the original one by Green is the best known. It simply consists of a number of vertical cast-iron tubes, each about 4 in. diameter and 5 ft. long, through which the feed water circulates.

The hot gases from the boiler pass round the tubes and give up some of their heat to the water. In doing so a certain amount of soot is deposited on the tubes, and this is removed at regular intervals by automatic scrapers.

The baffling of the hot gases and removal of some of their heat reduces the draught somewhat for a given height of chimney, but the heat recovered is valuable and the net result is a saving of about 15 % of the coal bill.

The economizer needs very little attention, as there are no moving parts except the scraper. The economizer can be built into the flue system out of the way.

The only point requiring care is the necessity of initially taking the "chill" off the feed water in order to prevent sweating and pitting of the outside of the tubes. With surface condensing plant this warm water is already provided, but otherwise it can be done by "short-circuiting" the whole economizer through a small pipe and valve, the effect being to allow a small portion of the heated water to back-travel to the pump side, and to impart the desired heat to the feed.

Feed Pumps.—The ordinary type of Worthington feed pumps, which for many years were the standard pattern in power stations, are extremely wasteful. Their place has therefore been taken by electrically driven pumps or high-class slow running steam pumps.

With electrically driven pumps the power required to supply feed water for a power station of average size is generally about 1 % of the total output. With good steam pumps it may be several per cent., whereas with the old type of Worthington it was anything up to 15 %.

Electrically driven feed pumps, although somewhat higher in first cost, have everything else to recommend their adoption. They work best with direct current from the bus bars of a direct current system, or from the exciter circuit of an alternating station and should be fitted with double wound

armature. A series parallel controller should be used so that the speed may be varied through wide limits without materially affecting the electrical efficiency.

Oil Separators.—Condensed water makes an excellent boiler feed because it is free from the chalky and other similar substances which form scales. As it leaves the air pump, however, it is charged with cylinder oil in a fine state of subdivision, and this cannot be entirely removed by mechanical filtration without previous treatment.

This takes the form of coagulating the oil particles until they become sufficiently large to be arrested by a mechanical filter.

The electrical method of bringing this about is known as the Davis-Perrett, and it consists in putting a direct electric current through the oily water as it passes between two iron plates. The energy required is about $1\frac{1}{2}$ units per 1000 gallons treated, and the oil is separated off in a coagulated state which can easily be arrested by a sand, sawdust, or bath-towel filter.

The chemical treatment consists in adding a quantity of alumina ferric and soda to the oily water, an ordinary water-softener plant being easily adapted to add the proper proportions automatically. These chemicals have an affinity for oil and coagulate it ready for mechanical filtration. Perhaps the most successful plant in connection with the chemical treatment is that known as the Harris-Anderson. The only drawback to this method is the possibility of *over treatment* and the liability of the excess chemicals finding their way back into the boilers, and after continued circuit of the water accumulating in inconvenient and harmful quantities.

Pipework.—When planning the steam piping, care should be taken to provide for sectioning, so that in case of accident, the faulty pipes, the boiler, or the generating unit, can be immediately shut off.

There should be no water pockets, whilst efficient drainage and ample allowance for expansion and contraction should be allowed.

Efficient lagging should be provided, as every square foot of steam-pipe surface left bare is responsible for the consumption of something like half-a-ton of coal per annum. Of recent years special box fittings have been placed on the market which enable flanges and valves to be covered in a neat and permanent fashion. As regards the covering material itself, each of the different varieties has its advantages, and the only remark which need be made here is that the use of silicate cotton should be avoided in all situations where there is the least likelihood of its glass-like fibres finding their way into the

pipes when the joint is broken. Their effect on valve and cylinder faces is disastrous.

Ring Main.—A few years ago central stations were provided with a main steam range in the form of a ring, Fig. 166, with double connections to each boiler and generating plant. This

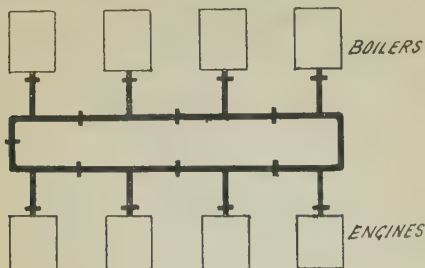


FIG. 166.

arrangement has been found to be quite unnecessary, and being expensive both in first cost and maintenance has fallen out of favour. The usual plan adopted in recent practice is to provide each steam set and each boiler with a single connection pipe leading into a main header (Fig. 167), running the whole length of the building. This main has a section

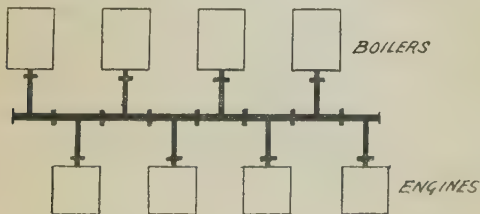


FIG. 167.

valve between each pair of adjacent connection pipes. Any joint or length of pipe can then be isolated without much inconvenience.

Material used in the construction of central station piping. Steam pipes are of lap-welded steel with flanges either riveted or welded on. The branches are provided for, either by pieces

riveted on, or by the insertion of special pipes of cast steel. Bends are copper.

Feed pipes are usually of cast iron with expansion lengths of copper. Fig. 168 shows a typical feed pipe layout.

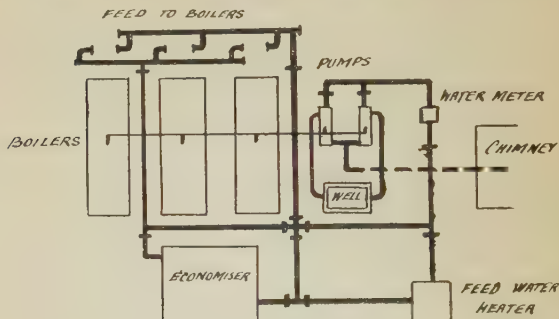


FIG. 168.

Valves.—The very best valves can be rendered almost useless by lack of attention. In installing new pipework care should be taken to free the pipes of all grit and sand, or the valve faces may be badly scored the very first time they are used, and then leak freely. Also every valve should be opened and shut at least once a week when not in regular use, in order to keep the surfaces clean and free from corrosion. In the larger sizes of steam valves the addition of a small cylinder oil lubricator is a great advantage.

Of recent years the most striking improvement has been the Ferranti-Hopkinson valve, which by an ingenious device of first throttling and then expanding the steam enables the valve to go into much smaller space than formerly.

Superheaters.—In order to improve the slow speed compound engine, steam having high superheat has been used. By high is meant a superheat of 350° F., and to give this a fuel fired heater is required. For most purposes, however, a superheater of 100° F. is sufficient, and this is best obtained by a nest of extra tubes forming part of the boiler.

This superheat of 100° F. is sufficient to prevent steam-pipe and cylinder condensation, and at the same time it does not seriously affect the lubricating value of the cylinder oil. The gain is about 10 %.

Stations equipped with special engines capable of working

with steam having a high superheat give great economy of steam, as low as 9 lb. per 1 h.p. hour having been obtained. This, however, is to some extent offset by the special and expensive cylinder oil that is required and the special care that has to be taken with the metal of the engine valves.

It should be noted that only moderate superheat, if any, is used with turbines, because of the detrimental effect of the high temperature on the metal blades, etc.

Combined Refuse Destructor and Electricity Works.—A good many electricity works in the United Kingdom and the colonies are worked in conjunction with refuse destructors, that at Shoreditch, London, being one of the largest and best known.

Quality of refuse varies in different towns and different localities in a town. Those places where there is a large working class population and factories generally give good refuse. The best kinds of refuse are cinders from houses and wood refuse from factories, there being more of the former in the winter months than in summer, whereas refuse from factories is generally about the same average quality all the year round.

Many exaggerated statements have been made regarding the value of refuse destructors when combined with electricity works, and this has prejudiced the combined system. At the same time, the success of well-managed installations has amply demonstrated that it is often worth while to combine the two.

Modern Destructors.—As regards the design of the destructor itself, there are several well-known makes by Horsfall, Heenan, Meldrum, etc. In one of the latest the wet garbage is stored by itself as it is delivered, away from ashes and rubbish, so that the free water can drain away to the sewer. As in winter this may amount to nearly 10%, it means considerable saving in heat units.

The refuse is shot on to a substantial brick hearth which receives the impact, and there it is dried for some time before it is drawn forward on to the firegrate and burnt.

A modern improvement is provision for the air admitted to the firegrate to be previously heated by the hot gases on their way to the chimney.

One very important feature in the design of a refuse destructor plant is that the amount of air required for combustion should be several times greater than the cubic contents of the building. This ensures the renewal of the air in the building every few minutes, and prevents the escape of any gases except through the chimney.

STEAM ENGINES—STEAM TURBINES—CONDENSERS.

Slow Speed Steam Engines.—Many of the early power stations were equipped with slow speed engines, but the quick speed is the usual type. The starting of public electric supply synchronized with the introduction of the earliest successful quick speed engine, namely, that of Willans.

Slow speed engines may be considered as being those running at speeds below 125 revolutions per minute, although most of them run at less than 100, because that is the maximum speed at which a Corliss gear will work properly.

The slow running engine has a long life, and will take overloads very well; it is a suitable type for traction stations where the load varies considerably.

The highest steam economy attained by any steam engine has been in engines of the textile mill type, and these are met with in some of the large electric power stations.

The dynamo machine is frequently built on the crank shaft between the cranks of the high and low pressure cylinders, and where the power is very great, as at the Greenwich station of the London County Council, there are four cylinders, two being vertical and two horizontal. Arranged in this way there is more even turning and a shorter crank shaft. The slow speed of course incurs heavy and expensive generators.

Although the steam consumption on such engines is very low, it may be said that the slow speed steam engine has had its day so far as electric power stations are concerned.

Quick Speed Steam Engines.—Any speed over 250 revolutions per minute may be considered as quick speed. For a given output this type of engine goes into a fraction of the space occupied by a slow speed engine. Therefore if the cost of the two engines is the same there is great saving in first cost of buildings, foundations, and pipework. There is also a saving in running cost for oil, attendance, and steam-pipe condensation. Contrary to what might be expected, the wear and tear of a quick speed engine is not any more, if as much as that of a slow speed engine. This is due to the lesser weight of reciprocating parts, the ample proportions of the bearing surface, and also to the fact that the piston speeds are about the same. This is due to the short stroke of the engines.

The steam consumption is higher than with slow speed engines when each is working at full load, but on three-quarter load and less, the difference is not so striking. As a matter of fact, half to three-quarter load is the rule rather than the exception in electric power stations. With a mill engine, a full steady load all day is the rule.

When it comes to the dynamo machines for coupling to the

engines, the quick speed engine has an immense advantage, as the high speed means a smaller dynamo or alternator. For alternators the quick speed engine is most suitable, as it can be cheaply made with three cranks at 120 degrees apart. This gives very even turning, which is desirable for parallel running, and there is also less vibration.

To arrange three cranks on a slow-running engine would be much more expensive, so the usual plan is to provide a heavier fly-wheel. This, however, does not altogether solve the difficulty. So the alternators are fitted with damping coils.

Cost of Running Engine.—A 500 kw. quick speed engine and dynamo, using steam at 170 lb. pressure at a temperature of 550° F., will consume $16\frac{1}{2}$ lb. of steam per kw. hour if the vacuum is kept at 26 inches.

Now good Welsh coal, costing 21s. a ton, gives an evaporation of $9\frac{1}{2}$ lb. of water per lb. of coal, so the cost per kw. hour works out at—

$$\begin{aligned} 252 \text{ pence} &= (2,240 \times 9\frac{1}{2}) \text{ lb. of steam.} \\ \frac{(2,240 \times 9\frac{1}{2}) \text{ lb.}}{16\frac{1}{2} \text{ lb.}} &= 1,290 \text{ kw. hours,} \\ \frac{252}{1,290} &= \cdot 19\% \text{ per kw. hour.} \end{aligned}$$

Steam Turbines.—The prime mover which is finding most favour at the present time, at any rate for the large output power stations, is the "Steam Turbine."

There is very little difference in the fuel consumption of a steam turbine plant as compared with a quick speed steam engine equipment. The advantages in favour of the former are, however—

Simplicity.

Small space occupied.

Freedom from vibrations.

Less "warming up" required before starting.

No internal lubrication required.

The last-mentioned point in favour is perhaps appreciated less than it should be. It is a great advantage to be able to return the condensed steam direct to the boilers without any special treatment to free it from oil.

In connection with the small space occupied, the difference between a horizontal spindle turbo generator with its condensing plant and a slow speed engine and generator with its condenser is very striking, but the difference is still more marked when the turbine runs on a vertical spindle, as is the case with the Curtis type.

The Hon. C. A. Parsons is the best-known name connected with the steam turbine; indeed its position to-day is mainly

due to him. Since he has made it a success, however, others have come forward with improvements. For example, De Laval, Prof. Rateau, C. E. L. Brown and Zoelly on the Continent, and Curtis in the States. Several English makers have improved on the original Parsons turbine in detailed construction, the firm of Willans and Robinson being one of the most prominent in this respect.

One point to notice about the De Laval turbine is that its normal speed is exceedingly high, say 20,000 revolutions per minute, and its application as a turbine by itself has been handicapped on that account. Parsons, on the other hand, was able to reduce his speeds from 9,000 revolutions in the earlier designs to 3,000 revolutions or less.

The De Laval turbine patent covers the idea of an expanding nozzle which gives good steam economy at the high pressures. The Parsons, on the other hand, is very suitable for low pressures. Many modern turbines combine the two.

Cost of Running Steam Turbine.—Most steam turbine makers will give the following guarantee. A 500 kw. steam turbo dynamo, using steam at 150 lb. per sq. inch boiler pressure and a temperature of 500° F., will consume 18·6 lb. of steam per kw. hour.

If good Welsh coal is assumed to cost 21s. a ton, and the boiler is such that it will evaporate 9½ lb. of water per lb. of coal, then the cost works out as follows :—

$$\begin{array}{r} 252 \text{ pence} = (2,240 \times 9\cdot5) \text{ lb. of steam,} \\ (2,240 \times 9\cdot5) \text{ lb.} \\ \hline 18\cdot6 \qquad \qquad \qquad = 1,142, \\ \frac{252}{1142} = \cdot22d. \text{ per kw. hour.} \end{array}$$

Exhaust Steam Turbines.—It is a scientific fact that there is as much energy in steam between a pressure a few lb. above atmospheric pressure down to a good vacuum of, say, 28 inches, as there is between a few lb. above atmosphere to the full boiler pressure of, say, 160 lb. per square inch.

Now the ordinary condenser used with reciprocating engines adds considerably to the power, but it does not double it, and consequently it is becoming the practice to install an exhaust steam turbine as an auxiliary to reciprocating engines.

Another interesting development due to Prof. Rateau is that exhaust steam turbines can be worked with the exhaust from intermittently loaded engines like the winding engine of a colliery or the rolling-mill engine of a steel works.

The steam comes away from such engines in gusts, and it is equalized by passing it into a reservoir (generally an old

Lancashire boiler), partly filled with water and pieces of iron and steel.

When the fresh steam comes in contact with the water and iron it parts with its heat. At the same time the exhaust steam turbine takes a regular supply from the reservoir, and so keeps on reducing the heat all the time.

One case where this is done is at the Hucknall Torkard Colliery, where practically all the electric motors are supplied with current from an exhaust steam turbo alternator, the exhaust in this case coming from the winding engine. It is also used in several ironworks, including Sir Bernhard Samuelson's at Middlesbrough. Here the exhaust from the rolling-mill engines drives large turbo alternators, which give a supply to the power mains of the Cleveland and Durham Power Co.

It is becoming quite a regular practice in the north of England for steelworks, collieries, etc., to sell their exhaust steam from engines, and their furnace and coke-oven gases to a power company for general bulk supply in the district.

Condensers.—Twenty years ago, when Parsons first began working on steam turbines, he clearly foresaw that to get maximum economy it was essential that they should work condensing. He also saw that such condensing as was then considered good enough for the ordinary reciprocating engine was not sufficient for a turbine.

Consequently, as the turbine's design has improved, so also has that of condensers. The contra-flow condenser is one of the results; in it the hot steam and the cooling water flow in opposite directions, the hottest steam meeting the warmest water. A given amount of cooling surface is thus more effective than the older types.

Evaporative condensers are now much used instead of the separate arrangement of condenser and cooling tower. The Ledward type, as installed at Southend and Bridlington, consists of a number of cast iron corrugated tubes, laid horizontally, and built up into sets something like economizer tubes. Underneath is a large brick tank, from which motor-driven centrifugal pumps raise water to distributing perforated pipes over the top of the tubes. The lift is about 14 ft., whereas the average lift in a cooling tower is 30 ft., so less power is required for pumping.

Steam enters the cast iron tubes at the top, and as it is condensed it is trapped off at intervals from the pipes below. The hot water goes straight to the boilers through a rotary wet pump, and so a great deal of heat is saved, and the water has no chance to become aerated.

INTERNAL COMBUSTION ENGINES— GAS PRODUCERS.

Gas Engines.—For many years gas engines using towns gas have been employed for driving dynamos. When, however, it comes to using gas engines for large power stations, illuminating gas is much too expensive. Producers have therefore to be installed unless waste gas is available.

Large gas engines of several thousand horse power are working quite successfully with the waste gases from blast furnaces, coke ovens, etc.

The cycle most generally used is the Otto or four-cycle, as used in the smaller engines. Large fly-wheels are therefore necessary to give even turning, especially when the engines are used for driving alternators.

The two-cycle modification, due to Korting, is very successful. An examination of a gas engine indicator diagram shows that at the toe of the diagram very little work is being done. In the Korting engine the gas and air are compressed separately, and the last fifth of the forward stroke and first fifth of the backward stroke are utilized to clear out all the burnt gases and introduce a fresh charge. In this way two whole strokes are saved, and the engine works on a two-cycle.

Another type of engine, called the Oechelhauser, has two pistons working in opposite directions in the one cylinder, also two crank shafts. On firing the charge, the two pistons travel outwards at an equal rate, and the power of one is made to assist the other by having connecting rods between the crank shafts. Naturally this type takes up a great deal of room.

It happens that nearly all the successful engines of large power run at very slow speeds, and they are not only costly themselves, but, because of the slow speed, the dynamo machines are costly also.

Early in the history of the gas engine it was foreseen that the evolution of the design would most probably follow that of steam engines. Therefore several makers, including the British Westinghouse Co., designed two- and three-crank, quick speed, vertical gas engines, outwardly similar to the steam engines of Willans, Belliss, etc.

Quick speed vertical gas engines have been fairly successful, and some day they will, no doubt, be the standard type of reciprocating gas engine. Possibly, however, the cycle will be completed by the gas turbine making its appearance.

A good many designers are working on this already, and the difficulties are, apparently, the large amount of power required to compress the gas, and finding metals that will withstand the high temperature. The noise is also troublesome. The most

successful design so far, utilizes the exhaust heat to raise steam in a flash boiler, which steam then passes into the turbine and helps the gas with the rotation, whilst reducing the temperature.

Gas Producers.—Ordinary town illuminating gas is not only costly but it is also too explosive by containing too much hydrogen. It does not contain carbon monoxide gas, which is a principal constituent of producer gas.

There are several kinds of gas producer ; for example—

Pressure Gas Producer.

Suction " "

Water " "

Mond " "

The Pressure gas producer differs from the Suction gas type in that it supplies gas to a gasometer, and all the pipes, and joints, etc., are under pressure.

A Suction producer, on the other hand, simply makes the gas as it is required by the engine, and the pressure in the pipes is therefore, if anything, below atmospheric pressure.

As producer gas is charged with the non-odourless and very poisonous carbon monoxide gas, it will be easily seen that the Suction gas producer is safer for the attendants, because if there should be a leaky joint the air goes in, instead of the poisonous gas coming out. The fact that it will work without an expensive and somewhat dangerous gasometer makes it a favourite type for moderate powers.

Water gas producers are those in which steam is forced through a hot fire and split up into its gases, the oxygen combined with the carbon to form carbon monoxide, and some carbon dioxide and the hydrogen partly coming off free, and partly as carbon hydrogen gases. The flame given by water gas is intensely hot, and it is much used for welding steel tubes, etc.

It is necessary to mention this type, not because it is used for driving gas engines, but because in many producers a certain amount of steam is used, and so the gas has something of the water gas character about it.

The Mond gas producer was developed primarily to make sulphate of ammonia and coal tar products, and the gas is more or less the by-product.

There is a great future before this type for large power work, because the sulphate of ammonia is valuable, and at the same time the gas given off is of that slow burning character that best suits large gas engines.

Quality of Gas.—It will be seen from the above that the large internal combustion engine problem is bound up in the manufacture of and the quality of the gas.

With a steam engine there is no variation in the steam ; it is simply oxygen and hydrogen in a combined state, and the only difference between the steam from one boiler and another is that possibly one may have a little water in suspension.

With gas engines, however, the problem is entirely different, as the product of a producer is a very complicated mixture of gases. It varies from time to time according to heat of fire, quality of fuel, the care given to plant, etc.

This was partly the cause of the failure of the large Johannesburg gas power station. The engines were practically the same as those used successfully for blast furnace gas, but the producers were faulty, and they made a poor gas. The reduced barometric pressure at Johannesburg was also not allowed for, when designing the engines.

Cost of running Gas Engines.—As an example of the cost of running gas engines, the following figures may be useful :—

A 500 kw. gas engine and dynamo consumes about 80 cubic feet of gas per kilowatt hour, the gas yielding 175 British thermal units per cubic foot.

Now good stock coal costing, say, 11s. 6d. a ton, should produce 120,000 cubic feet of gas or

$$\frac{120000}{80} = 1,500 \text{ kw. hours.}$$

If the calculation is completed, this gives a fuel cost of

$$\frac{138}{1500} = .092d. \text{ per kw. hour.}$$

Of course, when the gas engine is not working, the stand-by losses are very low as compared with those of a steam plant.

Oil Engines.—Many engineers think that the internal combustion engine, using oil as fuel, is going to run the gas engine very close in the future.

It has the very great advantage of not requiring a producer, and the oil fuel can be easily stored. The objection at present is that refined oil is expensive, and engineers are therefore turning their attention to designs which will enable the engines to use crude oil.

One of the most successful designs, so far, is that of Diesel. By compressing the air to 500 lb. and forcing the oil in at 750 lb. per sq. inch, there is perfect combustion of very crude oil.

The engine has other points which differentiate it from ordinary oil engines, in the valves and governing gear, and in being made of the quick speed vertical type.

It is expensive because of cost of manufacture and the large royalties, but both these are tending to come down and, when they do, the oil engine using the Diesel cycle and burning crude oil will be a keen competitor of the gas engine.

DYNAMOS—ALTERNATORS—SWITCHBOARDS.

Dynamos.—Modern dynamos differ from the older types in being fitted with interpoles. These, whilst adding small extra field coils, enable a given dynamo to sparklessly commute much larger currents than would otherwise be the case. Interpoles have therefore considerably increased the output of dynamos and the power of motors.

Another change in design is that greater advantage is taken of the magnetic material. It has been found advantageous to work at very high values of magnetic lines per sq. inch, and then to get rid of the extra heat caused by hysteresis and eddy currents, by means of efficient ventilation.

Small points in design have also been elaborated, such as the winding of series turns on every alternate pole. Also, instead of binding wires to hold the armature coils in position, this is now effected by wedges driven into the top of the slots, and by bell-shaped castings over the ends of the coils where they project beyond the armature.

Armature reaction has been reduced by so shaping the pole pieces as to greatly increase the reluctance of magnetic circuit of the armature coils, whilst at the same time not interfering with the path of the main field magnetic circuit.

With steam turbo dynamos the output of the dynamo is limited. One firm claims to be able to make 1,000 kw. turbo dynamos as a single unit, but usually two dynamos are coupled to the one turbine for such an output. This is principally owing to the difficulty in making a good mechanical job of the commutator. Mica has necessarily to be used in the construction, and there are so many small parts that, running at high speeds, there is the greatest difficulty in getting it to remain true, also to keep the mica below the bars.

Most of the difficulties would disappear with the advent of a really satisfactory homopolar dynamo. Hitherto such machines have been of low efficiency, and the brushes, in particular, have not been satisfactory. The demand is, however, so great that there may be a solution forthcoming. It should be mentioned that every increase in size and increase in speed is in favour of the homopolar type.

Alternators.—Ordinary alternators for moderate and slow speeds always have the armature stationary and the poles revolving, because it is so much easier to connect the high voltage direct to the armature. The field coils have two slip rings. Some makers of steam turbo alternators, however, prefer to make the armature revolve and collect high tension on the three slip rings for three-phase, or four for two-phase.

Regarding the number of phases, the three-phase type is the

standard now-a-days, because it enables the fullest use to be made of both magnetic and electrical material. To build a single-phase alternator is almost equivalent to having a three-crank engine and only using one cylinder and one crank. Even when single-phase is required, as is the case with some electric railways, current is first generated as three-phase.

The voltage at which it is safe to generate in the alternator is limited by the difficulty of insulating the parts and getting coils of opposite potential sufficiently far away from each other.

It has been found that on voltages of 11,000 or so there is a static discharge between the alternator windings. This in time is liable to pierce the insulation, but the worst trouble is that nitrous oxide gas is formed, and this with moisture in the air forms nitric acid, which attacks the metal and insulation.

It is probable, therefore, that 11,000 volts will be considered the limit at which it is safe to generate in the alternator itself; at any rate, in places where there is much moisture in the air, as in coast towns, hydro-electric stations, etc.

The latest developments have been in the direction of enabling the alternator to raise its voltage as the load comes on, just as a compound wound dynamo does. Some alternators are also made partly or wholly self-exciting by rectifying part of the alternating current.

In the case of steam turbo alternators the difficulties have been principally in the direction of obtaining sufficient mechanical strength in the rotating part. Also when the armature is stationary in giving rigidity to the end parts of the armature coils, where they project beyond the core, so that sudden short currents shall not move the coils.

The speed is a serious difficulty, because steam turbo alternators have to work at the standard periodicities of 25 and 50 per second, and as the alternator must have either two, four, or six poles it will easily be seen that there is not much choice of speeds.

This is why steam turbines have been standardized for the speeds 3,000, 1,500 and 1,000, these speeds agreeing with 25 and 50 periods per second. The reason why there is such a big jump between 1,500 and 3,000 is because the number of poles must be even to give N. and S., and there cannot be less than two, and of course the next number is four.

This turbo-alternator is somewhat difficult to design, but, all things considered, it is perhaps easier to deal with a minimum number of poles than with an excessive number, 20 and 40 and so on. Such are required when the prime mover speed is very low, as is the case with Corliss valve engines, large

gas engines and with water turbines, working on low heads of 10 or 20 feet. In some cases the speed of a large water turbine may be so slow that there is only room for an exciting coil on every other pole.

Switch-board.—As the whole output of the power station is brought to and controlled at the switch-board, any mishap there, has far greater consequences than the failure of one engine or dynamo. It is necessary, therefore, to give great attention to the design of this part of a station equipment.

The switch-board should be absolutely fireproof, and should be fixed in a convenient position in the station. In many cases a wooden framework has been used, while wooden floors are numerous. They are of course quite wrong.

Very ample space should be allowed behind switch-boards having back connections, and this space ought on no account to be filled up with regulator resistances, transformers, and other apparatus, but should be left perfectly free. These adjuncts to the switch-gear are preferably accommodated under the platform or in fireproof cellar.

Convenience to the station staff is most important and must not be sacrificed to obtain low cost. Nothing is so confusing to a new switch-board hand, or more likely to cause a mishap, than a board having the gear to control a generator or feeder dotted here and there, and made bewildering by a multiplicity of labels.

The Position of Switch-board.—It is often found placed at the end of the engine room at which the first generating sets were laid down, and, as a consequence, after one or two extensions have been made, it is at the furthest possible distance from the most important plant. It is very much better that the first instalment of the switching apparatus should be placed in a raised position (preferably in a gallery with a resistance and cable cellar beneath) at the side of the engine room, and at the *furthest* end from the first generating set, so that when the station plant and buildings are extended its final position will be near the centre of the building.

The only other point in connection with switch-gear which need be mentioned here, is provision for **extension**. Every possible forethought should be bestowed on the arrangements for facilitating the additions and coupling of extensions to the gear. This will avoid any necessity for reconstruction or stoppage of supply.

Isolation of Switches.—In order to prevent any failure or accident in one part of the board affecting the rest of the gear, modern switch-boards are divided up into panels.

The panel system, in which each generator and feeder has the switches, fuses, regulators, and measuring instruments arranged

one above the other in the form of a panel, is the outcome of much experience.

The standard system in large alternating current stations is to enclose all high tension main switches in brick cells and make the final break of each switch under oil.

As these switches are heavy they are often actuated by electric motors or solenoids, and the starting and stopping is done by remote control apparatus.

In large modern power stations the switch-board attendant sits at a desk on which is traced, in coloured lines, an exact representation of the switch-board circuits. At the points where there are switches the remote control board is fitted with electric pushes, and the pressing of these sends a current to the respective switch motors or solenoids.

In this way the attendants do not require to go near live parts. In case of repair very elaborate precautions are taken to ensure that all the metal parts adjacent to the repairer are earthed and kept so until the repair or examination is completed.

Signalling gear is sometimes found useful in order to simplify communication between the switch-board attendant and the engine driver. An arrangement similar to a ship's telegraph is sometimes used, while in several instances illuminated signs have been installed. The latter are preferable, as the sign relating to each plant can be controlled from its particular panel on the switch-board, and no mistake is then possible.

Synchronizing.—Some switchmen strongly favour the use of a lamp, while others prefer a synchronoscope. The synchronoscope as made by Messrs. Everett, Edgecumbe & Co., is perhaps the best way of synchronizing large turbo alternators where plugging in at the exact moment is most important.

The gear should be placed in a convenient position in relation to the main switches, and the arrangement of a swinging panel at the end of the switch-board will be found most handy in this respect.

BALANCERS—STORAGE BATTERY.

Balancers.—When connecting consumers on to a three-wire system care has to be taken to balance the load on each side of the middle or neutral wire. Of course, it is quite impossible to keep a correct balance, and consequently some provision has to be made to slightly raise the voltage on that side which for the time being has the largest load.

This may be done automatically by means of a pair of balancing dynamos connected up as shown in Fig. 169.

Each of the balancing dynamos A and B are shunt wound, the field coils of B being connected between the positive outer

and the middle wire, whilst the field coils of A are connected between the negative outer and the middle wire. Each machine is of such capacity that it will carry continuously a little more than half the out-of-balance current.

For the sake of showing the direction of the currents, etc., the negative side is shown to have one more lamp than the

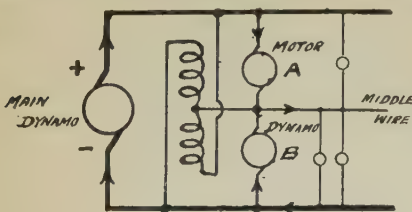


FIG. 169.

positive side, and as a result of this, current flows in the middle wire. Half of it passes through machine A, which for the time being acts as a motor, and half of it through machine B, which is therefore at the same moment a dynamo.

As there is a larger load on the negative side, the voltage on

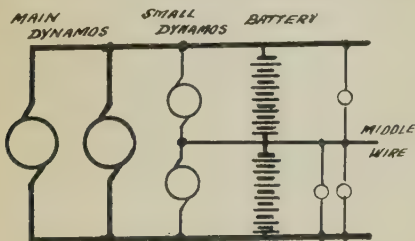


FIG. 170.

that side will tend to fall, and this will cause a weaker current to flow in the field of machine A. Such weakening of the field causes A to tend to speed up, and thus drive the other machine as a dynamo, raising the voltage of the latter. This tendency is also helped by the field of machine B being connected across the positive side.

Of course, if the positive side should happen to have the larger load, then B would be the motor and A the dynamo whilst that condition lasted.

As the output of such balancing dynamos is small when compared with the total output of the station, a balancer set is usually unable to cope with any unusual out-of-balance load, such as a short circuit or a heavy leak on one side. Some engineers, therefore, prefer to install a generating set having one engine driving two dynamos, and a large storage battery which is kept permanently connected across the mains, as shown in Fig. 170. Regulation is effected by the battery. The other generating sets are connected straight to the outers.

Storage Batteries. — The use of storage batteries is one of the chief advantages of a direct current supply. Both price and general efficiency have been so improved of late years that from being a luxury a battery having a large reserve of power has come to be looked upon as a necessity.

The advantages are :—

- (1) It enables the period of minimum load to be dealt with economically.
- (2) It forms a means of employing the generating plant with advantage during the period of medium load.
- (3) It acts both as a reserve and as a regulator during the period of maximum load.
- (4) It dispenses with the necessity for small and therefore inefficient generating plants.

The proportion which, under given conditions of price and capacity, the storage plant should bear to the generating plant in a direct current station depends almost entirely on the load factor and the cost of fuel. The higher the load factor and the lower the cost of initial generation the less the necessity for storage, and *vice versa*.

Maintenance. — Storage batteries are generally rated at their discharge capacity for three hours, and on this the capital cost of a storage battery is about equal to that of a generating set of same output.

As regards maintenance, leading manufacturers are willing to enter into 10 or even 15 years' maintenance agreement for an annual charge of from 5% to 6% on the initial outlay. This maintenance charge is no more than might be allowed on the generating plant, and a maintenance agreement should therefore be entered into with the makers, wherever possible, for a term of years. 100% of initial ampere hour capacity is the best basis of guarantee, but some makers only offer 60% or 70% of the capacity.

Under these maintenance agreements the only materials left

to be supplied by the generating station are acid and distilled water. The former is seldom required, while the latter may be obtained by the simple arrangement of collecting the discharge from steam traps draining the main steam pipes.

Charge and Discharge.—Although batteries can be purchased, which will easily withstand discharging in three hours, the usual safe charging rate is not more than half the discharge rate. Overcharge and overdischarge are the two points which are most to be guarded against, as they tend to ruin the cells.

In large batteries the maximum charge in ampere hours is generally about 115% of the discharge.

A battery meter of the Miller-Aron type, specially arranged to allow the extra charge, is a great help in preventing overcharging. It also saves the necessity for calculation.

Individual cells which have fallen off, owing to partial short circuits, have to be carefully watched for, and when found they are brought up to the mark again by receiving a separate charge from a small booster fitted with flexible connections.

The efficiency is reckoned by watts output as against input, and 85% has been obtained in special cases, but these figures were the results of tests of short duration, and made no allowance for milking, the cells, etc. Commercially, 70% may be taken as a fair and easily obtainable performance.

Small batteries are frequently installed in alternating current stations to work in parallel with the exciters. In this way the alternators cannot lose their field if the exciter should break down.

STATIC AND ROTATING TRANSFORMERS.

Static Transformers.—When the power station is for the purpose of transmitting electric energy at extra high tension, it is necessary to build a separate fireproof room for the step up transformers, etc.

Some transformers step up from 11,000 to 110,000 volts, this very high pressure being used by the Ontario Power Co., Canada. The terminals of extra high voltage transformers are of special condenser type, designed to reduce the stress on the insulator.

The transformers are placed in chambers filled with a special oil, which has no free acid or water. In order that the temperature of the oil may not get near the flash point, a cooling coil is submerged in the oil, through which cold water continually circulates.

For the smaller transformers, say 1,000 kw. and less, ordinary fan draught will carry off the heat, but it is necessary to see that the air going in, is quite free from dirt or moisture.

There are many uses for static transformers in power stations.

Thus all the measuring instruments on the switch-board have transformers, so that the instruments themselves can work at low tension.

Changing Phases.—It is sometimes necessary to change from two-phase to three-phase and *vice versa*. At Niagara, for example, the old power station on the American side has two-phase generators, whereas the new power house on the Canadian side has three-phase.

In order that each power house may be able to help the other in emergency, they are linked by cables and transformers, the connections being as devised by C. F. Scott (see Fig. 171). For

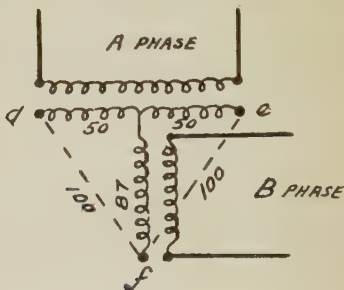


FIG. 171.

the supply to Buffalo two-phase is changed to three-phase at Niagara, and then at Buffalo it is changed back to two-phase.

Again, in California, most of the hydroelectric stations, some hundreds of miles apart, are linked up with three-phase transmission lines. Supply is, however, frequently given as two-phase, as it is found more convenient, and tends to less unbalancing. In this case, step down transformers which, at the same time, convert from three-phase to two-phase, are used.

Referring to Fig. 171, A and B represent each of the phases of a two-phase system. The transformer windings have the ratios as indicated by the numbers, and the consequence is that balanced three-phase connections can be made at *d*, *e* and *f*.

Rotating Transformers.—Motor generators, rotary converters and boosters may all be classed under rotating transformers. Although mostly found in sub-stations, there are few large power stations which have not some form or other of rotating transformer. For example, a high tension station may require low tension direct current for driving variable speed motors, charging a storage battery, or some local demand.

A motor generator or rotary converter is therefore installed for the purpose.

The motor generator may consist of an induction motor coupled to a dynamo, or it may consist of an alternator or, as it is called, a synchronous motor, coupled to the dynamo. The latter has the disadvantage of not being self starting from the A C side, but it has the advantage of giving better regulation, and of improving the power factor by specially exciting the motor. Over excitation tends to cause the current to lead, and as this is just the opposite effect of an inductive load the power factor is improved.

The **rotary converter** is cheaper and more efficient, being a self-contained machine instead of two machines coupled. It requires to have the alternating voltage adjusted to an exact amount suitable to the direct current side, and therefore static transformers are necessary to work with it. The voltage is not easy of variation, and so, whilst it is handy to have in a power station for supplying light and power, it is not so generally convenient. With a motor generator, the voltage is easily regulated to give any suitable pressure for charging storage cells.

COST OF SUPPLY—INITIAL COST—LOAD CURVES.

Cost of Supply.—In accordance with the Electric Lighting Act, all undertakers of electric supply have to file with the Board of Trade a statement of their yearly accounts divided under the following heads :—

1. Coal and other Fuel.
2. Oil Waste, Water, and Stores.
3. Wages on Generator and Distribution.
4. Repairs and Maintenance.
5. Rents, Rates, and Taxes.

An immense mass of statistics has consequently been accumulated, and the following gives an average of over 200 undertakings in pence per unit sold :—

Coal and other Fuel	. 69	} Works costs, 1·42
Oil Waste, Water, and Stores	. 09	
Wages	. 37	
Repairs and Maintenance	. 23	
Rent, Rates, and Taxes	. 17	
Management Expenses	. 44	

2·08 total costs.

The lowest cost is less than a halfpenny per unit sold, and the highest is over fivepence.

Large power stations, supplying current in bulk, do much better, their total costs being in the region of a farthing per unit sold.

Factors which Influence Cost.—The conditions under which electric supply is given in various districts vary greatly, the principal factors being —

1. Output.
2. Load factor of generating works. (This is taken as the ratio of the actual output, to the output possible if the maximum demand were continued throughout the year.)
3. Cost of fuel.
4. Efficiency of generating and distributing plant. (The term efficiency is used in its widest sense, and refers not only to steam and electrical efficiencies, but also to economical design and arrangement.)

The second, as a rule, depends to a great extent on the first, because, generally speaking, the **load factor** becomes more favourable as the number and variety of consumers increases. It is also controlled by the system of charging adopted, by the relative applicability of the particular system of supply to day uses, and by the extent to which the undertaking is responsible for public lighting and tramway supply.

The cost of fuel, except in special cases, depends on the distance of the undertaking from the coalfields, and on the cost of carriage.

Of the many branches of the **efficiency factor** most attention is, as a rule, paid to the test efficiency of the generating machinery, whereas the general arrangement of the whole plant, and the selection of the sizes of the various generating units relatively to the shape of the load curves, are points quite as important.

Initial Cost of Power Stations.—When electric power stations were first laid down the total cost per kilowatt of plant installed was exceedingly high. Even now, the actual costs of the 200 odd stations average out at over £60 per kilowatt of plant installed, or about £2 per 30-watt lamp connected. This is partly due to the large number of municipally owned plants. Of recent years the cost has come down very much, owing to the large generating units installed, also the more economical design of power stations. Company-owned stations cost about £15 per kilowatt of plant installed, and even below £10 is possible.

Load Curves.—The shape of the load curve has a great deal to do with the cost of production of the electric energy. Stations which give a supply for lighting only have very pronounced peaks. Those which also give a supply for tramways

still have peaks, but the daylight hour valleys become partly filled in.

The best load curves are given by bulk power stations which supply a great diversity of consumers, especially consumers who require power right through the twenty-four hours, such, for example, as electrochemical and electrometallurgical industries.

Fig. 172 shows typical **winter** (full line) and **summer** (dotted line) **load curves** for a power station which principally supplies energy for lighting.

The vertical lines are for the hours 12, 6 a.m., 12, 6 p.m., and 12, and it will be noticed that in winter there is a small peak

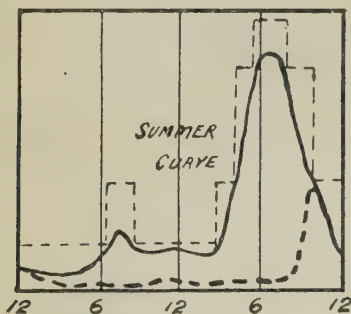


FIG. 172.

just after 6 a.m., which requires the starting of an extra generating set, represented by the rectangular dotted line.

At 6 p.m. there is a very high peak, requiring the running of four generating sets.

The station load factor is about 20%, which means that only that percentage of what the station is really capable of, is generated through the twenty-four hours.

The average load on the plant is 72%, which means that on an average right through the twenty-four hours only that percentage of the total output of the generating sets running was actually generated.

Effect of Storage Battery.—Now it is clearly beneficial to keep the generating sets always at full load, and this can be approximated to, by using a storage battery.

Fig. 173 shows how this is applied to the same 20% load factor curve. The black portion shows the battery discharge,

which is from midnight to nearly 12 noon, and again for a short time at the top of the 6 p.m. peak.

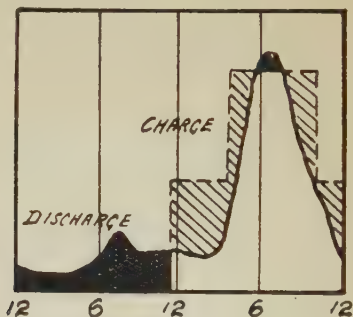


FIG. 173.

The shaded portion shows the battery charge, and it will be noticed that two generating sets are sufficient to carry the whole of the load, and they are always working at full power.

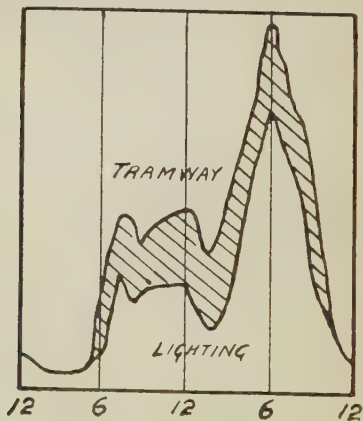


FIG. 174.

Fig. 174 shows the effect of adding a **traction load** on to a lighting and power load, the shaded portion representing the

energy required for the tramways. This improves the station load factor from 35 % to 42 %.

If the whole of the rectangle to the topmost peak represented power used right through the twenty-four hours, as might easily be the case for a power station supplying an electrometallurgical works, then the station load factor would approximate to 100 %.

SYSTEMS OF CHARGING FOR ELECTRICITY.

The ideal system of charging for the supply of electrical energy should comply with the following:—

It must have as its main object the popularization of the use of the supply all through the twenty-four hours.

It must be equitable to the consumers and remunerative to the undertakers.

It must introduce as few complications as possible on the consumer's premises and in the books of the undertakers.

It must be simple and easily grasped by the average consumer.

No system at present in use quite complies with all the above, but the following are some of the principal methods:—

SYSTEM I.

Set charge per unit for all purposes.

It will be seen at once that this system does not in any way comply with the first and most important qualification, as it does not in the smallest way encourage the use of the current during the daytime. It was very generally used in the early days of electric supply.

SYSTEM II.

Set charges per unit according to use of supply with no discounts.

This encourages to a certain degree the use of supply during the hours of daylight, but it has its disadvantages, as it necessitates two separate installations of wiring to enable the two uses of the supply to be kept distinct. It will readily be seen that to encourage the use of electric heating and small motive power work may sometimes be difficult under a system which compels the consumer to run fresh wires before he can have an opportunity of trying the effect of apparatus of which he knows very little.

SYSTEM III.

The same as I. and II., except that discounts are given in proportion to the quantity of energy consumed.

The basis for discounts is not quite equitable to the consumers, as the cost of supplying electricity depends on the

rate of supply and not on the *quantity supplied*. For instance, a consumer having ten lamps installed and burning them for five hours a day is a far more profitable customer than one having fifty lamps and burning them for one hour a day, although the energy consumed, and consequently the discount given, would be the same in both cases.

SYSTEM IV.

Set charges per annum per kilowatt of maximum demand plus a small charge per unit consumed.

This system encourages the use of the supply over long hours.

In Manchester a number of the consumers are charged on the basis of £7 per annum per kilowatt of maximum demand plus $1\frac{1}{2}d.$ per unit for all energy consumed.

Working this out, the consumer would pay for his supply somewhat as under :—

For one hour's use per day of all his lamps $6d.$ per unit.

„ two hours' „ „ „ „ „	$3\frac{3}{4}d.$	„
„ three „ „ „ „ „	$3d.$	„
„ four „ „ „ „ „	$2\frac{5}{8}d.$	„

SYSTEM V.

Maximum demand method, devised by Mr. Arthur Wright.

An instrument called the Maximum Demand Indicator, fixed on the consumer's premises, indicates the maximum rate at which energy has been supplied to him at any time during the quarter.

He is then charged as follows: $6d.$ to $8d.$ per unit for the number of units equivalent to the use of his recorded maximum for one or two hours a day as the case may be ; $1d.$ to $2d.$ per unit for all further consumption.

This gives enormous advantages to the long hour consumer, as will be seen from the following table :—

AVERAGE COST PER UNIT UNDER WRIGHT'S SYSTEM AT
 $8d.$ FIRST HOUR, $2d.$ AFTER.

No. of hours' daily use of maximum demand ...	1	2	3	4	5	6	7	8
Charge under Wright's system	$8d.$	$5d.$	$4d.$	$3\cdot5d.$	$3\cdot2d.$	$3d.$	$3\cdot86d.$	$2\cdot75d.$

The maximum demand system has the disadvantage that it is not easily understandable by the consumer. It also takes

no account of the time of the day at which supply is taken. For example: power users for several hours in the daytime could be profitably supplied at $1\frac{1}{2}d.$ a unit, but under the maximum demand they never get down to this figure.

SYSTEM VI.

Differential rates based on the time of the demand.

This consists of charging consumers a high price during the period of heavy load on the generating station in the evening, and a low price during the rest of the twenty-four hours. Motive power, heating, lighting, etc., are all placed on a level.

The registration of consumers' meters is controlled either by a special synchronome circuit, actuated by an electrical clock in the central station, or by separate time switches placed on consumers' premises, which so affect it that the energy consumed during certain evening hours is charged at the high rate, and the remainder at the reduced rate.

Time switches are in use at Bristol and Reading to a limited extent, while at Norwich the synchronome principle was used for several years, but has been abandoned.

Theoretically, the system is nearly perfect from the central engineer's point of view, but practical disadvantages unfortunately preclude its general adoption.

SYSTEM VII.

Fixed annual rate per lamp installed.

This is used for public lighting work only, the supply undertaking providing the necessary energy to light public lamps at a definite rate per lamp year. For all-night arc lamps, burning about 4000 hours per annum, the charges may vary from £16 to £26 a year, corresponding to an average charge per unit of rather over $2d.$

FEEDERS—DISTRIBUTORS—JUNCTION BOXES.

Feeders.—Although the title of this section, namely, Power Stations, indicates the actual building and its contents, it is usual to consider the feeder cables and distributors as coming under the heading.

There are two principal ways of arranging the cables, namely, *Underground*, which is the usual method in cities and towns, for the electric light power supply of towns, and *Overhead*, which is the method adopted where bulk power has to be conveyed over considerable distances.

Insulation.—For underground feeders and distributors, rubber insulation has been found unsuitable. The alternate wetting and

drying and the disintegrating action of the coal gas (which impregnates the ground) are frequent causes of breakdown.

Bitumen has been much used, and where the cables are laid solid in bitumen or pitch compound it appears to be the best insulation.

Paper has proved very successful, and it may be considered the standard form of insulation, especially for high voltages. The paper is made from pure wood fibre, laid on in many layers to break joint and lock back when the cable is bent and straightened. A special oil with which the paper is impregnated forms the chief insulation, and consequently the ends of paper-covered cables have to be kept carefully sealed.

Methods of Laying.—Underground systems may be conveniently classed under three heads: (a) cables laid direct in the ground, (b) cables laid solid with bitumen, and (c) cables drawn into conduits.

For *direct currents* the positive and negative cables can be laid separately, but with *alternating current* this is not so, because of induction troubles.

The cables for alternating current are either double and triple concentric, or else triple core. The latter is the favourite method of making three-phase high tension cable. Each conductor is clover-leaf shape, so that the three pack well together, thus reducing the amount of lead covering and armouring.

Cables laid Direct in the Ground.—Cables for the purpose of being laid direct in the ground without further protection generally consist of copper conductors served with paper, jute, or other compound type of insulating material. Over this a lead sheathing is drawn as a protection against moisture, and mechanical injury is prevented by wrappings of jute and steel armour over the lead.

In some cases vulcanized bitumen has been used as an insulating medium, and, being non-hygroscopic, it allows of the omission of the lead sheathing. This type of cable is, however, liable to give trouble owing to disintegration of the vulcanized insulation by the presence of coal gas, from which the soils of large towns are seldom free.

In the direct laid system one of the chief drawbacks is the liability of the metallic covering to corrosion and decay, either from the action of the chemical constituents of the soil or from what is now becoming a very prominent danger—electrolysis from electric tramway earth currents. Another disadvantage is the fact that a low resistance “earth” circuit is provided which, in the event of a fault developing, may allow a dangerously large fault current to flow, causing difficulty at the generating station in coping with the heavy additional load, and possibly damaging long lengths of the cable itself. This

disadvantage is increased in direct proportion to the extent to which "bonding" of the lead and armour has been resorted to.

Bonding may have some minor advantages, but it is better for all systems of armoured cables to be split up into sections, the metallic sheathing of each section being isolated and insulated from that of the adjacent section. This arrangement will have the effect of reducing the tendency for stray tramway earth currents to flow along the sheathing, and also of increasing the resistance in the earth circuit of which a fault current can take advantage.

It should be noted that when electrolysis occurs it always takes place at the point where current *leaves* a cable or its sheathing, and not where current enters.

Cables laid Solid.—When cables are to be laid solid they are usually insulated with vulcanized bitumen. A trench is excavated and wooden or stoneware troughing is laid along the trench. The cables are then laid in the troughing and kept in position by bridge pieces of stoneware or specially treated wood.

After the trough has been filled in with melted bitumen or pitch compound, tiles are laid over the top.

A pitch compound is sometimes used in the place of bitumen, but it is not so good, because there is a chance of its containing or developing free acids. It may also crack and let in moisture.

Cast iron troughing was formerly used instead of wood or stoneware, but it is expensive and liable to electrolytic action, especially where there are electric tramways.

The "Howard System" uses asphalt troughing, with bridge pieces of asphalt. This also is filled in with bitumen, and a protective layer of asphalt concrete finishes off the job.

The "Simplified System" has bare copper or aluminium conductors clamped on stoneware insulators in stoneware troughing. Then a special compound of high insulating quality is run in, and the top protection is asphalt or tiles.

Great care must be taken to prevent soil or moisture getting into the troughing during the filling process, but if this is properly carried out, there is no reason why this method of "manufacturing a cable on sight" should not be perfectly sound. Its simplicity and cheapness should at any rate make it worthy of the consideration of station engineers.

Cables drawn into Conduit.—For this method of laying, the cables must be very well insulated, generally by paper, and have a lead sheathing. The latter is sometimes served with jute braid, and compounded.

The ducts are generally of stoneware, with manholes at fairly frequent intervals. This is especially the case in crooked streets, in order that the cables may be easily drawn in.

A good method of making conduits is to use tubes of wood

fibre, treated with bitumen, and these are laid in concrete. There is first a layer of three or four, then more concrete is added, and another layer of tubes, more concrete, and so on for the required number.

The tubes are a good insulation in themselves, and water is kept out by the ends being coned and compounded together. The inner surface is quite smooth, so the cable draws in easily, without any danger of being torn. At the same time the concrete gives perfect mechanical protection to the tubes.

This method is used in Melbourne and some of the principal cities of the United Kingdom. It is also a favourite method in the United States.

Sizes of Feeder Cables.—The Board of Trade has no regulation regarding the voltage drop in a feeder cable, but the general practice is to allow 10%.

Theoretically the Kelvin law as to the most economical size should apply. At the starting of an electrical undertaking, however, when it is, that the sizes of feeders must necessarily be settled, there are so many variables and unknown quantities that it is difficult to apply the law.

The following are some of the variables :—

The load factor of the supply system; the cost of feeders laid complete, including opening out the ground and reinstating; the distance of the power station from the feeding points and the route taken; also the possibilities of an area changing its character, from a residential to a shopping or an industrial one. There is again the very important factor that it is most desirable to secure uniformity of sizes of cables, even where this means using a size too large or too small.

The usual sizes of feeder cables for 440 to 500 volts between outers are 1·5 sq. inch, 1·0 sq. inch, ·75 sq. inch, ·5 sq. inch, and ·3 sq. inch.

Distributing Cables.—A feeder is a conductor which is only tapped at its extreme end, whereas a distributor is a cable that is tapped at frequent intervals.

In the lighting and power supply of a town, the feeders are laid direct to points in the centres of the populated areas. Distributing cables then lead away through the streets, and are tapped for the house and factory services wherever required.

In a main street it may therefore happen that there are the two feeder cables lead and return besides the distributing cables down each side, each having lead and return.

As distributing cables have to be frequently tapped, it is necessary that they should be readily accessible, and therefore the solid system and the drawn in system of laying are not suitable.

The usual plan is to lay armoured cables direct in the ground,

unless the run from one district or factory to another is fairly long, say $\frac{1}{4}$ mile or so.

In the nature of things, distributors are smaller in cross section than feeders, and as they are frequently being opened out they are under more regular observation.

Sizes of Distributing Cables.—It is most important, both for the sake of uniformity of jointing material and for convenience in keeping spare cable, that as few sizes as possible should be employed. In the majority of supply areas three sizes should be ample.

It is also necessary that the sizes laid down in the first instance should not have to be increased for many years to come, and with this in view some estimate should be made of the lamp density which may eventually be arrived at in the various streets to be served.

In order to get some idea as to what size cables to lay down, it is usual to allow a certain number of 30-watt lamps per yard frontage of streets.

The majority of electrical undertakings in the United Kingdom give an average of twenty 30-watt lamps per yard frontage of streets in the central area, ten for an intermediate area, and two lamps per yard for outlying streets.

The Board of Trade regulations for securing a proper and sufficient supply of electrical energy allow a 4% variation on either size of the declared standard pressure of supply on ordinary distributing cables.

For a three-wire distribution with 440 to 500 volts across the outers, the following sizes of distributing cables have come to be considered as standard :—

	Outer cables.	Middle cables.
Central area . . .	·3 sq. inches.	·15 sq. inches.
Intermediate area .	·2 „	·1 „
Outlying area . . .	·1 „	·06 „

Given these sizes and the lamp density per yard of street frontage, also remembering that of the 4% voltage drop about 1% will be on the consumers' premises, the positions of the feeding points can be then mapped out.

EARTHING—DISCONNECTING BOXES.

Earthing.—The middle wire of all three-wire direct current systems working at 400 to 500 volts across outers, must be earthed at the power station in accordance with the following Board of Trade regulations :—

(a) The connection with earth of the intermediate conductor must be made at one point only on each distinct circuit, namely, at the generating station, sub-station, or transformer, and the insulation of the circuit must be efficiently maintained at all other parts.

(b) The distributing mains must be separated into sections corresponding approximately to the different feeders, and these sections may be interconnected only through suitable circuit breakers or fuses, arranged so as to be easily inspected.

(c) The current from the intermediate conductor to earth must be continuously recorded, and if it at any time exceed one-thousandth part of the maximum supply current, steps must be immediately taken to improve the insulation of the system.

(d) No fuses will be used on the branch connections on the intermediate main.

(e) All branched connections from the outer mains will be protected by suitable fuses, which will be so arranged as to be easily inspected, and so as to be completely free from all risk of causing fire by defective action, arcing, etc.

Earth Leakage.—There are various ways of recording earth leakage. One method is to fix a recording ammeter in the earth-wire circuit, whilst another is to have an ordinary ammeter, on which the switch-board attendant keeps a watchful eye.

The difficulty with all recording instruments is to obtain one that will be sufficiently sensitive to register the small leakage currents, and yet be heavy enough to withstand large momentary leaks. An automatic short-circuiting device can be used, but its time lag may prevent its coming into action until the mischief is done.

Sometimes a light fuse is placed in the earth-wire circuit, and shunted by a resistance so proportioned as to take the normal maximum current of the recording instrument at the greatest voltage possible on one side of the system. On the occurrence of a heavy leak the fuse will blow, leaving only a small current passing through the resistance.

The primary **object** of earthing the intermediate conductor is naturally to prevent the negative outer assuming the earth's potential by the attraction of moisture, and thereby raising the positive outer to an unpleasantly high potential *above* the earth. As the Board of Trade does not specify the minimum resistance of the connecting circuit to earth it is both convenient and advisable to make it an appreciable one (say, $1\frac{1}{2}$ to 2 ohms).

Disconnecting Boxes.—Whatever system of cable laying is adopted, a free use of disconnecting boxes will always be found to pay in the long run, as the ability to rapidly localize a fault to within 100 yards or so is most valuable. Every cable manufacturer has standard designs of disconnecting boxes to suit varying conditions, and in many cases special designs are prepared for a new scheme to suit the engineer's requirements. It is, therefore, only necessary to emphasize a few general points in connection with them.

It is a convenience to fit the lids of main disconnecting boxes with small plate-glass windows, and to leave the links bare. Small pieces of sealing wax are laid on them and on the connections, so that any loosening of connections and consequent

heating will be indicated by the melting of the wax. If the wax is right at the time of inspection through the window, then nothing need be disturbed.

Regarding moisture, this can be taken care of by placing calcium chloride inside the box.

Some makers fill the disconnecting boxes with resin oil, but this has the disadvantage of having to be extracted and the disconnecting links cleaned before any testing can be done. There is also a danger of electrolysis owing to impurities in the oil when the box is used for direct current.

The lids of disconnecting boxes should have a double petticoat joint, the outer joint being the lower of the two. All nuts or bolts which have to be regularly removed should be of gun metal, not steel.

Bitumen or compound glands are preferable to washer or wiped glands.

Disconnecting links should be arranged to lift out *vertically* in order to minimize dangers of short circuit when working on a live box. A hole drilled and tapped in each link is useful in conjunction with an insulated screwed bolt for lifting the links when the inspector is not using rubber gloves.

Switch Pillar.—Most disconnecting boxes are placed underground, but where possible a switch pillar above ground is a great convenience. The connections are thus out of reach of water in case of a flood, and being so much more accessible the time taken to split up a network of distributors or to disconnect a faulty feeder is very much less.

For tramway work switch pillars above ground are quite standard practice, and they are generally fitted with a telephone set, so that communication with the power station and the sub-stations is established directly anything goes wrong.

FAULTS AND THEIR LOCALIZATION.

Fault Signalling.—On high pressure cables unearthed, a rough guide as to the relative insulation resistances may be obtained by connecting each main to earth through an electrostatic voltmeter, a vacuum tube, or a gold-leaf electroscope. If this last-named instrument is used the leaves are connected to the cable and the core to earth. Any inequality in the effect on the two sides shows a corresponding inequality in the relative insulation resistances of the two mains.

On concentric cables with station-earthed outers there is no reliable method (apart from the creation of telephone disturbance) of signalling the development of an earth fault, unless some arrangement be made for measuring the potential difference between outer and earth at different points. If this

can be done, any decrease from the normal drop of pressure along the outer naturally indicates that a portion of the current is being shunted to earth through a fault.

Central Zero Ammeter.—For low pressure cables on the three-wire system having a station-earthed middle wire, an ammeter in the earth-wire circuit affords a **rough guide** as to the difference between the insulation resistances on the two sides. On a two-wire concentric system with station-earthed outer, the same method applies.

On two- or three-wire unearthed systems, a voltmeter or lamp connected from each main to earth shows the relative insulations, and there are several methods by which the actual values can be calculated. Perhaps the best of these for **large networks** is as follows :—

A central zero ammeter is connected between each main and earth and the readings noted. The insulation resistance is then calculated from the following formula :—

$$\text{Insulation resistance} = \frac{\text{voltage between mains}}{\text{sum of readings irrespective of direction}} - \text{ammeter resistance.}$$

The ammeter used should preferably be of low resistance compared with the insulation resistance of the mains.

Automatic Signalling.—On the Continent it is usual to signal a fault by stranding an *insulated pilot wire* in with the outer layer of wires forming the conductor. This pilot wire is connected, through a high resistance indicator having a “falling” shutter, to the station bus bars. The pilot wires on the negative cables are connected to the positive bus bars, and *vice versa*.

As any earth fault on the cable is accompanied by damage to the insulation of the pilot wire, it follows that a circuit is formed through the indicator, the shutter of which drops, and shows the switch-board attendant that something is wrong.

Inductive Method of Fault Localization on Live Mains.—This depends on the inductive effect of a current sent through the fault to earth, upon a coil of wire or a compass. Unfortunately, the wire or compass cannot be used with any accuracy on lead-sheathed cables, as the testing current is always shunted for some distance along the lead sheathing, or into adjacent gas and water pipes, causing very misleading results. For cables in stoneware conduits, or in iron pipes on which good metallic joints have not been made, good results can be obtained.

On an *alternating system* all that is necessary is to earth the sound main through a resistance at the station, and to “search” with a coil of wire connected to a telephone receiver. If it is arranged that the main is only fed into from the station end, a hum is heard in the receiver along the route of the cable, until the fault is reached, when the hum will very nearly cease.

A useful size of coil is a triangle having each side 2 ft. long and wound with 100 turns of copper wire double cotton covered. One side is held parallel to the cable.

In the case of rubber cables in pipes a solenoid 3 in. long by 2 in. in diameter, wound with 1,000 turns of No. 34 gauge wire, gives good results, the effect being accentuated by having a magnetic core of soft iron wires inside the coil. One advantage of this small coil is that it can be held close up to the cable in junction boxes.

The application of induction methods on live mains is somewhat dangerous if both cables lie in the same conduit, as the heat caused by the testing current at the fault is liable to destroy the insulation of the sound cable.

On a *continuous current system* the sound main may be earthed through a resistance and an interrupter, and the same method applied.

Compass Method.—This is useful for continuous current. The faulty main is fed from each end, the compass being carried along, over where the cable is laid. The reversal of the compass needle on passing the fault is very decisive. Not less than 10 amperes is required for the compass method, and this is sometimes impracticable.

A fault on a distributing main can sometimes be located to a particular length between two services by measuring fall of potential. The distributing main must be fed from one end only, and at a time of light load the sound main is earthed through a resistance at the feeding end. If the potentials between the fault side services and earth are taken with a low-reading voltmeter, the readings will fall from the feeding end until the fault is reached, after which they will remain stationary. If a good earth can be obtained on a consumer's premises, this method is fairly reliable, but it is doubtful whether it is worth using unless it is absolutely imperative to keep up the supply on that particular section.

Series arc-lighting mains having good insulation can often be dealt with satisfactorily by a similar fall of potential method. A static voltmeter is connected between each main and earth at the station. When no fault exists and the insulation resistance is constant along the line, the two readings obtained will be equal. If a fault occurs at any but the middle point of the circuit, the readings will become uneven, and will be in direct proportion to the number of lamps on either side of the fault. The fault can thus be localized to a particular length between two lamps. Its exact position can then be localized by taking the potential to earth at each end of the length with a low-reading voltmeter and working out the distances by simple proportion.

Fault Localization on Dead Mains.—It is nearly always possible in a well-sectioned network to shut down a faulty length of distributing main for an hour or so, some time during the twenty-four hours without seriously inconveniencing the few customers who take their supply from it. Also in the case of feeders it can generally be arranged to disconnect a faulty one at times of light load without the supply being affected at all.

Murray's Loop.—The two principal and most practical methods of localizing an earth fault, are Murray's Loop method, and the fall of potential method.

Fig. 175 shows the method of applying Murray's Loop to an earth fault on a length of concentric cable.

One end of the cable is short circuited, and at the other, in

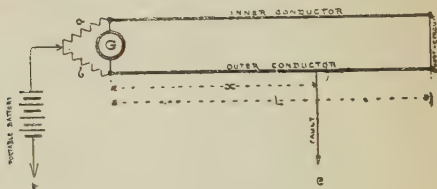


FIG. 175.

the case of a high resistance fault (1,000 ohms or so), a high resistance galvanometer and a double-arm resistance box are coupled across the two conductors. A portable 100 or 200 volt testing battery is connected between the junction of the resistance arms and earth, and the resistances having been varied until a balance is obtained, the distance of the fault is calculated by the following formula (which is merely an application of the Wheatstone Bridge law):—

$$x = 2L \left(\frac{b}{a + b} \right)$$

where x = distance of fault from testing station.
 L = length of cable.

a and b = values of resistance arms when a balance is obtained.

If the fault is of low resistance (say under 20 ohms) a slide wire resistance of fairly large section should take the place of the resistance box, a five- or ten-cell portable accumulator the place of the testing battery, and a centre-reading millivoltmeter the place of the high resistance galvanometer. In this case it is

necessary to allow for the resistance of the leads connecting the ends of the slide wire and the conductors. When using the loop method it is always advisable to test from both ends of the cable, and to take the mean of the positions of the fault so found in order to make as much allowance as is practicable for any inequalities of conductivity caused by joints, etc.

Fall of Potential Method.—If an unused length of cable next to the faulty length is available, very accurate results may be obtained by the use of a direct fall of potential method, providing the resistance of the fault is fairly low and constant.

Fig. 176 shows the method as used in connection with two

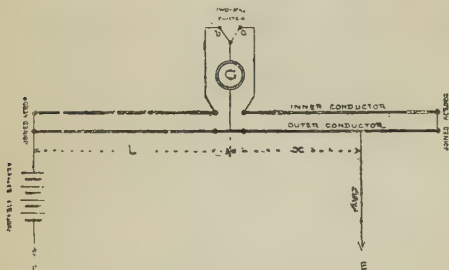


FIG. 176.

lengths of concentric cable of the same section, one of which has a low resistance earth fault at the "outer."

The two extreme ends are short circuited by pieces of wire, and the "inner" is disconnected at the junction of the two lengths. A portable accumulator is connected between the extreme end of the sound cable and earth, and one or two amperes allowed to pass. A millivoltmeter is connected as shown, and a number of readings are taken alternately on either side as quickly as possible and the readings averaged.

The length x can then be found from the single formula —

$$x = \frac{a}{b} L$$

where x and L represent the fault distance, and the total length of sound cable respectively, and a and b the average readings obtained on the faulty and sound, on the cable respectively.

This method is very reliable and practical if the current

passing along Lx can be kept fairly constant. Like Murray's Loop, it can be applied to two lengths of cable of different section by calculating the equivalent lengths, and it is of course equally applicable to single cables.

Fault Conductivity.—In the case of a pick or pinhole fault in the main cable, it sometimes happens (especially with a concentric "outer" fault) that only a few of the wires forming the strand "take up" the fault current for the first few yards. This has the effect of *increasing* the apparent distance of the fault from the testing-point, and it is therefore always safer to make a small allowance for this, say about 2 % or 3 % of the fault distance.

Battery Connections.—It should always be arranged for the fault to be in the battery circuit and *not* in the galvanometer circuit, as by so doing the potential of any couple formed at the fault is rendered incapable of introducing any error. It will be noticed that the accuracy of Murray's Loop test is not affected at all by the **resistance of the fault**, while the fall of potential method becomes unreliable when the resistance is very variable.

Instruments.—The centre-reading millivoltmeter for Murray's test on low resistance faults, is provided with a trial key for reducing the readings. For the fall of potential method a moving coil millivoltmeter gives admirable results owing to its extremely dead-beat action.

Resistances.—In view of the occurrence of faults, data should be ready to hand of the actual proportional resistances of the different sizes of cable in use, based on actual tests. Taking the largest cable as unity, a multiplier can be worked out for all the other sizes by which their equivalent lengths can be found.

For the resistances used with Murray's test a P.O. bridge box can be used for the high resistance faults, while for the low resistances a stretched wire bridge, a few yards in length, can be improvised. It is best, however, to keep one ready stretched over a scale marked in inches and fixed to a portable framework.

Short circuit Faults.—Short circuit faults are not as frequent as earth faults, and fortunately so, as their accurate localization is less easy.

(1) In the case of a short circuit in a length of concentric cable or between two single cables, an adaptation of Murray's Loop method can be applied, as shown in Fig. 177. The apparatus used depends, as in an earth test, on the fault resistance.

One of the conductors takes the place of "earth" in Fig. 175, and the loop is completed by a length of testing cable

laid along the surface of the ground. (That is, if a third underground conductor is not available.)

The length of the testing cable must of course be calculated out to the equivalent length of the faulty cable section, and what is more difficult, the *temperature* error must be allowed for if its temperature is likely to be greatly in excess of that of the faulty cable. For each 5° F. its "tested" length will be increased by 1.05%, and on a hot sunny day a difference of several yards may have to be allowed for, even in a short length of main.

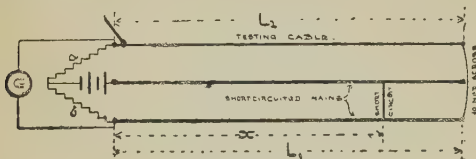


FIG. 177.

If L_1 represents the length of the faulty cable and L_2 the equivalent length of the testing cable (after allowing for difference of section and temperature)—

then

$$x = L_1 + L_2 \left(\frac{b}{a + b} \right)$$

(2) For a "dead" short circuit, in which it may be assumed that the conductors have been fairly well burnt together, a simple fall of potential test may be made from each end with a combined ammeter and voltmeter, as shown in Fig. 178.

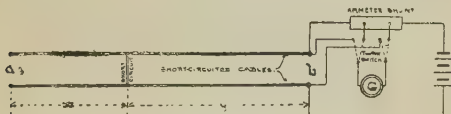


FIG. 178.

If milliampere and millivolt readings are taken from each end, and the ohmic resistances " a " and " b " calculated therefrom, then it is evident that the resistance of the short circuit fault " S " will be:—

$$S = \frac{a + b - R}{2}$$

Where R is the ohmic resistance of the two conductors when sound—

then
$$\frac{\text{length } x}{\text{length } y} = \frac{a - S}{b - S},$$

and the relative lengths from each end are found by simple proportion.

(3) **In a length of distributor** with frequent services, the fault can be localized to the length between two service mains by passing a small current, say 3 amperes, along the main from one end, and then taking millivoltmeter readings on each consumer's connection.

In Fig. 179 the readings would diminish from service F to



FIG. 179.

B , which latter would give the same potential differences as service A (*i. e.* merely the drop of potential across the fault). For this test it is of course essential that the fault resistance remains nearly constant, and that all the consumers' installations are disconnected.

(4) **Feeder with pilot wires.**—Mr. A. T. Cooper's method is

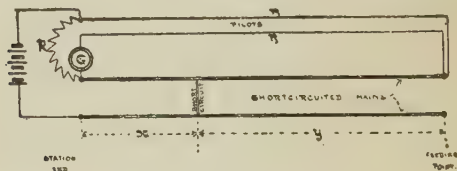


FIG. 180.

shown in Fig. 180. The ohmic resistance of each pilot wire is first taken by joining them together at the feeding-point, and testing the resistance of the ring so obtained. Dividing the result by 2 gives the resistance of each wire. Then in the connections shown, a bridge is formed, with pilot P_1 and a variable resistance R forming the two proportional arms, and lengths x and

y of one of the two short-circuited conductors forming the other two arms. The remaining conductor acts as a battery lead to the fault, while pilot P_2 acts as a galvanometer lead. When a balance is obtained—

$$\frac{\text{length } x}{\text{length } y} = \frac{R}{\text{resistance of } P_1}$$

It is advisable to reverse the pilots and take the means of the results if they vary, in order to allow for any difference of conductivity.

(5) If a length of sound main is available at either end or the faulty one, a modification of the **fall of potential method** shown in Fig. 176 can be used. Fig. 181 shows the connections.

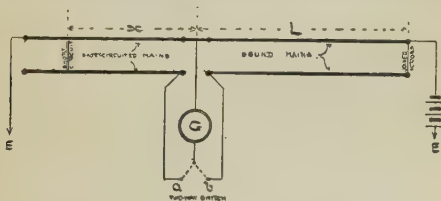


FIG. 181.

As in the test for an earth fault, the distance " x " can be found from the formula—

$$x = \frac{a}{b} L$$

The length of sound main may be a coil of cable if an underground conductor is not available, but in this case care must be taken to avoid temperature errors.

GENERAL LEGAL INFORMATION.

By PERCY M. BURTON, of the Inner Temple, Barrister-at-Law.

EMPLOYERS' LIABILITY.

At common law a workman could not maintain an action against his employer for injury received in the course of his employment unless he could prove personal negligence on the part of the employer.* The **Employers' Liability Act, 1880**, gives the workman a remedy in cases where the injury is caused by any defect in the condition of ways or machinery arising from negligence on the part of the employer or of a servant whose duty it was to see that the ways and machinery were in proper condition ; or where the injury is caused by the negligence of a person whose orders the workman was bound to obey ; or by the act of any person acting in obedience to orders given with the authority of the employer ; or where the injury is caused by the negligence of any person in the service of the employer having charge of any signals, points, or train on a railway.

Under the above Act the word **workman** does not include menial servants, but includes any person engaged in manual labour † employed to execute any work. The workman must give notice of the injury within six weeks of the accident.

The **Workmen's Compensation Act, 1906**, greatly extends the liability of employers, as* under it they are obliged to compensate their employees in respect of almost all accidents arising out of and in the course of the employment. It is not necessary for the workman to prove any negligence on the part of his employer, and it is no defence for the latter to show any carelessness on the part of the employee. Former Acts, being more or less experimental in character, only included certain classes of workmen, but the present Act includes all persons who are not expressly excluded from its benefits, and, though some of its provisions are by no means easy of interpretation, the object of the legislature having been to make the scope of the Act as wide as possible, these will probably be construed as far as reasonably possible in favour of the workman. The chief persons excluded from the benefits of the Act are: (1) persons paid over £250 a year ; (2) outworkers, *i. e.* persons who execute work in their own homes or in other premises in no way under the control or management of the person who has given out the work to be done ; (3) members of the employer's family dwelling in his house ; (4) casual labour employed otherwise than for the employer's trade or business—thus an extra hand taken on at

* *Priestly v. Fowler*, 3 M. and W. 1.

† *Employers' and Workmen's Act*, 1875.

works for an hour or so is clearly entitled to compensation if he meets with an accident during such employment. The employer is not, however, liable in respect of injuries which do not disable the employee for at least one week from earning full wages, or where the injury is attributable to the serious and wilful misconduct of the employee, an exception, however, being made in the latter case where death or permanent disablement results from the accident. Notice of the accident, giving the name and address of the employee and a full description of the accident and injuries, must be sent to the employer at once, and, except under special circumstances, no claim for compensation will be good if made more than six months after the injury, or death, as the case may be.

Where a **sub-contractor** is employed for the execution of any work by a principal, the workman can sue either the sub-contractor or the principal, the only exceptions being that where machinery is being employed for threshing or other agricultural work, and also in cases where the work is being done on the sub-contractor's own premises, then the sub-contractor alone can be sued.

In the event of the bankruptcy of the employer, the workman ranks as one of the preferential creditors under the Preferential Payments in Bankruptcy Act, 1888, to the extent of £100, and with the other creditors for any balance.

Numerous forms and regulations as to procedure, and also as to claims, awards, arbitration, medical examination of claimants, and various other matters connected therewith, are contained in the schedules to the Act and in the Workmen's Compensation Rules, 1907.

The scale of compensation is given in the first schedule to the Act. Contracting out of the Act is only permitted where the Registrar of **Friendly Societies** has certified that a scheme of compensation is proposed not less favourable to the workman than that provided by the Act.

COMPENSATION FOR BREACH OF CONTRACT.

In all contracts, in which the contractor agrees to execute certain works for the employer, the contractor will be liable to compensate his employer, in the event of the **works not being completed** in accordance with the terms of the contract.

In the **absence of any agreement** as to damages for breach of the contract, the damages will be such as may fairly and reasonably be considered as the probable result of the breach, either arising naturally from the breach itself, or such as may reasonably be supposed to have been in the contemplation of both parties at the time they made the contract.

The parties may, however, agree that, in the event of any particular term of the contract being broken, a **fixed sum** shall be payable as liquidated damages for the breach of that term, and in this case the fixed sum agreed on would be recoverable. It does not necessarily follow, however, that because a fixed amount is mentioned as payable on breach of the contract, that the fixed amount can always be recovered. The Court may decide that, according to the real intention of the parties, the sum named is not to be treated as **liquidated damages**, but is to be treated as a **penalty**, and in this case only the actual damage can be recovered. The use of the words "liquidated damages" will not be conclusive against the sum being held to be a penalty, nor will the use of the word penalty be conclusive against the sum being held to be liquidated damages: the Court will in all cases inquire what was the true **intention of the parties**, as gathered from the contract.

In Engineering contracts it is customary to fix a **date for the completion** of the work, and a clause is usually inserted imposing penalties for delay. The clause provides that a fixed sum shall be forfeited per week or otherwise after the contract time has expired, until the work is complete. This would be treated as liquidated damages and the sum agreed on would be recovered. On the other hand, a fixed sum may be mentioned as payable by way of compensation on the non-fulfilment of one or more or all of **several terms**, the breach of some of which may occasion serious, and others less serious, damage; in such a case the sum would be treated as a penalty. If it is desired to assess beforehand the damages to be paid for non-fulfilment of the contract so that a fixed sum shall be payable as liquidated damages, **separate sums** should be named as payable, on breach of separate terms; otherwise the agreed damages might be treated as a penalty and the fixed amount would not necessarily be recovered.

If the acts of the **employer cause**, or contribute to, the **delay** or breach of any other term of the contract, this fact will be taken into account in fixing the compensation to be paid, whether the damages were originally fixed as liquidated damages or were left to be assessed on the occurrence of any breach.

ELECTRIC LIGHTING.

By the **Electric Lighting Acts, 1882 and 1888**, the Board of Trade may grant to the local authority in any area, or any person, or company a **Provisional Order** or a licence to supply electricity within that area. The Provisional Order requires the consent of Parliament before it becomes effective; this is not necessary in the case of a licence, but no licence has been granted for a con-

siderable time and it may be assumed that the procedure will now in all cases be by Provisional Order.

The consent of the local authority is necessary where any person or company other than the local authority is the undertaker, unless the Board of Trade decide that such consent may be dispensed with.

The procedure on application for a Provisional order is governed by rules made by the Board of Trade in 1899 under the Act of 1882. It is provided by the Act that the undertakers may acquire lands and generally do such things as are necessary for the supply of electricity; they can break up streets for the purpose of laying wires and can alter the position of pipes, etc., which interfere with the necessary work. Overhead wires must not be erected without the express consent of the local authority, and streets cannot be broken up without the consent of the authority by whom they are repairable; compensation must be given for all damage, any dispute as to the amount of compensation is to be settled by arbitration.

Section 26 of the Act makes special provision for the protection of telegraph wires belonging to the Post Office.

The Provisional Order authorizes the supply of electricity for any public or private purposes. Public purposes include only the lighting of streets, churches, theatres, halls or buildings belonging to public authorities, and public places: private purposes any purpose whatever to which electricity may be applied except the transmission of telegrams.

The schedule to the **Electric Lighting (Clauses) Act, 1899**, contains elaborate regulations with regard to the execution of works for which a Provisional Order is granted, including the breaking up and subsequent repair of streets, provisions for the protection of telegraphs and telephones, conditions designed to secure a sufficient supply of energy to owners and occupiers within the area of the Provisional Order requiring it to be given them, the system and mode of supply, the appointment of electric inspectors and the duties of the latter in regard to the inspection and testing of the electric lines and works authorized and the supply of energy given, empowering them also to settle disputes between undertakers and consumers, and also providing for the certifying and examination of meters. The regulations in the schedule to the Act are to be deemed to be incorporated with every Provisional Order granted outside the County of London since October 1, 1899, except so far as any of them may be expressly varied by the Order. As regards Provisional Orders in London, however, it only applies to the extent that it is expressly incorporated in such Provisional Orders.

If the undertakers are using a system not approved or doing

work which appears not to be safe, or are interfering with telegraphs belonging to the Post Office, the Board of Trade may require discontinuance, or in default impose a penalty of not more than £20 a day until the defect is removed.

The **Board of Trade** also issue general **Regulations** (see also p. 489) to be followed by the undertakers for ensuring the safety of the public and the proper supply of electricity; those at present in force were first issued in November 1895, and have been slightly amended since; they are reimposed in every case. These Regulations * deal with the pressure allowed in the mains and at the consumers' terminals, the size of wires and insulation required, and also rules to be observed in laying overhead and underground lines; there are different directions as to high pressure lines, that is, lines on which the potential is 500—3000 volts continuous or 250—3000 volts alternating; lines on which the pressure is above 3000 volts are under special regulations.

The Regulations for ensuring a proper supply limit the variation of pressure allowed in the mains and at the consumers' terminals, and provide against unreasonable stoppage of the supply for repairs or any other cause.

Penalties may be imposed for the breach of these Regulations.

By section 4 of the Act of 1888 the Board of Trade are given power to make rules regulating electric lines not laid down under any Provisional Order, Licence, or Act of Parliament, and not altogether enclosed in buildings.

Pursuant to this section Regulations were issued in 1897 and are practically the same as those for insuring the safety of the public mentioned above, but a limit of pressure of 3000 volts is imposed, and the supply must not be more than 100,000 watts, or 50,000 watts if above-ground wires are used.

Nothing in a Provisional Order shall be deemed to exonerate the undertakers from any proceedings for nuisance. If the work of the station is so carried on as to cause a nuisance by smoke, vibration, or in any other manner, the undertakers may be restrained by injunction and are liable for damages.†

Electrical Stations, which include any premises in which electrical energy is generated for supply by way of trade, or for lighting purposes, are factories within the meaning of the Factory and Workshop Act, 1901, and the provisions of section 10 of that Act as to fencing machinery must be complied with.

* The "General Regulations" can be obtained from any corporation, or other undertaker, as these are compelled to print and sell the same (at sixpence or less). In addition to the observance of these Regulations, the "system of supply" has to be submitted to the Board of Trade for approval. The Board has thus a fairly free hand, and is not tied down by any hard and fast rules.

† *Shelfer v. City of London Electric Lighting Company*, L. R. 1895, 1 Ch. 287.

Notice of any accident disabling the employee so as to be absent at least one whole day must be given to the Inspector of Factories for the district, and where due to electricity, machinery, etc., to the certifying surgeon for the district as well (Notices of Accidents Act, 1906).

TRAMWAYS AND LIGHT RAILWAYS.

The **Tramways Act, 1870**, provides that the Board of Trade may make Provisional Orders authorizing the construction of tramways in any district, and make Rules as to any matter for the purpose of carrying the Act into effect.

The procedure for obtaining a provisional order is governed by **Rules under the Tramways Act, 1870**, issued by the Board of Trade (Railway Department).

The Provisional Order is not effective until the consent of Parliament has been obtained, and the Act confirming the Order usually provides that the tramway shall not be used except by the consent of and subject to the **Board of Trade Regulations**, which are designed to secure the public safety and also the proper working of the traffic, while provisions are usually inserted for the protection of the Post Office.

The Board of Trade have Model Forms of these Regulations and Bye-laws for adaptation to the needs of each Provisional Order with respect to—

- (1) The use of Steam or any mechanical Power on tramways.
- (2) The use of Electric Power, including the conduit system.
- (3) The Overhead Trolley System, which Form, with the exception of certain clauses, also applies to the surface contact system.
- (4) Cable Traction.

When the regulations for a particular system have been finally approved and sanctioned they are printed in the Statutory Rules and Orders for the current year with a distinctive number, and can then be obtained from Wyman & Sons, Ltd., Fetter Lane, E.C., or through any bookseller; the usual price is one penny.* The Board of Trade from time to time introduce minor improvements suggested by experience in the Model Forms—thus in the Regulations for the Overhead Trolley System, the minimum height of overhead conductors permitted from the surface of the street has recently been raised to 20 feet, and where unprotected telegraph or telephone wires cross above the conductors, efficient guard wires have to be erected.

The Regulations now in force for the use of electric power provide for the use of continuous currents, and give directions

* *c. g.* Statutory Rules and Orders, 1908. No. 46. Walsall Corporation Tramways.

as to the position in which the electric lines must be laid, and there are also elaborate requirements as to insulation.

The Regulations also provide that certain records must be kept and forwarded to the Board of Trade on demand. The records must give particulars of the maximum working current and potential used, and the results of tests made periodically as to leakage, current, insulation, condition of earth connections, and conductance of joints to pipes.

The Regulations for the working of the tramways also require that every motor carriage used shall have the wheels fitted with brake blocks, and in addition shall have an adequate electric brake and a slipper or other track brake of approved pattern. The speed allowed varies according to local circumstances. Where any accident by explosion or fire occurs, which has, or might have, caused loss of life, notice must be given to the Assistant Secretary, Railway Department, Board of Trade, S.W.

There being no special model form of light railway Order, the regulations as to the use of electrical power on **light railways** authorized by orders under the Light Railway Act, 1896, are the same as those in force with respect to tramways, and these are re-issued in each case under the Order authorizing the construction of the railway.*

With regard to **electric railways** (see also pp. 442 and 489), such, for example, as the Central London Railway, the Board of Trade impose regulations under the Act authorizing the railway in each particular case. The regulations now imposed are applicable only to railways constructed underground in metal-lined tunnels, and in which continuous currents are employed. These regulations are directed to the protection of pipes and wires, etc., belonging to parties other than the railway company, and deal chiefly with the insulation of the lines. The regulations also provide for the keeping of records as to the number of trains running and the maximum current and pressure employed, and as to the state of the insulation. These records must be produced to the Board of Trade when required.

The Board of Trade will consider applications as to the use of **alternating currents** on tramways and light railways, but no regulations as to this have as yet been issued.

LOCOMOTIVES ON HIGHWAYS.

By the **Locomotives Acts, 1861, 1865**, and the Highways and Locomotives (Amendment) Act, 1878, Part II., the speed of locomotives on highways was limited to 4 miles an hour and certain restrictions were placed on the construction of locomotives.

* *e. g.* Statutory Rules and Orders, 1907. No. 944. Coy. Hertford Lt. Ry.

These were as follows :—

A locomotive of weight less than 3 tons must have the tyres not less than 3 inches in width, with an additional inch for every ton or fraction of a ton above the first 3 tons.

A locomotive drawing a waggon must have the tyres 2 inches in width for every ton weight of the locomotive ; or if the diameter of the wheel is greater than 5 feet, the width of tyre may be reduced in proportion as the diameter is increased, but in such case must not be less than 14 inches.

A locomotive must not exceed 9 feet in width or 14 tons in weight. The driving wheels must be cylindrical and smooth soled or shod with diagonal crossbars not more than $\frac{3}{4}$ inch thick and not less than 3 inches wide, and the space between the crossbars must not be greater than 3 inches.

Breaches of these regulations are punishable by a fine not exceeding £5.

The **Locomotives on Highways Act, 1896**, exempts **light locomotives** from the above restrictions.

A light locomotive is defined as a vehicle propelled by mechanical power and weighing not more than three tons unloaded, or a locomotive drawing a single vehicle of total weight not more than four tons unloaded.

A limit of speed of fourteen miles per hour was imposed by this Act.

Under the **Motor Car Act, 1903**, the speed limit is raised to twenty miles per hour for motor cars, the definition of motor cars being the same as that of light locomotives in the Act of 1896.

The Local Government Board are empowered to make regulations under the Locomotives on Highways Act, 1896, and also under the Motor Car Act, 1903. Those now in force are found in the Motor Cars (Use and Construction) Order, 1904. This Order rescinds the regulations made under the Act of 1896, and imposes conditions as to construction as follows :—

- (1) The motor car, if it exceeds in weight unladen five hundredweight, shall be capable of being so worked that it may travel either forwards or backwards.
- (2) The motor car shall not exceed 7 feet 2 inches in width, such width to be measured between its extreme projecting points.
- (3) Except where pneumatic or elastic tyres are used, the tyre of each wheel of the motor car should be smooth and shall, where the same touches the ground, be flat and of the following width, viz. :—
 - (a) If the weight of the motor unladen exceeds fifteen hundredweight, but does not exceed one ton, not less than $2\frac{1}{2}$ inches ;

- (b) If such weight exceeds one ton, but does not exceed two tons, not less than 3 inches ;
- (c) If such weight exceeds two tons, but does not exceed three tons, not less than 4 inches.
- (4) The motor car shall have two independent brakes in good working order and of such efficiency that the application of either to the motor car shall cause two of its wheels on the same axle to be so held that the wheels shall be effectually prevented from revolving, or shall have the same effect as if such wheels were so held.

In cases where the car has less than four wheels, the application of each brake must cause one wheel to be so held.

The motor car and all fittings thereof must be in such condition as not to cause, or to be likely to cause, danger to any person on the motor car or on the highway, and the motor car must be so constructed as to enable the driver, when the motor car is stationary otherwise than through an enforced stoppage owing to necessities of traffic, to stop the action of any machinery attached to or forming part of the motor car so far as may be necessary for the prevention of noise.

The same regulations as to construction apply to vehicles drawn by motor cars, with the exception of those as to brakes. In the case of drawn vehicles weighing less than two hundredweight, no brake is necessary, but where the weight exceeds two hundredweight the drawn vehicle must be provided with a brake capable of holding two wheels as in the case of the self-driven motor car, and must carry a person competent to apply it unless it can be applied from the motor car.

In calculating the weight of a vehicle unloaded, the weight of any water, fuel, or accumulators used for the purpose of propulsion need not be included.

Under section 12 of the **Motor Car Act, 1903**, the Local Government Board have power as respects any class of vehicle to increase the maximum weights of three tons and four tons mentioned in the Locomotives on Highways Act, 1896, subject to any conditions which may be imposed as to the use and construction of the vehicle. This section therefore gives the Local Government Board power to increase the weight allowed for light locomotives and motor cars.

The Motor Car (Registration and Licensing) Order, 1903, deals with the issue of licences, identification marks, etc.

The Home Secretary issued an important circular in 1903, drawing attention to the necessity of keeping petrol in sound and properly closed vessels, and to the risk of the highly inflammable vapour given off by it becoming ignited, and to the latter's explosive qualities when mixed with air, and

at the same time regulations were issued as to the storing of it.

The Heavy Motor Car Order, 1904, provides special regulations for commercial and similar "heavy motor cars," defined in the Order to mean motor cars exceeding 2 tons in weight unladen; these are permitted to be used on the highway, if registered as such, and if not exceeding 5 tons in weight unladen, or together with a trailer unladen, $6\frac{1}{2}$ tons.

Upon registration the owner must declare (1) the weight of the car unladen, (2) the axle weight of each axle, and (3) the diameter of each wheel, and the owner must bring the car to the registering authority, which is either the county council or the council of the county borough, as the case may be, for their officer to inspect the car and certify the correctness of the declaration. These particulars, together with the width and material of the tyre of each wheel, and the highest rate of speed at which in conformity with the order, the motor may be driven without a trailer are entered in the register of motor cars. The registered weight and axle weight of the car must be painted on the off side, and the rate of speed permitted on the near side of the motor, in letters not less than one inch in height.

The speed of a heavy motor car with pneumatic or elastic tyres is restricted to 12 miles an hour, unless the registered axle weight exceeds 6 tons, when only 8 miles an hour is allowed. Where the tyres are not of such a description the speed is restricted to 8 miles, unless the weight of the motor unladen exceeds 3 tons, or unless it is drawing a trailer, in which case the maximum speed permitted is 5 miles an hour.

If the heavy motor is of 3 tons weight unladen, both it and its trailer are permitted to be of a width of 7 ft. 6 in., and both a motor and its trailer must be constructed with springs between each axle and the frame. There are also somewhat similar regulations in the Order dealing with trailers, the axle weight of each axle of which must not exceed 4 tons. The weight of the trailer and, where the axle weight exceeds 1 ton, the axle weight must be painted on the trailer. The minimum width of the tyre of a heavy motor is 5 inches, and of a trailer 3 inches. A heavy motor car used for carrying passengers for hire must not draw a trailer. There are also rules in the Order as to registration, mode of ascertaining weights and axle weights, and restricting the right to use bridges where the bridge authorities exhibit notices stating they are insufficient to bear specified weights.

PATENTS, DESIGNS AND TRADE MARKS.

By W. GRYLLS ADAMS.

A. PATENTS. (UNITED KINGDOM).

THE law as to Letters Patent for inventions in the United Kingdom and the Isle of Man is contained in the Patent and Designs Act, 1907, and in the Patent and Designs Rules, 1908. Patents are granted for a term of 14 years from the date of the application, but in special cases may be extended for a further term of seven years, and in exceptional cases 14 years.

Any person whatever may apply for a patent, and a patent may be granted to two or more persons jointly, provided one of them is the first inventor. The word "Inventor" includes a person introducing an invention from abroad, and such a person will be able to obtain a good patent if the invention has not been previously published in this country.

Any new manufacture is a patentable invention. For instance, any new article or new process of manufacturing new or old articles, or any chemical process which is new at the date of application. An idea or principle cannot be patented, but where the inventor has discovered a new principle he may patent some practical way of making use of the principle, and if his patent is properly drawn he will not be restricted to only the particular method which he has described, and the patent will enable him to stop infringers who are using the principle in a manner analogous to that described in his specification, although there may be considerable differences in the details of the method and apparatus which they employ. The number of patents granted for inventions involving entirely new principles is small, and most patents are granted for improvements in old articles or processes of manufacture. In all cases, in order to obtain a valid patent there must be some exercise of ingenuity. The application of well-known apparatus or principles to purposes analogous to those for which they have been previously used is not subject matter for a patent, and slight alteration, such as the substitution of mechanical equivalents, cannot form the subject of a valid patent unless some advantage which it requires ingenuity to foresee results from the alteration.

The applicant for a patent must leave at the Patent Office a declaration that he is the inventor, or that he claims to be the inventor within the United Kingdom.

Several forms are given in the Patent Rules made under the Act, and the particular forms suitable for the various cases must be selected. The ordinary form for inventors in this

country is the Form No. 1. Forms 1*a*, 1*b*, 1*c* and 1*d* are the forms applicable to patents imported, patents of addition and secret patents.

With the application form the applicant for a patent must leave either a provisional or a complete specification. This, in the case of a provisional, must be in Form No. 2, and in the case of a complete in Form No. 3. If the patentee has previously obtained or applied for a patent in a foreign country which is a party to the International Convention, he can obtain in this country the priority of the first such foreign application by applying under the International Convention on Form 1*b*, and his British patent will then date back to the first foreign application, if that is within 12 months before the application in this country.

The fees are £1, payable on application, if a provisional specification only is filed, and £4 if a complete specification is filed in the first instance. Where a provisional is filed a further fee of £3 will be required on filing the complete specification. A further fee of £1 is payable on sealing the patent on completion of the grant. The renewal fees now in force are £5 before the end of the fourth year from the date of application, £6 before the end of the fifth year, £7 before the end of the sixth year, etc., to £14 before the end of the thirteenth year in respect of the fourteenth year of the term. If the renewal fees are not paid the patent will lapse, but under Section 20 of the Act of 1907 may be restored on payment of a fee of £20, if the omission to pay the fee at the proper time was unintentional. The application to restore the patent may, however, be opposed.

A specification filed on the application for a patent commences with the title, which must sufficiently indicate the subject matter of the invention, for example, "Improvements in Dynamo Machines," and the specification must not contain anything which does not come within the title. The specification should only include one invention, although several claims may be made in a single specification. The provisional specification should describe the nature of the invention, but need not go into any detail as to the method of carrying the invention into effect; it should, however, mention any new points which the applicant intends to claim in his complete specification, and which he thinks to be new. If the invention is a great novelty, as in the case where the patentee has discovered a new principle, and devised one practical method of carrying it into effect, a very general description is all that is necessary. If, however, he has only invented a modification of a known process or machine, it will be necessary in the provisional specification to describe exactly the difference

between the new process or machine and those previously known. The object of the provisional specification is to allow the inventor to obtain protection from accidental publication of his invention during the time necessary for working it out to such a state of perfection that he can describe fully some method of carrying it out in a manner that the ordinary workman conversant with the art to which the invention relates will understand.

Unless an applicant has filed a complete specification in the first instance, within six months of the filing of the provisional specification he must file a complete specification, in which he must completely describe and ascertain the nature of the invention and the manner in which it is to be performed. The complete specification must also contain a short statement of what the inventor considers to be new. This statement takes the form of claims at the end of the specification, and it is on these claims that the amount of protection to which the patentee is entitled depends. If necessary to fully show the method of carrying out the invention, drawings must be filed, which must be in the form prescribed by the Patent Rules, 1908. In the case of chemical inventions, the Patent Office may require that samples or specimens of the new substance produced by the process shall be left at the Patent Office with the complete specification before the latter is accepted. The description required in a complete specification should be sufficient to enable persons of ordinary intelligence, and acquainted with the common knowledge of the trade to which the invention relates, to produce the result claimed by the patentee without exercising invention on their own account.

The claims should be very carefully drawn, and care should be taken that they are sufficiently broad to include everything which is new in the specification. Amendment can be made after the specification is filed, if it is found that the original claims include matters which are found to be old.

After the application and specification are filed, the Comptroller General refers the application papers to an Examiner; in the case of a provisional the Examiner merely sees that the application and specification are in correct form, and that the nature of the invention is fairly described. In the case of a complete specification, the examination is continued further than the mere formal examination in the case of a provisional, and a search is made by the Patent Office as to the novelty of the invention claimed. The search is continued back for 50 years before the date of the application, but is confined to specifications filed in this country. If the applicant does not leave a complete specification with the application the search

is not made until the complete specification is left. If the patentee finds it impossible to complete the application within six months, he may, on payment of a fee of £2, claim an extension of one month within which he can file his complete specification.

The Examiner, on completion of the search, or if he finds a prior specification which he thinks completely anticipates the subject matter of the invention claimed, reports to the Comptroller. This report is not published, but the result of the search and any formal objection which requires amendment are communicated to the applicant, with a statement that in the opinion of the Examiner the invention claimed is wholly or in part described in certain prior specifications. The applicant can then amend the specification so as to be clear of what has been disclosed by the search, and if the Comptroller is satisfied with the amendment, the specification is accepted, and will be sealed in due course.

If the Comptroller is still not satisfied that the invention claimed is not wholly or in part described, he may require further amendments, and if the applicant does not meet the requirements the Comptroller may appoint a verbal hearing and determine what references to prior specifications shall be inserted in the specification, or if he is of opinion that the invention claimed is wholly described in a prior specification, he may refuse to grant a patent altogether.

Among the matters reported on by the Examiner are the questions as to whether the invention described in the complete specification is the same as that described in the provisional. If the Examiner reports that part of the invention claimed in the complete is not described in the provisional, the Comptroller may allow the original application to proceed so far as the invention included both in the original and complete specifications is concerned, and may allow the part of the invention included in the complete specification and not included in the provisional to be claimed as if in a separate application for that part made on the date on which the complete specification was left. If the Comptroller refuses to accept the specification, there is an appeal to the Law Officer, whose decision is final.

Under Section 8 of the Act of 1907 the investigation may be extended to specifications not published at the date of the application, but which will when published bear date prior to that of the application.

After the complete specification is accepted, the Comptroller advertises the acceptance, and the application and specifications, together with the drawings, are open to public inspection. Under Section 11 of the Act of 1907, any person may within

two months of the date of advertisement of acceptance of a complete specification give notice of opposition on any of the following grounds:—

- (a) That the applicant obtained the invention from him.
- (b) That the invention has been claimed in any complete specification for a British patent of prior date to the patent which is opposed, not more than 50 years before the date of application.
- (c) That the nature of the invention, or the manner in which it is to be performed, is not sufficiently or fairly described and ascertained in the complete specification.
- (d) That the complete specification describes or claims an invention other than that described in the provisional, and that such other invention forms the subject of an application made by the opponent in the interval between the leaving of the provisional and the leaving of the complete specification.

Notice of the opposition is given to the applicant, and after two months the Comptroller decides the case after hearing the applicant and the opponent. An appeal lies from the decision of the Comptroller to the Law Officer. If the Comptroller or Law Officer decides that the patent may be granted, he makes an order to that effect, and the patent is then sealed. If there is no opposition the patent must usually be sealed within 15 months from the date of the application. If there is an opposition, the time for sealing may be extended, or, if there has been an extension of the usual 12 months limited for acceptance of the complete specification, a further four months' delay of sealing may be allowed. The sealing completes the grant of the patent and dates back to the date of application, but no damages will be given in an action for infringement in respect of anything done before the date of acceptance of the complete specification, that is, before the date on which the specification was published.

Under Section 26 of the Act of 1907, any person who would have been entitled to oppose the grant or succeeds to the interest of a person who was so entitled, may within two years from the date of the patent apply to the Comptroller for revocation on any of the grounds on which the grant of the patent might have been opposed, but when there is an action for infringement or any proceedings for revocation are pending, this provision is only applicable by leave of the Court. Revocation under this Section is made after hearing the parties, and the Comptroller may give the patentee an opportunity of amending the patent to meet the objections.

Section 15 of the Act of 1907 provides that a patent shall not be invalidated by an application in fraud of the true and first inventor, and also provides that where a patent has been

revoked on the ground of fraud the Comptroller may, on the application of the true inventor, grant him a patent in lieu of and bearing the same date as the patent so revoked.

Section 16 provides that where two or more provisional specifications have been filed by an inventor for inventions which are cognate or modifications one of the other, and the Comptroller considers that the whole of them may fairly be considered one invention, the applicant may file a single complete specification in respect of the whole of these and the Comptroller may grant a single patent thereon. The patent granted under this Section bears the date of the earliest of the provisional applications, but in considering validity, the various claims must be considered with the various provisional specifications to which they respectively relate. That is to say, each claim must be treated as to novelty as if the invention which it claims had been applied for on the date of the corresponding provisional.

Section 19 of the Act provides for the grant of patents of addition in respect of improvements or modifications of an invention claimed in a prior patent, and in the case of patents applied for under this Section no renewal fees are payable; the patent, however, determines at the same time as the patent to which it is additional.

Extension of the term of a patent may be obtained under Section 18 of the Act of 1907 by petition to the Supreme Court. The petition must be presented at least six months before the expiration of the ordinary term of the patent, and any person may give notice of objection to the extension and be made a party to the proceedings. The Comptroller is entitled to appear and may be directed to appear by the Court. In considering whether extensions shall be granted, the Court will consider the nature and merit of the invention in relation to the public, and will also consider the profits made by the patentee from the patent. No extension will be granted if the patentee has made any considerable profits, nor will it be granted unless the invention is of exceptional merit, the test of merit for this purpose being the benefit which accrues to the public from the use of the invention.

Prior to 1908 no working of the invention in this country was compulsory. Section 27 of the Act of 1907, however, provides that at any time not less than four years after the date of a patent, and not less than one year after the passing of the Act, that is, after August 1908, any person may appeal to the Comptroller for revocation of a patent on the ground that the patented article or process is manufactured or carried on exclusively or mainly outside the United Kingdom. If this is proved to be the case, the Comptroller may revoke the patent,

unless the patentee gives satisfactory reasons why the article or process is not manufactured or carried on in the United Kingdom. It is provided that this order may, as the Comptroller may direct, either come into force immediately, or at such time as the Comptroller may allow, to give the patentee the chance of manufacturing in the United Kingdom. Any decision of the Comptroller is subject to appeal to the High Court. It is expressly provided in the Section that no order shall be made which is at variance with any treaty, convention, or arrangement with any foreign country, or British possession, so that this Section may be used in order to secure mutual benefits as regards working arrangements between this country and others in which working is compulsory.

The patentee may at any time obtain leave to amend his specification or drawings by way of disclaimer, correction or explanation, so as to cure any defect arising from faulty description or from claiming more than he is entitled to in view of the state of the art at the date of his application. If the claim includes anything which is published not only in prior specifications but in any publication in this country before the date of the provisional specification, the patent as a whole is invalid and, until the defect is cured, is useless for the purpose of stopping infringers.

If it comes to the knowledge of the patentee that other persons are using his invention without his authority his proper course is to first warn them of the existence of his patent, and if they do not come to terms begin an action for infringement, in which he can claim damages and an injunction to restrain them from infringing. Under the Act of 1907 it is provided that the patentee shall not obtain damages for infringement of a patent granted after the first of January 1908 from any defendant who proves that at the date of the infringement he was not aware and had no reasonable means of making himself aware of the existence of the patent. This Section of the Act, however, does not affect the granting of injunctions, so that a patentee may well succeed in the action and obtain the injunction even if the defendant proves that he was ignorant of the existence of the patent. It is provided under the same Section that the marking of an article with the word "Patent" or "Patented" does not constitute notice to an infringer of the existence of a patent, unless these words are accompanied by the number and date of the patent or patents under which the article is made. It is therefore advisable for any one manufacturing under a patent to mark all articles not only with the words "Patent," or "Patented," but with the number and date of the patent. If the defendant in the infringement action is of opinion that the patent is invalid, the invalidity, if it can be

proved, is a complete answer to the action, and the defendant may if he desires make a counter-claim asking that the patent be revoked. This considerably simplifies the procedure of getting rid of invalid patents. If it appears in an infringement action that the patent is invalid for the reason that something claimed in it is old, the patentee may, by leave of the Court, amend the specification, and it appears that the action might then go on as regards the claim for injunction. No damages, however, will be given unless the patentee shows that his original claim was made in good faith and with reasonable skill and knowledge. The Court has also power to impose terms in allowing the amendment, and it is probable that one of the terms in a case of this kind would be that no damages should be given in respect of any act of the defendant before the date on which the amendment of the specification was allowed. Under no circumstances is an amendment allowed which increases the scope of the patent.

Apart from want of novelty other grounds on which a patent may be held invalid are, that some other person than the patentee was the true inventor, that the invention is not useful, or that the specification is insufficient, or describes processes or apparatus which will not produce the result claimed.

A patentee may, if he desires, grant licences to others to use his invention, he may assign his patent either in whole or in part, and generally deal with it as personal property. If a patent is granted to two or more persons jointly, for the purpose of devolution of the legal interest, they are treated as joint tenants, but each can use the invention for his own profit without accounting to the others. The co-patentees cannot, however, grant licences without mutual consent.

Section 38 of the Act of 1907 introduces certain new provisions as regards the legality of contracts in licensing clauses, making certain conditions void as being in restraint of trade.

Protection is given to an inventor whose invention has been published without his consent by Section 41 of the Act, which provides that if the patentee proves that the publication was made without his knowledge and consent, and that the matter published was obtained from him, and that he applied for and obtained protection with reasonable diligence after learning of the publication, such publication shall not invalidate the patent.

Section 42 of the Act abolishes the old plea of disconformity, and provides that differences between the provisional and complete specifications shall not invalidate the patent, if the invention claimed in the complete specification was not patented in a patent of date prior to the filing of the complete specification.

Section 45 deals with the exhibition of inventions at Industrial or International Exhibitions, and provides that on application giving notice to the Comptroller such publication by exhibition shall not invalidate a patent applied for within six months of the date of the opening of the Exhibition.

The public are to a certain extent protected from persons who make their patents a means of unfair dealing. Section 36 of the Act of 1883 provides a remedy against patentees who threaten with legal proceedings rival traders who are not infringing any valid patent. Any person aggrieved by such threats may bring an action to restrain the patentee from issuing threats, and may obtain compensation for the damage done thereby.

Under the Act of 1907 any person interested in the manufacture to which the patent relates, may present a petition to the Board of Trade alleging that the requirements of the public with respect to the patented invention have not been satisfied, and may obtain the grant of a compulsory licence if he proves that, by reason of the default of the patentee to manufacture to an adequate extent and supply the patented article on reasonable terms, any trade, or the establishment of any new trade, or any industry in the United Kingdom is unfairly prejudiced.

The Board of Trade may refer such a petition to the Court, and the Court will decide whether the reasonable requirements of the public have been satisfied; and if such requirements have not been satisfied, may grant a compulsory licence on such terms as it may consider reasonable, or if the grant of a compulsory licence does not meet the circumstances of the case, may revoke the patent. Revocation of the patent may also be obtained on petition to the Court on any ground on which the patent might be held invalid in an action of infringement. The petition for revocation may be presented by any person who alleges—

- (1) That the patent was obtained in fraud of his rights, or the rights of any person under or through whom he claims.
- (2) That he or any other person under or through whom he claims, was the true inventor of any invention included in the claim of the patentee.
- (3) That he or any person under or through whom he claims an interest in any trade, business or manufacture used or sold within this realm before the date of the patent, anything claimed by the patentee as his invention.

The petition for revocation may also be presented by the Attorney-General, or any person authorized by him.

Under the Patent Rules of 1908 the authorization of the

Attorney-General may be obtained by a petition alleging the grounds on which the person applying seeks to revoke the patent, and the slightest interest is sufficient to obtain the authority.

The patentee may also offer to surrender his patent and so obtain revocation under Section 26 and so avoid the expense which may arise from the possession of an invalid patent.

Under Section 89 various acts are made offences under the Act ; the chief of these is the selling of articles having stamped or impressed thereon, or in any way applied thereto the words "Patent," "Patented," "Registered" or any other word implying that the article is subject to patent or registered design when such is not the case.

B. PATENTS (FOREIGN).

There are a few important points which should be considered by the inventor before applying for a foreign patent. In certain countries it is necessary to work the invention, but on the whole the foreign working required is not so stringent as that introduced into this country by the British Act of 1907. The obtaining of a patent, however, in certain foreign states is considerably more difficult than it is in England. For instance, in Germany and in the United States, a thorough search through all literature dealing with the subject matter of the application is made and no patent will be granted unless the applicant satisfies the Officials at the Patent Offices that the invention claimed is really new and such as is allowable according to the Patent Laws of the respective countries.

Certain countries also are not included in the International arrangements, and in these it is necessary to apply for a patent before the publication in any foreign state takes place. As a general rule, the British applicant will be fairly safe if he applies for foreign patents before acceptance of the British complete specification. In the United States, however, he should apply within twelve months of his first British or foreign patent, as the United States require that the patent shall be applied for before the expiration of twelve months from the first foreign patent or else that the United States patent shall be the first to issue.

C. DESIGNS.

Designs are dealt with by Sections 49 to 61 of the Patents and Designs Act, 1907, and the Design Rules made under the Act. Any person claiming to be the proprietor of any new or original design not previously published in the United Kingdom

may apply for registration. For the purpose of designs, goods are classified into various classes, and the design should be registered in one or more of these classes. Registration of the design gives the registered proprietor copyright in the design for five years from the date of registration, but the period of copyright may be extended for a further period of five years on payment of the prescribed fee. A third period of five years may also be obtained on payment of a fee. The protection given by the possession of a registered design is a very narrow one, and only covers exact copies of the actual design which is on the Register. The question as to whether a design is an infringement of the copyright registered is a matter which is decided by the eye alone; it is entirely a matter of the appearance of the article to which the design is applied. There is no question as to utility, and a design is of an entirely different nature and gives an entirely different protection from that given by a patent.

In order to preserve the copyright of any design there are certain provisions which the registered proprietor must carefully consider.

Firstly, before delivery or sale of any article to which the design is applied, the proprietor must furnish to the Comptroller the required number of representations or specimens of the design as specified in the Design Rules made under the Act. Before selling articles to which the design is applied, he must also cause each article to be marked with words or figures denoting that the design is registered. If he fails to do this he will not be able to recover any penalty or damages for infringement of his copyright, unless he shows that he took all proper steps to ensure the marking of the article, or unless he shows that the person infringing had knowledge of the existence of the copyright at the date of the infringement.

As regards designs as well as patents, it is provided that accidental disclosure shall not invalidate a subsequent registration, so that the disclosure of a design in breach of good faith by a person other than the proprietor shall not interfere with the rights which can be obtained.

A registered design is not open to public inspection until the period of copyright expires. After this period copies of the design may be taken by any person on payment of the fee prescribed by the Design Rules.

Publication of the design in an Industrial or International Exhibition does not destroy the copyright.

D. TRADE MARKS.

The Law as to Trade Marks is contained in the Trade Marks Act, 1905, and the Trade Mark Rules, 1906, made under that Act.

Any person being the proprietor can apply for registration of a trade mark. The trade mark must be registered for particular goods or classes of goods. The classification will be found in Schedule 3 of the Trade Mark Rules, 1906. A trade mark to be registered must contain one of the following particulars :

- (1) The name of the Company, individual or firm represented in a special or particular manner.
- (2) The signature of the applicant for registration or some predecessor in his business.
- (3) An invented word or invented words.
- (4) A word or words having no direct reference to the character or quality of the goods and not being according to the ordinary signification a geographical name or surname.
- (5) Any other distinctive mark ; but a mark, signature, or word or words other than such as fall within the description in the above paragraphs 1, 2, 3, and 4 shall not, except by order of the Board of Trade or the Court, be deemed a distinctive mark.

There are special provisions for the registration of trade marks which were in use before 1875.

Application for registration of a trade mark is made by writing to the Registrar of Trade Marks at the Patent Office.

The Registrar advertises the application and any person can give notice of opposition within one month or within such further time as the Registrar may allow, not exceeding three months from the date of advertisement. The fees are 5s. on application for registration covering one or more articles in one class, and £1 for registration. The fee for continuance of the registration after a period of 14 years is £1.

A trade mark after registration can be assigned and transmitted only in connection with the goodwill of the business concerned in the goods for which it has been registered, and is determinable with that goodwill. The proprietor of a registered trade mark may assign the right to use the trade mark in any British possession or foreign country in connection with the goods for which it is registered, provided he assigns therewith the goodwill of the business in that country in such goods.

Trade marks may be registered in any colour or colours, and may contain in addition to the distinctive feature required by Section 9, matter common to the trade and of a non-distinctive character, but in case of such marks the Registrar may require the proprietor to disclaim any right to the exclusive use of the parts which are common to the trade.

In addition to the Law as to Registered Trade Marks contained in the Act of 1905, there are a considerable number of decisions based on the Common Law, and according to these

decisions damages may be obtained by any person who is aggrieved by a rival trader attempting to pass off goods as and for the goods of the person aggrieved. It is provided in the Act of 1905 that any such Common Law right shall not be affected by anything in the Act, so that quite apart from registration, a protection very similar to that given by the trade marks may be obtained by traders whose goods become known by means of a particular form of wrapper, or other distinctive device. This Common Law protection is frequently of more importance than that obtained by the use of trade marks registered under the Act.

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